AN OPTIMUM SPACE-TIME MTI PROCESSOR FOR AIRBORNE RADAR

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ABSTRACT

This paper presents an optimum space-time moving target indication (MTI) processor for the airborne radar. The optimization is based on a stochastic target model, rather than deterministic target models adopted in most space-time MTI processor designs. The optimum solution that maximizes the improvement factor yielded by the processor is shown to be the generalized eigenvector corresponding to the smallest generalized eigenvalue of the signal and clutter covariance matrices. A suboptimal, but computationally simpler solution to this problem is also derived. This approach requires the solution of a linearly constrained minimum variance (LCMV) problem. Unlike typical LCMV problems, our solution also calculates the response vector specifying the frequency response along the look direction. Experimental results demonstrating the usefulness of our methods are included in the paper. The results indicate that the suboptimal solution does not suffer from significant performance loss.

1. INTRODUCTION

Detection of slowly moving targets by an airborne radar is often limited by echoes from the stationary ground. Performance degradation due to effects such as terrain masking can be significant in airborne radar because of the platform motion [1],[2]. The echoes received by a moving platform along all possible directions from the ground are spread over a Doppler band. A two-dimensional MTI processor, usually referred to as a space-time processor, is often employed to cope with the motion-induced clutter bandwidth. By utilizing both spatial and temporal information, the radar system can detect targets by distinguishing it from the clutter signal in the azimuth-Doppler spectral domain. Space-time processors based on an antenna array have been the subject of considerable interest for a long time [3],[4]. The problem of finding the optimum weight vector of the space-time MTI processor for the clutter cancellation provides different solutions depending on whether a deterministic target or stochastic target model is employed. The most common optimization technique for the space-time MTI processing is concerned with the detection of a deterministic signal. The amplitude and Doppler frequency is known *a priori* in problem involving the detection of deterministic signals. In the case of stochastic target models, only the covariance matrix of the target signal is known. MTI techniques involving prefiltering and/or Doppler analysis are known to be efficient in enhancing targets from strong background clutters [4]. The optimization procedure for one-dimensional (temporal only) MTI processors under the stochastic target assumption is well-known [5]. However, little work has been done for finding the optimal solution for the space-time MTI processor. The main objective of this paper is to present an optimization procedure for a space-time MTI processor for detecting targets modeled as random variables having uniform distribution over the frequency of the radar. We will show in this paper that the optimum space-time MTI processor that maximizes the improvement factor is characterized by the generalized eigenvalue decomposition (GEVD) of the signal and clutter covariance matrices. This solution is computationally complex, and therefore we also introduce a suboptimal solution to the problem and study its properties. The suboptimal processor is obtained as the solution of a linearly constrained minimum variance (LCMV) optimization problem. However, unlike ordinary LCMV problems [6], our solution also calculates the response vector specifying the frequency response along the look direction. Experimental results as well as theoretical calculation of the detection probabilities of the two processors indicate that the performance loss in the computationally simpler solution is not significant in most situations.

This work was supported by the Microwave Applied Research Center.

2. OPTIMIZATION OF SPACE-TIME MTI PROCESSOR

2.1. Problem Formulation

Consider a linear array that consists of N uniformly spaced antenna elements and M taps on each element. Let $x_i(k-l)$ denote the received signal at the ith element and lth tap at time k. Define the MN-dimensional stacked snapshot vector $\mathbf{x}(k)$ and the weight vector $\mathbf{w}$ in the space-time MTI processor as $\mathbf{x}(k) = [x_0(k), x_1(k) \ \cdots$ $x_{N-1}(k-M+1)]^T$ and $\mathbf{w} = [w_{0,0} \ \cdots \ w_{N-1,M-1}]^T$. Then, the output at time k is given by $y(k) = \mathbf{w}^H\mathbf{x}(k)$, where the superscript H indicates the complex conjugate transpose operator. The design of the space-time processor involves the selection of the coefficient vector such that the improvement factor of the MTI processor is maximized. The improvement factor is defined as the signal-to-clutter ratio at the output of the system divided by the signal-to-clutter ratio at its input, averaged uniformly over all target velocities of interest [5]. The average improvement factor of the MTI processor can be shown to be [7]

