AN EFFICIENT BLIND IDENTIFICATION ALGORITHM FOR MULTICHANNEL FIR SYSTEMS USING LINEAR PREDICTION

Yifeng Zhou[1], *Henry Leung*[2] *and Patrick C. Yip*[3]

[1]Telexis Corporation, 2427 Holly Lane, Ottawa, Ontario, Canada K1V 7P2
[2]Surface Radar Section, Defence Research Establishment Ottawa, Ottawa, Ontario, Canada K1A 0Z4
[3]Communications Research Lab, McMaster University, Hamilton, Ontario, Canada L8S 4K1

ABSTRACT

In this paper, an efficient blind identification algorithm for multichannel FIR systems was proposed based on a deterministic model of the channel input. By decoupling the multichannel identification, the proposed method was able to estimate individual channel responses separately without having to solve for the augmented channel responses. The algorithm was implemented using linear prediction techniques. It was computationally efficient and suitable for real-time applications. Computer simulations were used to demonstrate the effectiveness of the proposed algorithm.

1. INTRODUCTIONS

Recently, the problem of blind channel identification has been extensively studied by researchers since the pioneering work by Sato [1]. By blind, we mean that the channel responses are estimated solely based on the sensor outputs without using the training sequences. Blind identification is attractive for many applications when the transmitted signals are not accessible.

For the last few years, it has received considerable attention in communications and signal processing [2]. Most of the earlier approaches to blind identification are based the use of higher-order statistics, which are known to suffer from many drawbacks. They require a large number of data samples and a heavy computational burden, making them unattractive for practical applications. A recent major progress was made by Tong, Xu and Kailath [3], in which they explored the cyclostationary properties of an oversampled communication signal to estimate the channel responses based on the second-order statistics of the sensor outputs. Since then, several approaches have been developed including the eigenstructure-based methods [4][5] and the least squares (LS) approach [6]. The LS approach is based on the deterministic modeling of the source input and does not require explicit statistical knowledge of the channel input. This is important for many practical applications where such information is usually not available. The LS approach uses the cross relation between each output pairs to estimate the augmented channel responses. When the channel order is relatively small, the computation cost may be affordable for real-time implementation. However, for some applications such as speech dereverberation and echo cancellation, since each channel order is usually at the level of several hundreds, the increased dimension of the augmented channel responses will lead to significantly increased computational burden, making it difficult for real time implementations. Also, since the LS approach deals with the augmented channel responses, it detects the maximum channel order instead of individual channel order. This may cause a severe deterioration in channel identification performance.

In this paper, we proposed an efficient blind identification algorithm for multichannel FIR systems. The approach was based on the deterministic modeling of the channel input. It was able to decouple the multichannel identification process and estimate individual channel separately. The channel responses were estimated by solving a set of linear prediction equations. The algorithm was computationally more efficient than the LS approach and practical for real-time applications. When the channel orders are unknown, we showed that the backward linear prediction can be combined to detect the channel orders and improve the channel response estimation performance. The identification performance can be further enhanced by exploiting the underlying structure of the data matrices of the linear prediction. Finally, computer simulations were used to demonstrate the effectiveness of the proposed algorithm. The results were compared to those of the LS approach.

2. PROBLEM FORMULATION

Consider an array of M sensors receiving data from a common source. Each channel is assumed to be a FIR system. The mth sensor output, $x_m(t)$, can be written as

$$x_m(t) = \sum_{i=0}^{L_m} h_m(t)s(t-i) = h_m(t) \odot s(t), \tag{1}$$

where $\odot$ denotes the convolution operator, $s(t)$ is the common source which is assumed to be unknown deterministic, and h_m is the FIR impulse response of the mth channel of order L_m. We assume that the number of sensor outputs is N. The objective of blind channel identification is to estimate the each channel responses $\{h_m(t)\}$ given the sensor outputs $\{x_m(t)\}$.

