A SIGN-ERROR ALGORITHM FOR BLIND EQUALIZATION OF REAL SIGNALS

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ABSTRACT

The two criteria most commonly used in blind equalization are Sato's cost function and Godard's cost function. In this paper we analyze a sign-error cost function for real signals which gives an error term that can be viewed as the sign of either the Sato or the Godard error. We show that the conventional definition of equalizer convergence is not suitable for analyzing this cost function. A more realistic definition of convergence for low to medium SNR situations is presented and used to analyze this sign-error cost function. The performance of this cost function is evaluated via simulations and shown to have excellent performance as compared to the Godard cost function, with substantially less complexity.

1. INTRODUCTION

Blind equalization for digital communication systems subject to intersymbol interference (ISI) has been an active research area for a number of areas. One of the application areas is digital television (DTV) transmission where the signal is subject to multipath conditions caused by reflections off buildings, towers, etc. The equalizer in such a DTV receiver is fairly long due to the nature of the multipath channel. For instance, the prototype DTV receiver built by Zenith had a 256 tap equalizer. One of the main considerations when implementing such a long equalizer is the complexity of the blind adaptation algorithm. Conventional Godard [1] or Sato [2] errors involve a multiplication of the error signal with the data for each tap update which leads to increased area due to the large number of equalizer taps. Hence, a sign-error algorithm would greatly reduce the complexity.

In [3] a sign algorithm for QAM transmission is explored where it is shown by simulation that the sign version performs very close to the Godard's algorithm. However the cost function proposed there has the flaw that, for real signals, the derivative with respect to the equalizer taps is not equal to zero at convergence, as it is, for example, with the Godard and Sato cost functions.

In this paper, we analyze the sign-error cost function for real signals only and show that by defining "equalizer convergence" in a more realistic manner, one can choose the cost function such that the derivative with respect to the equalizer taps is zero. Section 2 briefly recaps the Godard and Sato cost functions and presents the sign-error cost function and shows how it can be viewed as the sign version of either of the above cost functions. Section 3 presents the new definition of convergence and derives the sign-error cost function that has a zero derivative at convergence. Section 4 presents simulation results comparing the performance of the sign-error cost function to the Godard cost function. Finally, conclusions are presented in Section 5.

2. COST FUNCTIONS FOR BLIND EQUALIZATION

In this paper we will consider only real signals. The case of complex signals will be mentioned in Section 5.

Let a_k be the transmitted real symbol stream. In the following, for clarity, we will assume that a_k is drawn from a 8-level PAM constellation. This does not affect the generality of the result. The received signal r_k after multipath distortion and added noise can be written as:

$$r_k = \sum_{i=0}^{L_h-1} h_i a_{k-i} + n_k \tag{1}$$

where h_i, $i = 0, \cdots L_h - 1$, is the multipath channel of length L_h and n_k is the additive noise, assumed to be white and gaussian. The received signal is processed by an equalizer, which in the most general case is a decision feedback equalizer (DFE). The output of the equalizer $\tilde{a}_k$ is an estimate of the transmitted signal and is expressed as:

$$\tilde{a}_k = \sum_{i=0}^{L_f-1} f_i r_{k+d_f-i} + \sum_{i=1}^{L_b} b_i \hat{a}_{k-i} \tag{2}$$

where f_i, $i = 0, \cdots L_f - 1$ are the forward equalizer taps, b_i, $i = 1, \cdots L_b$ are the feedback taps, d_f is the delay through the forward equalizer and $\hat{a}_k$ is the constellation point closest to $\tilde{a}_k$.