$$IF = \frac{\mathbf{w}^H\mathbf{M}_s\mathbf{w}}{\mathbf{w}^H\mathbf{M}_x\mathbf{w}} , \quad (1)$$

where $\mathbf{M}_x$ denotes the covariance matrix of $\mathbf{x}(k)$ under the hypothesis that the output of the antenna elements contains clutter plus noise only, and $\mathbf{M}_s = E\{\mathbf{s}\mathbf{s}^H\}$ represents the covariance matrix of the normalized target signal steering vector $\mathbf{s}$. The received signal from the target may differ from $\mathbf{s}$ by a complex attenuation factor. The maximization of the improvement factor is equivalent to the following constrained optimization problem [5] :

$$\underset{\mathbf{w}}{minimize} \quad \mathbf{w}^H\mathbf{M}_x\mathbf{w}, \quad subject\ to\ \mathbf{w}^H\mathbf{M}_s\mathbf{w} = c \ , \quad (2)$$

where c is a real and positive constant.

Suppose that the target signal is incident from azimuth angle ϕ_t that is measured with respect to the normal direction of the array and elevation angle θ_t. We take the first element of the array as the reference point and assume that the propagating wave disturbance is a narrow-band signal. Let us define a spatial steering vector $\mathbf{a}(\vartheta)$ as $\mathbf{a}(\vartheta) = [1\ e^{-j2\pi\vartheta}\ \cdots\ e^{-j2\pi(N-1)\vartheta}]^T$ and a temporal steering vector $\mathbf{b}(\varpi_t)$ as $\mathbf{b}(\varpi) = [1\ e^{-j2\pi\varpi}\ \cdots\ e^{-j2\pi(M-1)\varpi}]^T$, where $\varpi_t = \frac{2v_t}{\lambda_0 PRF}$ is the normalized Doppler frequency of the target and $\vartheta_t = \frac{d}{\lambda_0} sin\phi_t cos\theta_t$ is the spatial frequency of the target. In this definition, v_t and λ_0 denote the relative target speed and the wavelength of the carrier frequency, respectively. The space-time steering vector $\mathbf{s}$ can then be expressed as

$$\mathbf{s} = \mathbf{b}(\varpi_t) \otimes \mathbf{a}(\vartheta_t), \tag{3}$$

where $\otimes$ denotes the Kronecker matrix product operation.

Using the above definition for $\mathbf{s}$, the covariance matrix $\mathbf{M}_s$ can be expressed as

$$\mathbf{M}_s \ = \ E\left[\{\mathbf{b}(\varpi_t)\mathbf{b}(\varpi_t)^H\} \otimes \{\mathbf{a}(\vartheta_t)\mathbf{a}(\vartheta_t)^H\}\right] \ . \tag{4}$$

We assume that the normalized target Doppler frequency is a random variable distributed uniformly over [0,1]. Then, we can show that $E[\mathbf{b}(\varpi_t)\mathbf{b}(\varpi_t)^H] = I_M$, where I_M is the $M \times M$-dimensional identity matrix. Therefore, the signal covariance matrix $\mathbf{M}_s$ can be expressed as

$$\mathbf{M}_s = \mathbf{C}\mathbf{C}^H \ , \tag{5}$$

where $\mathbf{C} = \mathbf{I}_M \otimes \mathbf{a}(\vartheta_t)$.

Let us now define a transformed coefficient vector $\mathbf{h}$ as $\mathbf{h} = \mathbf{C}^H\mathbf{w}$. The elements of the vector $\mathbf{h}$ can be thought of as the coefficients of the antenna signals after they have been aligned in the direction of interest. With the help of the above definition of $\mathbf{h}$, we can reformulate the optimization problem in (2) as

$$\underset{\mathbf{w}}{minimize} \quad \mathbf{w}^H\mathbf{M}_x\mathbf{w}, \ \ subject\, to\ \mathbf{C}^H\mathbf{w} = \mathbf{h}\, and\, \mathbf{h}^H\mathbf{h} = c\ . \tag{6}$$

If the vector $\mathbf{h}$ is given *a priori*, the above problem corresponds to a linearly-constrained minimum variance (LCMV) problem [6]. The vector $\mathbf{h}$ specifies M scalar constraints on the response of the array in the direction of interest. However, an important point to note is that, unlike the ordinary LCMV problem, the response vector $\mathbf{h}$ is not known *a priori* and must be designed to satisfy the quadratic constraint in (6).

