THE BLIND QRD-DMS BEAMFORMER AND ITS VLSI SYSTOLIC DESIGNS FOR DS/CDMA SYSTEMS

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ABSTRACT

In this paper, we present a blindly adaptive beamforming algorithm which is based on the second order interference estimation to maximize the received SINR. Using the desired signature code and the orthogonal to it code, a new code filter is introduced to decompose the receiving signals into two part : desired information and interference. The above method motivates us to develop the QR-decomposition based dominant eigen mode search (QRD-DMS) algorithm which is more numerically stable than the DMS one. The corresponding wavefront systolic architecture is also proposed for the VLSI implementation. Compare to the families of minimum mean square error (MMSE) algorithms which need training sequences, we have completed the maximum SINR families, as shown in Table 1, by proposing the QRD-DMS method which not only blindly updates the beamforming weights and converges as fast as the QRD-RLS method.

1. INTRODUCTION AND SYSTEM MODEL

In recent years, blindly adaptive beamforming techniques have arised much attention in military or commercial applications. One approach uses the principle generalized eigenvector of the signal and interference matrix pair to maximize the signal to interference and noise ratio (SINR) [1, 2, 3, 4]. The advantages of the above method is nonsensitive to array geometry and has the fastest convergence speed among the blindly adaptive algorithms. By the fact that the power iterations [5] converges faster than the estimation of correlation matrices, we have introduced the dominant eigen-mode searching (DMS) method [6] to search the principle eigen mode economically. To keep from amplifying the quantization errors of the finite-digit sampling in the data vectors motivates the study of updating the beamformer directly from the data sequences. In this paper, we propose a novel QRdecomposition DMS (QRD-DMS) algorithm which updates the weights directly from the data sequences and offers a numerically stable DMS beamformer. Similar to the QRD-RLS systolic architecture, a wavefront QRD-DMS systolic array is also introduced for high efficient VLSI designs.

We consider the scenario of the uplink transmission in a single cell. P array sensors receive the information from total K users which are uniformly distributed in azimuth around the base station. At the mobile transmitters, we assume the source data is modulated by the balanced-DQPSK. The inphase (I) and quadrature (Q) signals are spread by the user's long user code and short I-Q channel code. The channel is assumed to be multipath propagation medium with total Lresolvable path groups. To decompose the received data into the signal and the interference, we divide the post-correlation data into two parts. The first part $\alpha(n)$ is obtained by integrating the post-correlated results from $[0, T_b/2]$; and the second part $\beta(n)$ is from $[T_b/2, T_b]$.

Beamforming Method	MMSE (Training seq.)	Max. SINR (Blindly adaptive)
Gradient based	LMS	Maxmin [7]
$\operatorname{Deterministic}$	RLS	DMS [6]
QRD based	QRD-RLS	$QRD-DMS^*$

Table 1: The beamforming algorithms of the MMSE and Max. SINR families. * : being proposed in this paper.

$$\begin{aligned} \alpha_{k,l}(n) &= \int_{\tau_{k,l}+(n-1/2)T_b}^{\tau_{k,l}+(n-1/2)T_b} \mathbf{r}(t) \star C_k(t-\tau_{k,l})dt, \\ \beta_{k,l}(n) &= \int_{\tau_{k,l}+(n-1/2)T_b}^{\tau_{k,l}+nT_b} \mathbf{r}(t) \star C_k(t-\tau_{k,l})dt, \end{aligned}$$
(1)

where $\tau_{k,l}$ denotes the time delay of the desired signal; $C_k(t)$ denotes the signature code's waveform. The operator ' \star ' denotes the remultiplications of the signature code in the I and Q channels. We notice that the original post-correlated results have been modified in our system. In the present DS/CDMA system, like IS-95, the post-correlated signal $\mathbf{y}_{k,l}$ is obtained by integrating one symbol's long and equals to the summation of the above two parts.

$$\mathbf{y}_{k,l}(n) = \alpha_{k,l}(n) + \beta_{k,l}(n)$$

= $G \cdot s_k(n) \mathbf{a}_{k,l} + \mathbf{u}_{k,l}(n).$ (2)