Since a training sequence is unavailable to a blind equalizer during the adaptation period, the mean squared error (MSE) at the equalizer output cannot be used in a stochastic gradient algorithm for adapting the equalizer. Instead, some kind of "cost function" that depends only on the equalizer output and the known statistics of the transmitted symbol stream is required in order to adapt the equalizer coefficients. Two widely used cost functions are the Godard (C_G) and Sato (C_S) cost functions defined as follows:

$$C_G = \mathrm{E}\left[\left||\tilde{a}_k|^2 - R_G\right|^2\right] \text{ and } C_S = \mathrm{E}\left[||\tilde{a}_k| - R_S|^2\right]$$

The above cost functions give the following error terms for use in the tap update algorithm:

$$e_G(k) = \tilde{a}_k \left(|\tilde{a}_k|^2 - R_G\right) \quad (3)$$

$$e_S(k) = \mathrm{sgn}(\tilde{a}_k)\left(|\tilde{a}_k| - R_S\right) \quad (4)$$

where the tap update algorithm is given by:

$$\underline{f}(k+1) = \underline{f}(k) + \mu e_X(k)\underline{r}(k)$$

$$\underline{b}(k+1) = \underline{b}(k) + \mu e_X(k)\underline{\tilde{a}}(k)$$

Here μ is the step size, $\underline{r}(k)$ and $\underline{\tilde{a}}(k)$ are the data in the DFE filters at time k and $e_X(k)$ is the error term (could be $e_G(k)$, $e_S(k)$ or something else).

The values of R_G and R_S are determined by setting the derivative of C_G and C_S respectively with respect to the filter taps equal to zero **when the equalizer has converged, i.e. when** $\tilde{a}_k = a_k$. This gives us:

$$R_G = \mathrm{E}\left[|a_k|^4\right]/\mathrm{E}\left[|a_k|^2\right], \quad R_S = \mathrm{E}\left[|a_k|^2\right]/\mathrm{E}\left[|a_k|\right]$$

For a 8-PAM signal with signal points $\pm1, \pm3, \pm5, \pm7$, the value of R_G is 37 and R_S is 5.25.

Now, let us consider the following sign-error cost function:

$$C_{SE} = \mathrm{E}\left[||\tilde{a}_k| - R_{SE}|\right] \quad (5)$$

This cost function has the following error term for use in the tap update algorithm above:

$$e_{SE}(k) = \mathrm{sgn}(\tilde{a}_k)\mathrm{sgn}(|\tilde{a}_k| - R_{SE}) \quad (6)$$

Comparing equations (3), (4) and (6) we see that if $R_{SE} = \sqrt{R_G}$ then $e_{SE}(k)$ is equal to $\mathrm{sgn}(e_G(k))$ and if $R_{SE} = R_S$ then $e_{SE}(k)$ is equal to $\mathrm{sgn}(e_S(k))$. Hence, depending on the choice of R_{SE}, the sign-error cost function can be viewed as the sign version of either the Godard or Sato cost functions. The implementation advantage of such a cost function is immediately apparent - the tap-update step does not have a multiplication between the error term and the data vector.

3. DERIVATION OF OPTIMUM R_{SE}

In order to determine the optimum value of R_{SE}, if we follow the same procedure as before for the Godard and Sato cost functions, i.e we set the derivative of C_{SE} with respect to the equalizer taps equal to zero when $\tilde{a}_k = a_k$, we get $\mathrm{E}[\mathrm{sgn}(a_k)a_k\mathrm{sgn}(|a_k| - R_{SE})]$ equal to zero. However, it can be easily shown that if a_k belongs to a 8-PAM constellation, this function does not become zero for any value of R_{SE}. The best that we can do is choose R_{SE} between 5 and 7 to get the minimum value of the derivative which is -0.5 in this case. This does not tell us what value of R_{SE} is the optimum choice.