2.2. The Optimal Solution

The optimal solution to the constrained minimization problem in (2) can be obtained using the method of Lagrange multipliers, and can be shown to be the generalized eigenvector that satisfies

$$\mathbf{M}_x\mathbf{w} = \nu_{min}\mathbf{M}_s\mathbf{w} \ , \tag{7}$$

where ν_{min} is the smallest generalized eigenvalue of the matrix pencil $(\mathbf{M}_x, \mathbf{M}_s)$. In this work, we find the corresponding optimal solution for the coefficient vector $\mathbf{h}$ for a given look direction ϕ and the matrix $\mathbf{C}$ that is defined by the look direction of the antenna array.

If the vector $\mathbf{h}$ is given *a priori*, it is well-known [6] that the optimal solution of the minimization problem described in (7) is

$$\mathbf{w}_o = \mathbf{M}_x^{-1}\mathbf{C}\left(\mathbf{C}^H\mathbf{M}_x^{-1}\mathbf{C}\right)^{-1}\mathbf{h}\ . \tag{8}$$

To have the complete solution to the problem, the response vector $\mathbf{h}$ must also be specified properly. We can substitute (8) in (2), and reformulate the optimization problem for designing the MTI processor as

$$\underset{\mathbf{h}}{minimize} \quad \mathbf{h}^H\left(\mathbf{C}^H\mathbf{M}_x^{-1}\mathbf{C}\right)^{-1}\mathbf{h} \ \ subject\, to\, \mathbf{h}^H\mathbf{h} = c\ . \tag{9}$$

This minimization problem is equivalent to that of minimizing the Rayleigh quotient of the vector $\mathbf{h}$, defined by $R(\mathbf{h}) = \mathbf{h}^H(\mathbf{C}^H\mathbf{M}_x^{-1}\mathbf{C})^{-1}\mathbf{h}/(\mathbf{h}^H\mathbf{h})$. Let $\mathbf{u}_{min}$ be the eigenvector associated with the minimum eigenvalue γ_{min} of the matrix $(\mathbf{C}^H\mathbf{M}_x^{-1}\mathbf{C})^{-1}$. It is well-known [8] that $R(\mathbf{h})$ is minimized when the coefficient vector is given by

$$\mathbf{h}_{opt} = \mathbf{u}_{min}\ . \tag{10}$$

By substituting (10) in (8), we can find the optimum weight vector maximizing the improvement factor of the space-time MTI processor to be

$$\mathbf{w}_{opt} = \gamma_{min}\mathbf{M}_x^{-1}\mathbf{C}\mathbf{u}_{min}\ . \tag{11}$$

Finally, the maximum improvement factor possible for the space-time MTI processor is obtained as

$$IF_{opt} = \frac{\mathbf{u}_{min}^H(\mathbf{C}^H\mathbf{M}_x^{-1}\mathbf{C})(\mathbf{C}^H\mathbf{M}_x^{-1}\mathbf{C})\mathbf{u}_{min}}{\mathbf{u}_{min}^H(\mathbf{C}^H\mathbf{M}_x^{-1}\mathbf{C})\mathbf{u}_{min}} = \gamma_{min}^{-1}\ . \tag{12}$$

2.3. A Suboptimal Solution

The optimal coefficient vector obtained in the previous subsection chooses the response vector $\mathbf{h}$ so as to achieve the maximum cancellation of the input clutter signal in the look direction. However, this solution involves computation of the eigenvector corresponding to the minimum eigenvalue of the matrix $(\mathbf{C}^H\mathbf{M}_x^{-1}\mathbf{C})^{-1}$, and therefore its design can be computationally demanding. In most applications, the covariance matrix M_x is estimated online, and the task of computing the optimal solution each time may make real-time implementation difficult. In this subsection, we provide a suboptimal solution that is computationally less demanding.