The interference $\mathbf{u}_{k,l}(n)$ is uncorrelated with desired data $s_k(n)$ if we assume mutually uncorrelated sources and random signature code. $\mathbf{a}_{k,l}$ denotes the $P \times 1$ channel vector, and G means the processing gain. We introduce another post-correlated signal which is remultiplied by the orthogonal code.

$$\mathbf{z}_{k,l}(n) = \alpha_{k,l}(n) - \beta_{k,l}(n),$$

$$= \int_{\tau_{k,l}+(n-1)T_b}^{\tau_{k,l}+nT_b} \mathbf{r}(t) \star C_{k\perp}(t-\tau_{k,l})dt, \quad (3)$$

which null out the desired information. The orthogonal code can be expressed as the element-wise multiplication of desired code to a Hadamard-Walsh orthogonal sequence.

$$C_{k\perp}(t) = C_k(t) \, .* \, [\underbrace{T_b/2}_{t+++\dots++}, \underbrace{T_b/2}_{t--\dots--}].$$
(4)

Let us first assume that $\mathbf{\tilde{r}}(t)$ represents the received signal $\mathbf{r}(t)$ without the desired information. So the interference $\mathbf{u}_{k,l}(n)$ can be viewed as the post-correlation results of $\mathbf{\tilde{r}}(t)$.

$$\mathbf{u}_{k,l}(n) = \int_{\tau_{k,l}+(n-1)T_b}^{\tau_{k,l}+nT_b} \tilde{\mathbf{r}}(t) \star C_k(t-\tau_{k,l}) dt,$$

$$= \bar{\alpha}_{k,l}(n) + \bar{\beta}_{k,l}(n).$$
(5)

where $\{\bar{\alpha}_{k,l}, \bar{\beta}_{k,l}\}$ represents the signal $\{\alpha_{k,l}, \beta_{k,l}\}$ without the desired information. Since the orthogonal code $C_{k\perp}(t)$ filters out the desired signal $s_{k,l}(t)$, $\mathbf{z}_{k,l}(n)$ does not contain the desired information anymore. Reader can check the following equality

$$\mathbf{z}_{k,l}(n) = \alpha_{k,l}(n) - \beta_{k,l}(n) = \bar{\alpha}_{k,l}(n) - \bar{\beta}_{k,l}(n), \quad (6)$$

which gives the following lemma:

Lemma 1 If the spreading code is an independent Bernoulli process, the correlation matrix of the estimated interference equals to which of the exact interference, i.e $\mathbf{R}_{uu} = \mathbf{R}_{zz}$.

Proof: The data $\bar{\alpha}_{k,l}(n)$ and $\bar{\beta}_{k,l}(n)$ are uncorrelated because they are contributed by the independent spreading code from the mutually disjoint first and second $T_b/2$ time interval. From equations (5) and (6), we can find that \mathbf{R}_{uu} equals to \mathbf{R}_{zz} by direct calculations. \Box

In time varying environment, we can only assume pseudo time invariant channels within a finite sampling window, so the estimation error $\mathbf{E}(n) = |\mathbf{R}_{uu}(n) - \mathbf{R}_{zz}(n)|^2$ always exists. However, the above lemma is valid for both synchronous or asynchronous CDMA systems even if the correlator are unperfectly synchronized to the desired signal.

2. THE QRD-DMS ALGORITHMS

Based on the above interference estimation, we will introduce the algorithm which blindly maximize the averaged signal to interference and noise ratio (SINR) at the output of the beamformer. After the beamforming combination, the resulting signals can be expressed as

$$\hat{s}_{k}(n) = \mathbf{w}^{H} \cdot \mathbf{y}_{k,l}(n)$$

= $G \cdot \kappa_{w} \cdot s_{k}(n) + u_{k,l}(n).$ (7)

where the constant $\kappa_w = \mathbf{w}^H \cdot \mathbf{a}_{k,l}$. The $u_{k,l}(n)$ is uncorrelated to the desired data $s_k(n)$ if we assume uncorrelated sources. The resulting SINR can be expressed as the ratio of the signal power $E[|\kappa_w s_k(n)|^2]$ to the noise power $E[|u_{k,l}(n)|^2]$. From the above equation, we can compute the SINR as

$$SINR = \frac{E \|\mathbf{w}^H \mathbf{y}_{k,l}(n)\|^2}{E \|\mathbf{w}^H \mathbf{u}_{k,l}(n)\|^2} - 1 = \frac{\mathbf{w}^H \mathbf{R}_{yy} \mathbf{w}}{\mathbf{w}^H \mathbf{R}_{uu} \mathbf{w}} - 1. \quad (8)$$