3. ESTIMATION OF CHANNEL RESPONSES

Let $H_m(z)$ denote the z transform of $h_m(t)$. To ensure the unique identification of the channel responses, we assume that $\{H_m(z),\ m = 1, 2, \ldots, M\}$ are coprime, i.e., they do not share any common roles [7]. In the z plane, we have

$$X_m(z) = H_m(z)S(z), \tag{2}$$

where $X_m(z)$ and $S(z)$ are the z transforms of $x_m(t)$ and $s(t)$, respectively. For a pair of sensor outputs, $x_m(t)$ and $x_n(t)$, their z transforms are related by

$$H_m(z) = \frac{X_m(z)}{X_n(z)} H_n(z). \tag{3}$$

Since $h_m(t)$ is of finite length of $L_m + 1$, $H_m(z)$ is a polynomial of order L_m. It is known that a polynomial of order L_m can be accurately interpolated by $N \geq L_m + 1$ points on the unit circle on the z plane [8]. To interpolate $H_m(z)$, we can choose the uniformly distributed points $z_l = W_N^l, l = 0, 1, \ldots, N-1$, where $W_N = \exp j2\pi/(N-1)$ as the interpolation points. The choice of uniformly distributed interpolation points has some useful properties. First, this choice gives equal emphasis to the entire frequency range of $h_m(t)$ which is appropriate especially when we do not have any *a prior* information on the channel spectra. Second, using uniformly distributed points will lead to the minimum norm interpolation of $H_m(z)$ [9]. The minimum norm property implies that if the error in the information of $H_m(z)$ on the unit circle is uniform, the interpolated error has minimum norm on the unit disk [10]. It follows that the error of the time sequence corresponding to the interpolated $H_m(z)$ is also minimum norm. From the computational point of view, since the uniformly distributed points on the unit circle are in the form of the discrete Fourier transform (DFT), the time sequence associated with the interpolated polynomial can be computed using fast Fourier transform (FFT) algorithms directly. The inverse DFT of $\{H_m(z_l);\ l = 0, 1, \ldots, N-1\}$ is given by

$$\begin{cases} h'_m(t) = h_m(t) & \text{for } t = 0, 1, \ldots, L_m \\ h'_m(t) = 0 & \text{for } t > L_m \end{cases} \tag{4}$$

When N points are used to recover $h_m(t)$, by the inverse DFT formula, we have

$$h'_m(t) = \sum_{i=0}^{L_n} \left[\frac{1}{N} \sum_{l=0}^{N-1} \frac{X_m(W_N^l)}{X_n(W_N^l)} W_N^{(t-i)l} \right] h_n(i). \tag{5}$$

Define $Y_{mn}(k) = X_m(W_N^k)/X_n(W_N^k)$ and let $\{y_{mn}(t);\ t = 0, 1, \ldots, N-1\}$ be the inverse DFT of $\{Y_{mn}(k),\ k = 0, 1, \ldots, N-1\}$. We use L to denote the maximum channel order. Since $h'_m(t) = 0$ for $t > L_m$, then

$$\tilde{A}_{mn} \underline{\tilde{h}}_n = 0, \tag{6}$$

where $\underline{\tilde{h}}_n = [h_n(0), h_n(1), \ldots, h_n(L_n)]^T$ and

$$\tilde{A}_{mn} = \begin{bmatrix} y_{mn}(L+1) & \ldots & y_{mn}(L-L_n+1) \\ y_{mn}(L+2) & \ldots & y_{mn}(L-L_n+2) \\ \vdots & \vdots & \vdots \\ y_{mn}(N-1) & \ldots & y_{mn}(N-L_n-1). \end{bmatrix} \tag{7}$$

When the sensor outputs are noise-free, $\underline{\tilde{h}}_n$ is in the null space of $\tilde{A}_{mn}$ which can be determined up to a scaling factor. In practice, since the sensor outputs are usually corrupted with noise, $\underline{\tilde{h}}_n$ can be solved in the least squares sense by

$$\min_{\underline{\tilde{h}}_n} \| \tilde{A}_{mn} \underline{\tilde{h}}_n \|_F^2, \tag{8}$$

where $\underline{\tilde{h}}_n$ is subject to certain nontrivial constraint. We usually choose to constrain that $h_n(0) = 1$. For an array of M sensors, the criterion for solving $\underline{\tilde{h}}_n$ becomes

$$\min_{\underline{\tilde{h}}_n} \| \tilde{A}_n \underline{\tilde{h}}_n \|_F^2, \tag{9}$$

subject to $h_n(0) = 1$, where

$$\tilde{A}_n = [\tilde{A}_{1n}^T, \tilde{A}_{2n}^T, \ldots, \tilde{A}_{n-1,n}^T, \tilde{A}_{n+1,n}^T, \ldots, \tilde{A}_{Mn}^T]^T. \tag{10}$$

Note that the estimation of channel responses has been decoupled and each channel responses can be estimated separately.