Our approach is based on a suboptimal solution commonly employed in conventional MTI processors [5]. Instead of solving the optimization problem in (9), we choose $\mathbf{h}$ to be the solution of the linearly constrained minimum variance estimator formulated as

$$\underset{\mathbf{h}}{minimize} \quad \mathbf{h}^H\left(\mathbf{C}^H\mathbf{M}_x^{-1}\mathbf{C}\right)^{-1}\mathbf{h} \quad subject\, to\, \mathbf{f}^T\mathbf{h} = 1\ , \tag{13}$$

where $\mathbf{f}$ is the $(M \times 1)$-element constraint vector. If we set one of the elements of the constraint vector to be one and set all the others to be zero, the optimization results in the linear prediction solution. Such a solution attempts to provide maximum clutter cancellation in the absence of the target signal. We can solve the optimization problem using Lagrange multipliers. The response vector $\mathbf{h}_{LP}$ in this case is given by

$$\mathbf{h}_{LP} = \frac{(\mathbf{C}^H\mathbf{M}_x^{-1}\mathbf{C})\mathbf{f}}{\mathbf{f}^T(\mathbf{C}^H\mathbf{M}_x^{-1}\mathbf{C})\mathbf{f}}\ . \tag{14}$$

Note that the formulation of the objective function in (13) employed the relationship between the coefficient vectors $\mathbf{w}$ and $\mathbf{h}$ as given in (8). Substituting (14) in (8) gives the desired weight vector based on the linear prediction approach to be

$$\mathbf{w}_{LP} = \frac{\mathbf{M}_x^{-1}\mathbf{C}\mathbf{f}}{\mathbf{f}^T(\mathbf{C}^H\mathbf{M}_x^{-1}\mathbf{C})\mathbf{f}}\ . \tag{15}$$

We emphasize that the linear prediction approach is a suboptimal technique since the optimization problem in (13) is not equivalent to the problem of maximizing the improvement factor as formulated in (2). However, this approach provides significant computational simplicity. To operate the space-time MTI processors in inhomogeneous backgrounds, they should be adaptive. The optimal approach in this environment requires adaptive techniques for the estimation of the minimum eigenvector, which demands significant complexity in its computation, whereas the suboptimal linear prediction approach is implementable with simpler techniques such as the sample matrix inversion (SMI) procedure [3],[4]. A similar method to the linear prediction approach in this paper was suggested by Klemm [9]. In [9], a space-time adaptive filter was designed as a simple extension to the one-dimensional prediction error filter. The output of the clutter filter was processed by a beamformer vector to steer the array processor toward the direction of interest. However, no optimality or suboptimality of the approach was indicated in [9]. Our analysis shows that the linear prediction approach is a suboptimal solution to the space-time MTI processor design problem.

3. PROBABILITY OF DETECTION OF THE SPACE-TIME MTI PROCESSORS

In this section, we analyze the detection performance of the two-dimensional MTI processors derived in the previous section. Such analyses can be performed for given clutter covariance matrices.

To evaluate the detection probability P_D and the false alarm probability P_{FA}, it is necessary to determine the probability density function of the array output $y(k)$ for two alternative hypotheses in the MTI problem. Let H_0 represent the hypothesis that the input signal to the antenna array contains only clutter plus noise. Similarly, let H_1 denote the hypothesis that the input signal contains the target signal in addition to the clutter and noise. We assume that the target on boresight is modeled as a Swerling II, *i.e.*, fluctuations are independent from pulse to pulse, and the received signal is Gaussian with zero mean value. The covariance function of the input signal may now be modeled as approximately equal to the signal-to-clutter-plus-noise ratio (SCNR) during the interval between pulses, and to be approximately zero for lag times longer than the pulse interval [10]. Under hypothesis H_1, the covariance matrix $\mathbf{M}_x$ can then be expressed as