Since both correlation matrices \mathbf{R}_{yy} and \mathbf{R}_{uu} are positive definited, for any weight vector $\mathbf{w} \neq \mathbf{0}$, we have [8]

$$\lambda_{max} \ge \frac{\mathbf{w}^H \mathbf{R}_{yy} \mathbf{w}}{\mathbf{w}^H \mathbf{R}_{uu} \mathbf{w}} \ge \lambda_{min} \tag{9}$$

where $\lambda_{max} \geq \lambda_2 \geq \ldots \geq \lambda_{min} > 0$ are the ordered general eigenvalues of the matrix pair $(\mathbf{R}_{yy}, \mathbf{R}_{uu})$. The SINR criterion is maximized when the weight vector equals to the principle eigenvector of the above matrix pair, that is,

$$\mathbf{R}_{yy} \cdot \mathbf{w}_{SINR} = \lambda_{max} \cdot \mathbf{R}_{uu} \cdot \mathbf{w}_{SINR}. \tag{10}$$

Equation (10) depicts a deterministic way to find the optimal beamformer. From the result of *lemma 1*, we can replace \mathbf{R}_{uu} by \mathbf{R}_{zz} and still have positive eigenvalues. Power iterations offers an efficient and economic way to search the dominant mode (DMS) in matrix $\mathbf{R}_{zz}^{-1}\mathbf{R}_{yy}$. For the fact that the correlation matrix's eigen-field varies slowly and power iterations eventually converge faster than the correlation matrices, we

can modify the weight updating equation to execute power iterations once per symbol's duration.

$$\mathbf{w}_{SINR}(n) = \mathbf{R}_{zz}^{-1}(n)\mathbf{R}_{yy}(n) \cdot \mathbf{w}_{SINR}(n-1).$$
(11)

Just like the RLS method, the DMS suffers from truncation errors of finite-digit sampling when it computes the data vectors' outer products to estimate the correlation matrices. This motivates us to develop a numerically stable QRD-DMS method which updates the weights directly from the data sequences.

In time varying environment, the most popular method to estimate the data and interference correlation matrices is to take the time average of the vectors' outer products over an exponentially decay window.

$$\hat{\mathbf{R}}_{yy}(n) = \mathbf{Y}(n)^{H} \mathbf{Y}(n), \hat{\mathbf{R}}_{zz}(n) = \mathbf{Z}(n)^{H} \mathbf{Z}(n),$$
(12)

where the data matrices $\mathbf{Y}(n)$ and $\mathbf{Z}(n)$ represent the received data over the exponentially decay window. The only known method to solve the above eigenvalue decomposition problem in data domain is to solve the generalized singular value decomposition (GSVD) problem [5] toward matrix pair $(\mathbf{Y}(n), \mathbf{Z}(n))$. However, to do the GSVD is very computational intensive while doing the cosine-sine (CS) decomposition. Furthermore, we only interest in dominant eigen-mode computations rather than finding all the other eigen-modes or orthogonal matrices. The above GSVD approach is not practically satisfied in our beamforming problem, and we propose the following approach by using QRD to the joint data matrices.