4. ALGORITHM IMPLEMENTATION

When $\underline{h}_n(0) = 1$, the constraint optimization problem (8) can be written as the least squares solution of the following linear equations

$$A_{mn} \underline{h}_n = \underline{b}_{mn}, \tag{11}$$

where $\underline{h}_n = [h_n(1), \ldots, h_n(L_n)]^T$, $\underline{b}_{mn} = -A[:,1]$ is formed by the first column of A_{mn}, and

$$A_{mn} = \begin{bmatrix} y_{mn}(L) & \ldots & y_{mn}(L-L_n+1) \\ y_{mn}(L) & \ldots & y_{mn}(L-L_n+2) \\ \vdots & \vdots & \vdots \\ y_{mn}(N-2) & \ldots & y_{mn}(N-L_n-1). \end{bmatrix}. \tag{12}$$

Correspondingly, criterion (9) can be written as

$$A_n \underline{h}_n = \underline{b}_n, \tag{13}$$

where $A_n = [A_{1n}^T, \ldots, A_{n-1,n}^T, A_{n+1,n}^T, \ldots, A_{Mn}^T]^T$ and

$$\underline{b}_n = [\underline{b}_{1n}^T, \ldots, \underline{b}_{n-1,n}^T, \underline{b}_{n+1,n}^T, \ldots, \underline{b}_{Mn}^T]^T. \tag{14}$$

It can be observed that that $\{h_n(t)\}$ forms a linear prediction relationship among $\{y_{mn}(t)\}$. Equations (13) are known as the normal equation in least squares terminology. Many existing linear prediction techniques [11] can be applied for solving for the solutions of (13). There are also the adaptive algorithms such as the multichannel recursive least squares (RLS) approach [12]. These adaptive algorithms have the ability to make the identification adapt to the time-varying channel environments. The proposed approach can be summarized as follows.

1. Compute the FFT of the sensor outputs $\{x_m(t);\ m = 1, 2, \ldots, M\}$.
2. Use the inverse FFT to compute $\{y_{mn}(t)\}$ and form the data matrix A_n.
3. Solve the linear prediction problem (9) for each channel.

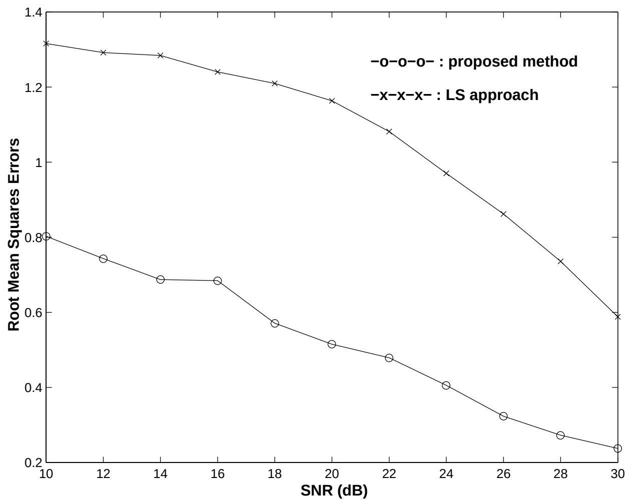

Figure 1. Variation of the RMSE of the first channel response estimates via the SNR.

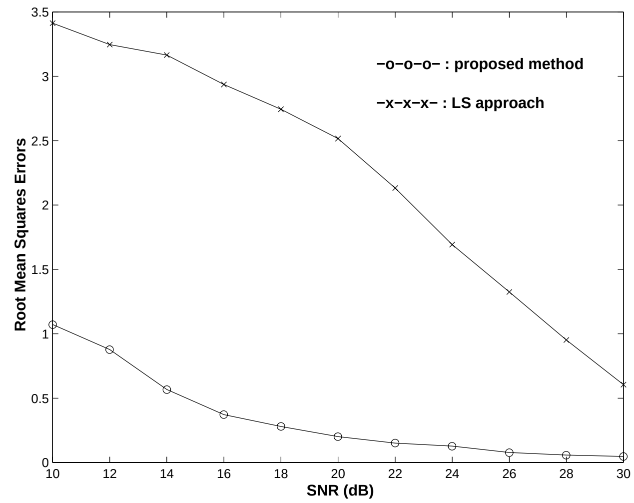

Figure 2. Variation of the RMSE of the second channel response estimates via the SNR.