Let us now change the definition of "equalizer convergence". Previously, it was assumed that at convergence we had $\tilde{a}_k = a_k$, i.e. convergence was defined as when the equalizer output is equal to the transmitted stream. Clearly, in an ISI channel with finite length equalizers and additive noise, this is not a valid assumption. Instead, let us assume that when the equalizer has converged, the output is expressed as:

$$\tilde{a}_k = a_k + N_k \quad (7)$$

where N_k is the sum of additive noise through the filter and left over intersymbol interference. In general, this noise is not gaussian, but the central limit theorem can be used to make the reasonable assumption that N_k is gaussian. Hence, let us assume that N_k is gaussian with mean zero and variance σ_N^2. With this assumption, let us define the derivative of the cost function C_{SE} at convergence, $d(R_{SE})$, as follows:

$$d(R_{SE}) = \mathrm{E}\left[a_k\mathrm{sgn}(\tilde{a}_k)\mathrm{sgn}(|\tilde{a}_k| - R_{SE})\right] \quad (8)$$

$$= \mathrm{E}\left[a_k\mathrm{sgn}(a_k + N_k)\mathrm{sgn}(|a_k + N_k| - R_{SE})\right] \quad (9)$$

$$= \sum_{a\in(\pm1,3,5,7)} a\ \mathrm{p}(a) \int_{-\infty}^{\infty} \mathrm{sgn}(a+N). \mathrm{sgn}(|a+N| - R_{SE}) \frac{e^{-\frac{N^2}{2\sigma_N^2}}}{\sigma_N\sqrt{2\pi}} dN \quad (10)$$

$$= \sum_{a\in(\pm1,3,5,7)} a\ \mathrm{p}(a) \left[-\int_{-\infty}^{-R_{SE}-a} \frac{e^{-\frac{N^2}{2\sigma_N^2}}}{\sigma_N\sqrt{2\pi}} dN + \int_{-R_{SE}-a}^{-a} \frac{e^{-\frac{N^2}{2\sigma_N^2}}}{\sigma_N\sqrt{2\pi}} dN - \int_{-a}^{R_{SE}-a} \frac{e^{-\frac{N^2}{2\sigma_N^2}}}{\sigma_N\sqrt{2\pi}} dN + \int_{R_{SE}-a}^{\infty} \frac{e^{-\frac{N^2}{2\sigma_N^2}}}{\sigma_N\sqrt{2\pi}} dN \right] \quad (11)$$

$$= \sum_{a\in(\pm1,3,5,7)} a\ \mathrm{p}(a) \left[1 + 2Q\left(\frac{R_{SE} - a}{\sigma_N}\right)\right.$$

SNR (dB)	R_{SE}	SNR (dB)	R_{SE}
16.0	5.5680	21.0	5.3458
17.0	5.5350	22.0	5.3071
18.0	5.4899	23.0	5.2733
19.0	5.4392	24.0	5.2434
20.0	5.3900	25.0	5.2169

Table 1: SNR Vs. R_{SE} for 8-PAM constellation

$$-2Q\left(\frac{-a}{\sigma_N}\right) - 2Q\left(\frac{R_{SE}+a}{\sigma_N}\right)\Bigg] \quad (12)$$

$$= \frac{1}{4}\sum_{a\in(1,3,5,7)} a\left[1+2Q\left(\frac{R_{SE}-a}{\sigma_N}\right)\right.$$

$$-2Q\left(\frac{-a}{\sigma_N}\right) - 2Q\left(\frac{R_{SE}+a}{\sigma_N}\right)\Bigg] \quad (13)$$

where it is assumed that all the points in the constellation are equally probable (i.e. p$(a) = 1/8$) and the Q function is defined as follows:

$$Q(a) = \frac{1}{\sqrt{2\pi}}\int_a^\infty e^{-\frac{x^2}{2}}dx \quad (14)$$

In order to obtain the optimum value of R_{SE}, we need to solve $d(R_{SE}) = 0$. This can be done numerically to give a value of R_{SE} that depends on σ_N^2, or the output SNR of the equalizer defined as $10\log(\sigma_a^2/\sigma_N^2)$. In this respect R_{SE} differs from R_G or R_S of the Godard and Sato cost functions, because those are independent of the SNR. However, it is fairly straightforward to calculate R_{SE} for various SNR's beforehand. Table 1 gives the optimum value of R_{SE} for various SNRs for the 8-PAM constellation. Figure 1 shows how $d(R_{SE})$ varies with R_{SE} and SNR. As expected, for large SNRs, e.g. SNR = 50 dB, $d(R_{SE}) = -0.5$ for R_{SE} between 5 and 7. For SNRs of interest, between 15 dB and 25 dB, the optimum value of R_{SE} is between 5.6 and 5.2. Hence, the optimum value of R_{SE} for 8-PAM is neither the Sato value of 5.25 or the square-root of the Godard value of 6.08, but some value depending on the SNR. The results obtained above can be carried over to any level PAM signal by using the appropriate signal set and probability distribution in the summation in equations (10) to (13).