2.1. The Proposed Theory

Theorem 1 Assume $\mathbf{Y}, \mathbf{Z} \in \mathcal{C}^{n \times p}$, and their joint matrix is non-null, i.e. $null(\mathbf{Y}) \cap null(\mathbf{Z}) = \{0\}$. The generalized eigenvalue decomposition problem in the correlation domain

$$(\mathbf{Y}^{H}\mathbf{Y})\cdot\mathbf{W} = (\mathbf{Z}^{H}\mathbf{Z})\cdot\mathbf{W}\cdot\mathbf{\Lambda},$$
(13)

equals to solving the following problem in the data domain

$$\mathbf{H} \cdot \mathbf{W} = \mathbf{R} \cdot \mathbf{W} \cdot \mathbf{\Lambda} \ . \tag{14}$$

where the $P \times P$ complex matrices **R** and **H** can be obtained by the following 3 steps:

1. Do a simple data transformation.

$$\mathbf{A} = \frac{(\mathbf{Y} + \mathbf{Z})}{2}, \quad \mathbf{B} = \frac{(\mathbf{Y} - \mathbf{Z})}{2}. \tag{15}$$

2. Do the QRD of the first joint data matrix.

$$\begin{bmatrix} \mathbf{A} \\ \mathbf{B} \end{bmatrix} = \mathbf{Q} \begin{bmatrix} \mathbf{R} \\ \mathbf{O} \end{bmatrix}.$$
 (16)

3. Apply the same rotations to the second matrix.

$$\begin{bmatrix} \mathbf{H} \\ \mathbf{F} \end{bmatrix} = \mathbf{Q}^{H} \begin{bmatrix} \mathbf{B} \\ \mathbf{A} \end{bmatrix}.$$
(17)

The eigenvalues are transformed by $\Lambda^{'}=(\Lambda-I)/(\Lambda+I).$

The above theory says that we can solve the generalized eigenvalue problem directly from the data matrix pair (\mathbf{R}, \mathbf{H}) . Let us see a useful lemma first which will be used in the proof of the theory.

$\begin{bmatrix} \mathbf{R}^{(n)} \end{bmatrix}$:	$\mathbf{H}(n)$:	$\mathbf{B}^{-H}(n)$] [$\mu^{1/2}\mathbf{R}(n-1)$	$\mu^{1/2}\mathbf{H}(n-1)$	÷	$\mu^{-1/2}\mathbf{R}^{-H}(n-1)$
	÷	"	÷	it (11)	$=\mathbf{T}(n)$	Ο	: #	÷	#
0		#	:	#		$\alpha^{H}(n)$	$\beta^{H}(n)$	÷	0
$\begin{bmatrix} 0_{2\times P} \end{bmatrix}$	•	#	·	#		$eta^{H}(n)$	$\alpha^{H}(n)$	÷	0

Lemma 2 If $null(\mathbf{Y}) \cap null(\mathbf{Z}) = \{0\}$, then $null(\mathbf{A}) \cap null(\mathbf{B}) = \{0\}$. The upper triangular matrix \mathbf{R} is non-singular. **proof:** we will prove it from the opposite way. If $null(\mathbf{A}) \cap null(\mathbf{B}) \neq \{0\}$, then there must exist a vector $\mathbf{w} \neq \mathbf{0}$ such that

$$\begin{array}{c} \mathbf{A} \\ \mathbf{B} \end{array} \right] \mathbf{w} = \mathbf{0}, \qquad \Rightarrow \left[\begin{array}{c} (\mathbf{A} + \mathbf{B}) \\ (\mathbf{A} - \mathbf{B}) \end{array} \right] \mathbf{w} = \left[\begin{array}{c} \mathbf{Y} \\ \mathbf{Z} \end{array} \right] \mathbf{w} = \mathbf{0}.$$
(18)

Therefore, the joint matrix of **Y** and **Z** are null ($null(\mathbf{Y}) \cap null(\mathbf{Z}) \neq \{0\}$), which contradicts the assumption. The nonsingularity of the matrix **R** comes from the result that matrix

It is a necessary condition to have \mathbf{R}^{-1} exist not only for the following proof but also for the systolic array's applications.