In practice, the channel orders are usually unknown and need to be detected. There are different approaches for detecting the channel orders. One of the commonly used approach is the Akaike information criterion (AIC) [13] which determines the order by minimizing an information criterion. A more elaborate approach is to use an overestimated channel order and combine the backward linear prediction [14]. When overestimated channel orders, it can be shown that the data matrices in (13) has a rank that is equal to the channel order. It follows that channel order can be detected as the number of the principal eigenvalues. In [14], a minimum norm solution was proposed which only included the eigenvectors associated with the principal eigenvalues. Those eigenvectors associated with the small eigenvalues are ignored in the solution because they could introduce considerable fluctuations of along their directions, amplified by the reciprocal of small noise eigenvalues, eventually resulting in degraded estimation performance. The channel zeros can be identified by incorporating the backward linear prediction technique. It is known that spurious zeros occur when an overestimated channel order is used. However, since the spurious zeros are generated by the prediction errors, they usually do not change when the process is time reversed and tend predominantly to stay within the unit circle. Thus, by examining the zeros estimated from the both the forward and backward prediction processes, we choose those zeros which occur in reciprocal positions along a common radius as the FIR channel zeros and discard the remaining ones as spurious ones.

Since the proposed approach deals with each channel separately, it would be computationally more efficiently than those method based on solving the augmented channel responses. To discuss the computational complexity problem, we assume that each channel has a known order of L. When direct methods (Gauss reduction or elimination) are used, the computational complexity of the LS approach is $M^3 \cdot O(L^3)$ while the proposed algorithm only requires $M \cdot O(L^3)$ operations. If we use the Levinson's recursive method [15], the total computational complexity can be further reduced to $M \cdot O(L^2)$. It should be pointed out that, although the algorithm requires additional computations in forming the data matrix A_n, they only involve fast Fourier transform algorithms and will not introduce any significant computation cost.

5. NUMERICAL EXAMPLES

Two sensors were considered. Each channel channel responses were characterized by $h_m(t) = \exp\{-\alpha_m t\}, \quad t = 0, 1, \ldots, L_M$, where we chose $\alpha_1 = 0.25$, $\alpha_2 = 0.1$, $L_1 = 8$ and $L_2 = 12$. The sensor noises were simulated as independent white Gaussian processes with zero mean.

First, we considered the case in which the channel orders were assumed to be known. Figures 1 and 2 showed variations of the root mean squares error (RMSE) of the channel response estimates via the sensor SNR for the two channels. The comparisons were made with the LS approach. The number of sensor samples was chosen as $N = 200$. Each test is repeated 100 times to obtain the averaged results. We can see that the proposed method outperformed the LS approach in the sense that it produced smaller RMSE that that by the LS approach at each SNR.

In the next example, we assumed that the channel orders were unknown and the forward and backward linear prediction techniques were used. We chose $N = 200$ and $SNR = 40$dB. An overestimated channel order $L_E = 14$ was used. Using the eigendecomposition of the data matrices, we determined each channel order as $L_1 = 8$ and $L_2 = 12$, respectively. The minimum norm solution of both the forward and backward linear predictions were obtained using the principal eigenvectors. Figure 3 showed the distribution of the estimated zeros of each channel. In Figure 3(a) and (c), we plotted the estimated zeros of the two channels by forward and backward prediction. In the figure, labels 'x' and 'o' denote the estimated zeros by forward and backward prediction, respectively. Figure 3(b) and (d) showed the actual zeros of the two channels. We identified those zeros which occur in reciprocal positions along a common radius as the channel zeros. The estimated channel

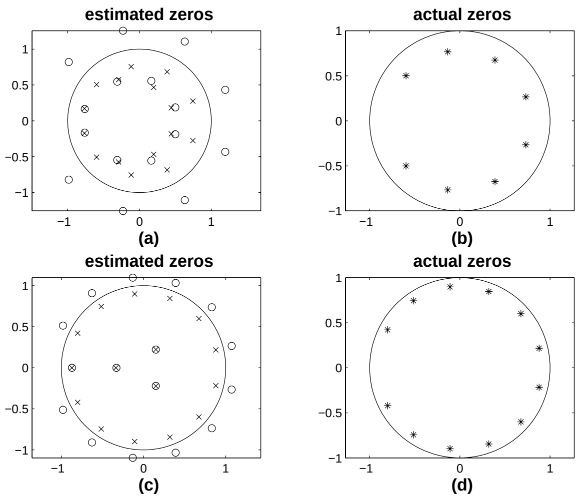

Figure 3. Distributions of the estimated zeros by the forward and backward linear predictions.