$$\mathbf{M}_x = SNR\,\mathbf{C}\mathbf{C}^H = SNR\,\mathbf{M}_s \ , \tag{16}$$

where *SNR* denotes the signal-to-noise ratio. The array output $y(k)$ is a Gaussian random variable with zero mean value. The envelope of $y(k)$ is Rayleigh-distributed for both hypotheses H_0 and H_1 and its probability density function conditioned on the hypothesis H_0 or H_1 is given by

$$p(|y(k)|\,|\,H_i) \;=\; \frac{|y(k)|}{\sigma^2_{y(k)|\,H_i}} exp\{-|y(k)|^2/(2\sigma^2_{y(k)|\,H_i})\} \ , \quad |y(k)| \geq 0, \;\; i = 0, 1 \ . \tag{17}$$

In the above equation, $\sigma^2_{y(k)|\,H_i}$ represents the variance of $|y(k)|$ under hypothesis H_i. Given the above information about the signals involved in the MTI problem, we can evaluate P_D and P_{FA} to be

$$P_D = exp(-T_d/2\sigma^2_{y(k)|\,H_1}) \ , \tag{18}$$

and

$$P_{FA} = exp(-T_d/2\sigma^2_{y(k)|\,H_0}) \ , \tag{19}$$

respectively, where T_d denotes the detection threshold employed by the procedure. The two values of the variances required to complete the evaluation of the detection and false alarm probabilities can be calculated in a straightforward manner for the optimal and linear prediction-based processors. The results are as follows.
Optimal Method :

$$\sigma^2_{y(k)|\,H_0} = E\{|y(k)|^2\,|\,H_0\} = \gamma_{min} \tag{20}$$

and

$$\sigma^2_{y(k)|\,H_1} = E\{|y(k)|^2\,|\,H_1\} = SNR \tag{21}$$

Linear Prediction :

$$\sigma^2_{y(k)|\,H_0} = \mathbf{w}^H_{LP}\mathbf{M}_x\mathbf{w}_{LP} = \frac{1}{\mathbf{f}^T(\mathbf{C}^H\mathbf{M}_x^{-1}\mathbf{C})\mathbf{f}} \tag{22}$$

and

$$\begin{aligned} \sigma^2_{y(k)|\,H_1} &= SNR\,\mathbf{w}^H_{LP}\mathbf{M}_s\mathbf{w}_{LP} \\ &= SNR\,\left\| \frac{\mathbf{f}^T(\mathbf{C}^H\mathbf{M}_x^{-1}\mathbf{C})}{\mathbf{f}^T(\mathbf{C}^H\mathbf{M}_x^{-1}\mathbf{C})\mathbf{f}} \right\|^2 \ , \end{aligned} \tag{23}$$

where $\|(\cdot)\|$ denotes the Euclidean norm of $(\cdot)$.

4. COMPUTER SIMULATIONS

This section presents the results of several simulation experiments that evaluated the performance of the space-time MTI processors. A uniform linear array with fourteen antenna elements spaced half a wavelength of the carrier frequency apart was simulated in all the experiments. The radar PRF is 625 Hz and 16 pulses are transmitted with a carrier frequency of 435 MHz. The platform altitude is 1 km, the platform speed is 100 m/s, and the range of interest is 2 km.

The clutter covariance matrix was estimated using the model described in [2] as superposition of 160 independent clutter sources that are evenly distributed in azimuth about the radar. Figure 1 shows the space-time spectrum of the clutter signal used in the experiment. The space-time weight responses of the MTI processor are shown in Figure 2. The results in Figure 2 indicate that both the optimal and the linear prediction technique enable the MTI processor to critically reject the incoming clutter signals throughout the entire azimuth angle. The detection performances of the MTI processors are also shown in Figure 3. In detection tests, we examined the experimental P_D for SNR values in the range of -10 to 25 dB in 0.5 dB increments. The P_D values were evaluated using (18) $\sim$ (23) at $P_{FA} = 10^{-5}$. Even though the optimal method yields higher detection performance than the linear prediction method for all $SNR's$, the difference is not significant. Additional comparisons between the performances of the two processors were made in terms of the improvement factor, and the results are summarized in Table 1. As one can expect, the optimal method always performs better than the linear prediction method. However, in most of the cases considered, the performance degradation in the linear prediction approach is not significant.