Proof (theorem) : From equation (15), we have $\mathbf{Y} = (\mathbf{A} + \mathbf{B})$ and $\mathbf{Z} = (\mathbf{A} - \mathbf{B})$. The generalized eigenvalue problem in equation (13) can be written as

$$(\mathbf{A} + \mathbf{B})^{H} (\mathbf{A} + \mathbf{B}) \cdot \mathbf{W} = (\mathbf{A} - \mathbf{B})^{H} (\mathbf{A} - \mathbf{B}) \cdot \mathbf{W} \cdot \mathbf{\Lambda}$$
$$\Rightarrow \qquad (\mathbf{A}^{H} \mathbf{B} + \mathbf{B}^{H} \mathbf{A}) \cdot \mathbf{W} = (\mathbf{A}^{H} \mathbf{A} + \mathbf{B}^{H} \mathbf{B}) \cdot \mathbf{W} \cdot \mathbf{\Lambda}'$$

$$\Rightarrow \quad [\mathbf{A}^{H}\mathbf{B}^{H}] \begin{bmatrix} \mathbf{B} \\ \mathbf{A} \end{bmatrix} \cdot \mathbf{W} = [\mathbf{A}^{H}\mathbf{B}^{H}] \begin{bmatrix} \mathbf{A} \\ \mathbf{B} \end{bmatrix} \cdot \mathbf{W} \cdot \mathbf{\Lambda}^{'}. (19)$$

From the definitions of the matrices \mathbf{R} and \mathbf{H} ,

$$\begin{bmatrix} \mathbf{R}^{H} \mathbf{O} \end{bmatrix} \begin{bmatrix} \mathbf{H} \\ \mathbf{F} \end{bmatrix} \cdot \mathbf{W} = \begin{bmatrix} \mathbf{R}^{H} \mathbf{O} \end{bmatrix} \begin{bmatrix} \mathbf{R} \\ \mathbf{O} \end{bmatrix} \cdot \mathbf{W} \cdot \mathbf{\Lambda}'$$
$$\Rightarrow \qquad \mathbf{R}^{H} \cdot \mathbf{H} \cdot \mathbf{W} = \mathbf{R}^{H} \cdot \mathbf{R} \cdot \mathbf{W} \cdot \mathbf{\Lambda}' \qquad (20)$$

Since the upper triangular matrix \mathbf{R} is non-singular, we can take it out at the both sides of the above equation.

$$\mathbf{H} \cdot \mathbf{W} = \mathbf{R} \cdot \mathbf{W} \cdot \mathbf{\Lambda} \quad (21)$$

where $\mathbf{\Lambda}^{'} = (\mathbf{\Lambda} - \mathbf{I})/(\mathbf{\Lambda} + \mathbf{I})$ is a diagonal matrix. Since $(\mathbf{Y}^{H}\mathbf{Y})$ and $(\mathbf{Z}^{H}\mathbf{Z})$ are all positive, $\mathbf{\Lambda}$ is positive and so is the $(\mathbf{\Lambda} + \mathbf{I})$. This means that $\mathbf{\Lambda}^{'}$ always exists and is finite. \Box

2.2. The QRD-DMS Algorithm

From the definition of $\alpha(n)$ and $\beta(n)$, we have

$$\alpha(n) = [\mathbf{y}(n) + \mathbf{z}(n)]/2,$$

$$\beta(n) = [\mathbf{y}(n) - \mathbf{z}(n)]/2.$$
(22)

Therefore, the matrices $\mathbf{A}(n)$ and $\mathbf{B}(n)$ can be written as

$$\mathbf{A}(n) = [\mu_{1/2}^{(n-1)}\alpha(1), \mu_{1/2}^{(n-2)}\alpha(2), \dots, \alpha(n)]^{H}.$$

$$\mathbf{B}(n) = [\mu_{1/2}^{(n-1)}\beta(1), \mu_{1/2}^{(n-2)}\beta(2), \dots, \beta(n)]^{H}.$$
 (23)



Figure 1: The QRD-DMS systolic array.

To have a easy way to update the joint data matrices, we update the post-correlated vectors $\alpha(n)$ and $\beta(n)$ to the bottom rows of the joint matrices $\begin{bmatrix} \mathbf{A}(n) \\ \mathbf{B}(n) \end{bmatrix}$ and $\begin{bmatrix} \mathbf{B}(n) \\ \mathbf{A}(n) \end{bmatrix}$, which gives the following proposition.

Proposition 1 Shuffling the rows of both joint matrices in the same order does not affect the validity of the theory.