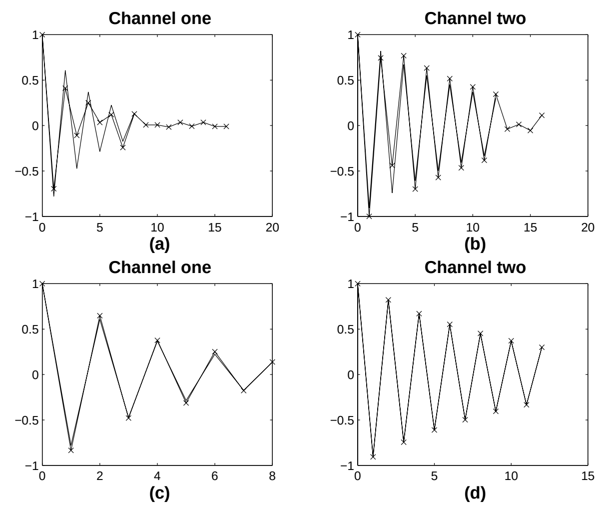

Figure 4. Estimation of the channel responses.

zeros were used to reconstruct the channel response estimates. Figure 4(a) and (b) showed the channel response estimates where we applied the proposed approach directly with the overestimated channel orders. Figure 4(c) and (d) were the estimation results by the forward and backward predictions. In Figure 4, the solid lines denote the actual channel responses, and the lines labeled with 'x' are the estimated ones. As can be seen, an improved performance was achieved by the application of the forward and backward linear predictions. It is interesting to note that even when an overestimated channel order is used, the performance of the direct solution is not affected seriously. This partially indicates that the proposed prediction approach is robust to the overdetermination of the channel orders.

6. CONCLUSIONS

In this paper, we have presented a blind identification approach for multichannel FIR systems. We showed that the proposed approach was able to decouple the estimation of channel responses successfully. The algorithm was implemented using linear prediction techniques. It was computationally more efficient than most existing methods based on solving for the augmented channel responses. The approach overcame the difficulties in detecting the channel orders and was practical for real-time applications. Computer simulations demonstrated the effectiveness of the proposed approach. Further statistical performance study and the robustness analysis are under investigation.

REFERENCES

[1] Y. Sato, "A method of self-recovering equalization for multilevel amplitude-modulation", IEEE Trans. Communi., vol. 23, pp. 679-682, 1975

[2] H. Liu, G. Xu, L. Tong and T. Kailath, "Recent developments in blind channel equalization from cyclostationarity to subspaces", Signal Processing, vol. 50, pp. 83-99, 1996

[3] L. Tong, G. Xu and T. Kailath,"Blind identification and equalization based on second-order statistics : a time domain approach", IEEE Trans. IT, vol. 40, pp. 340-350, 1994

[4] E. Moulines, P. Duhamel, J. Cardoso and S. Mayrargue, "Subspace methods for the blind identification of multichannel FIR filters", IEEE Trans. SP, vol. 43, pp. 516-525, 1995

[5] H. Z. Zeng and L. Tong, "Blind channel estimation using the second-order statistics : Algorithm", IEEE Trans. SP, vol. 45, pp. 1919-1997, 1997

[6] H. Liu, G. Xu and L. Tong, "A deterministic approach to blind identification of multi-channel FIR systems", Proc. ICASSP-94, vol. 4, pp. 581-584, 1994

[7] L. Ljung, *System identification*, Prentice Hall, Englewood Cliff, NJ, 1987

[8] A. V. Oppenheim and R. W. Schafer, "Discrete-time signal processing", Prentice Hall, Englewood Cliffs, NJ, 1989

[9] L. Brutman and A. Pinkus, "On the Erdos conjecture concerning minimal norm interpolation on the unit circle", SIAM J. Numer. Anal., vol. 17, pp. 373-375, 1980

[10] L. Brutman and A. Pinkus, "On the polynomial and rational projections in the complex plane", SIAM J. Numer. Anal., vol. 17, pp. 366-373, 1980

[11] J. Markhoul, "Linear prediction : A tutorial review", IEEE Proc., vol. 63, pp. 561-580, 1975

[12] S, Haykin, *Adaptive filter theory*, Prentice-Hall, Englewood Cliffs, New Jersey, 1986

[13] H. Akaike, "A new look at the statistical model identification", IEEE Trans. AC, vol. 19, pp. 716-723, 1974

[14] D. Tufts and R. Kumaresan, "Estimation of frequencies of multiple sinusoids : Making linear prediction perform like maximum likelihood", IEEE Proc., vol. 70, pp. 975-989, 1982

[15] E. A. Robinson, *Statistical communications and detection*, New York : Hafner, 1967