5. CONCLUSIONS

This paper presented an optimum space-time MTI processor for airborne radar. The optimization is based on a stochastic target model in which the target is modeled as a random variable having uniform distribution over the PRF of the radar. The optimum solution that maximizes the improvement factor yielded by the processor is given by the generalized eigenvector corresponding to the smallest generalized eigenvalue of the signal and clutter covariance matrices. A suboptimal, but computationally simpler solution based on the linear prediction approach was also derived in the paper. The performance of the two processors were evaluated in terms of the detection probability and the improvement factor in an airborne radar environment. Results of such evaluations showed that both the optimal and the linear prediction techniques critically reject clutter signals arriving from the azimuth angle of interest. The optimal technique provides higher improvement factor than the linear prediction technique. However, the performance difference between the two methods was relatively small in most of the cases considered. Consequently, the space-time MTI processor based on the linear prediction approach may be a good candidate for use in airborne radar systems.

6. REFERENCES

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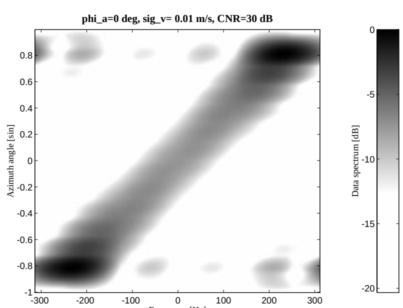
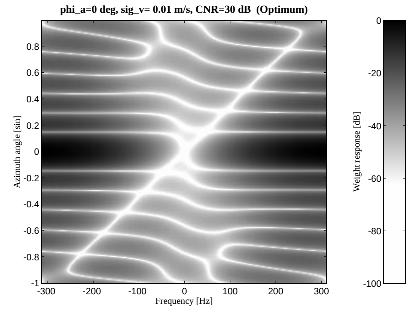
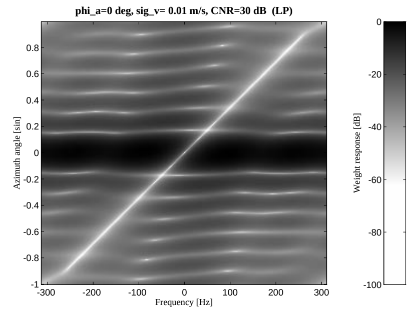


Figure 1: Space-time spectrum of the input data. Scenario : $\phi_a = 0^o, \sigma_v = 0.01m/s, CNR$=30dB, M=5.



(a) Optimal approach



(b) Linear prediction approach

Figure 2: Space-time weight responses of the space-time MTI processors.

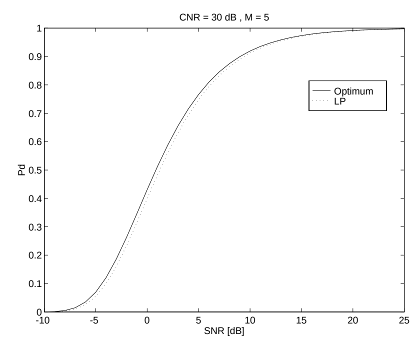


Figure 3: Probability of detection versus SNR.

Table 1: Comparison of the improvement factors due to the optimal and the linear prediction techniques.

ϕ_a $[deg]$	σ_v $[m/s]$	CNR $[dB]$	$N=5$		$N=7$	
			IF_{opt} $[dB]$	IF_{LP} $[dB]$	IF_{opt} $[dB]$	IF_{LP} $[dB]$
0	0.01	30	11.36	10.98	11.40	10.93
		60	11.24	10.38	11.32	10.48
	0.50	30	11.32	10.30	11.37	10.56
		60	11.13	10.06	11.23	9.75
60	0.01	30	11.13	10.54	11.24	10.53
		60	10.89	9.55	11.08	9.48
	0.50	30	11.11	9.05	11.22	9.18
		60	10.22	9.85	10.74	8.64