The proof is omitted here. To find the desire matrices $\mathbf{R}(n)$, $\mathbf{H}(n)$ and $\mathbf{R}^{-1}(n)$, we use a series of Given rotations [9], $\mathbf{T}(n)$, to null out the $P \times 1$ vectors $\alpha(n)$ and $\beta(n)$. The final matrix updating formula is shown at the top of this page. Fig. 1 shows the resulting QRD-DMS systolic array. It combines the given rotations and the power iterations together. Since we input two data per symbol, the systolic array operates at 2X symbol's rate, so the weight vector can be obtained by skewedly down sampled by 2. However, because of pipelining inputs, the present weight vector $\mathbf{w}(n)$ is powered updated by $\mathbf{w}(n-P)$. This reduces the convergence speed of power iterations by a factor of antenna number P. We multiplex and pipeline at least P user into the systolic array in one symbol's duration to solve this problem. The working frequency is therefore increased to at least 2PX symbol's rate. Another advantage of multiuser pipelining is that the processing time of the beamforming weights is shortened to less then 2 symbol's durations, so we can use coherent beamforming combination $\hat{s}(n) = \mathbf{w}^{H}(n) \cdot \mathbf{y}(n)$ to enhance the performance instead of the post combinations $\hat{s}(n) = \mathbf{w}^H(n-m) \cdot \mathbf{y}(n)$, where m > 0.

3. SIMULATIONS AND CONCLUSIONS

In this section, we compare the performance of different beamforming algorithms by extensive computer simulations.



Figure 2: Compare the uncoded bit error rate of the QRD-DMS algorithm, the upper bound and lower bound.

We consider the uplink transmission in a pico cell CDMA network which use DQPSK modulation scheme at 2.5 GHz. The processing gain equals to 16 and the user's long PN code is generated by a decided loop with period 2^{23} . The received E_b/N_0 equals to 6dB. A uniformly linear array is implemented and each element is distanced by a half of the central carrier wavelength $\lambda_c/2$. We simulate the multipath propagation model of L = 2 multipath groups and M = 10 DOA partitions in each group [6]. The arriving angle diversity is uniformly distributed over $[-10^{\circ}, 10^{\circ}]$.

We compare the proposed algorithm to the other two well-know beamformer methods : Equal gain diversity combining and QRD-RLS algorithm. Since the diversity combining does not need weight's computations, it is considered as the performance lower bound. While the QRD-RLS algorithm assumes that training sequence is available all the time, and the coherent combining $(\hat{s}(n) = \mathbf{w}^{H}(n) \cdot \mathbf{y}(n))$ is performed. The QRD-RLS's performance is always better than the Wiener's solution since we equivalently use the sequence $s_k(1), \ldots, s_k(n)$ to estimate the desired signal $s_k(n)$. Fig. 2 shows the uncoded BER of QRD-DMS beamformer when antenna element number P = 5 and active users N =6. The exponential windowing factor for QRD-DMS and QRD-RLS algorithms are $\mu = 0.99$. As we can see, the convergence rate of our blindly adaptive algorithm is very fast, in fact, it converges as fast as the unblind QRD-RLS beamformer. Fig. 3 shows how good our estimation of interference matrix is. The vertical axis shows the percentage of the estimation error $\|\hat{\mathbf{R}}_{zz} - \hat{\mathbf{R}}_{uu}\|_F^2 / \|\hat{\mathbf{R}}_{uu}\|_F^2$, while the horizontal axis shows the interations. $\|\mathbf{R}\|_F$ denotes the Frobenius norm of the matrix **R**. We see the interference estimation converges at the same speed as the proposed beamformer's performance. This implies the correlation matrix's estimation is the critical process of the convergence, and it is reasonable to use power iteration once per symbol in the beamforming updation.

In this paper, we proposed an accurate estimation of the interference matrix for the DS/CDMA systems. Based on this interference's estimation, a simple and novel QRD-DMS algorithm is introduced to update the beamformer directly



Figure 3: The estimation error of the interference matrix versus iterations.

from the data sequences. The QRD-DMS method offers numerical stability and the local connections for VLSI implementations. We can further improve the hardware efficiency and the beamforming performance by multiuser multiplexing and coherent weight's combining. Since the interference estimation is valid even if the desired signals are not perfectly synchronized, the families of maximum SINR algorithms can be easily extended to the jointly space-time processing in DS/CDMA networks.

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