4. SIMULATION RESULTS

Simulation result were carried out with a 8-PAM signal through the channel impulse response shown in Figure 2. The equalizer parameters used in the simulation were: $L_f = 20$, $d_f = 14$ and $L_b = 34$. The input SNR was 20 dB. Figure 3 shows the average MSE at the end of each "segment" where a segment is defined to be a block of 832 8-PAM symbols, averaged over 100 different simulation runs. For the Godard equalizer, μ was 10^{-7} and for the sign-error equalizer μ was 5x10^{-6}. The sign-error algorithm was run with 3 different values of R_{SE}: 5.25, 6.08 and 5.57, which are the values for the Sato error, square-root of the Godard error and the value of R_{SE} from Table 1 for an output SNR of approximately 16 dB, respectively. It is clear that the sign-error algorithm delivers the lowest MSE with $R_{SE} = 5.57$ as compared to the other 2 values. Also, for the same final MSE, the sign-error algorithm converges slower as compared to the Godard algorithm. However, one can use a larger μ during the initial phase to speed convergence. In Figure 4, the sign-error algorithm was run with $R_{SE} = 5.57$ and μ = 2x10^{-5} for the first 100 segments and μ = 5x10^{-6} thereafter. This step-size selection makes the sign-error algorithm very similar in convergence as compared to the Godard equalizer. Other step-size selection schemes mentioned in [3] could also be used to speed up convergence.

Since the choice of R_{SE} depends on the SNR at the output of the equalizer, a practical scheme has to be devised at the receiver in order to pick the optimum R_{SE}. One such scheme could be that Table 1 is precomputed and stored in memory for a given constellation. The equalizer starts with a default value which is chosen by some knowledge of the expected SNR range, and then updated by measuring SNR at the equalizer output and looking up the corresponding value of R_{SE} from Table 1.

5. CONCLUSIONS

We have analyzed a sign-error cost function for the blind equalization of real signals with a new definition of convergence for low to medium SNR conditions. This cost function was shown to perform very favorably as compared to the Godard cost function, but with greatly reduced complexity. The same derivation for the optimum value of R_{SE} (equations (8) to (13)) can be carried out for complex signals as well.

6. REFERENCES

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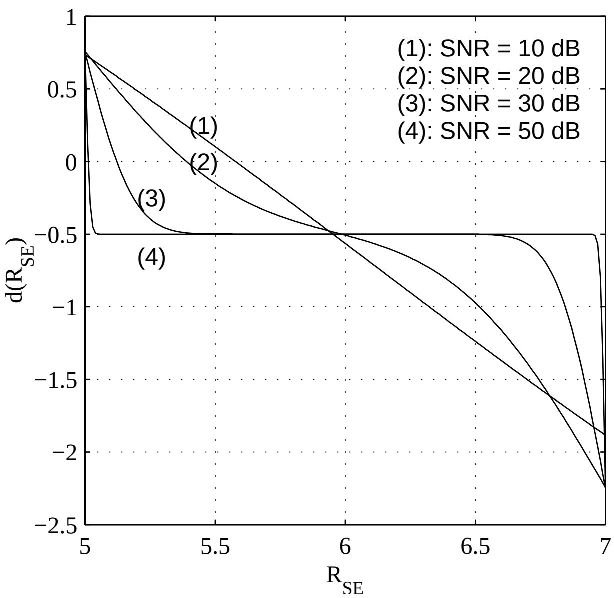

Figure 1: Derivative at convergence, $d(R_{SE})$, Vs. R_{SE} for different values of SNR.

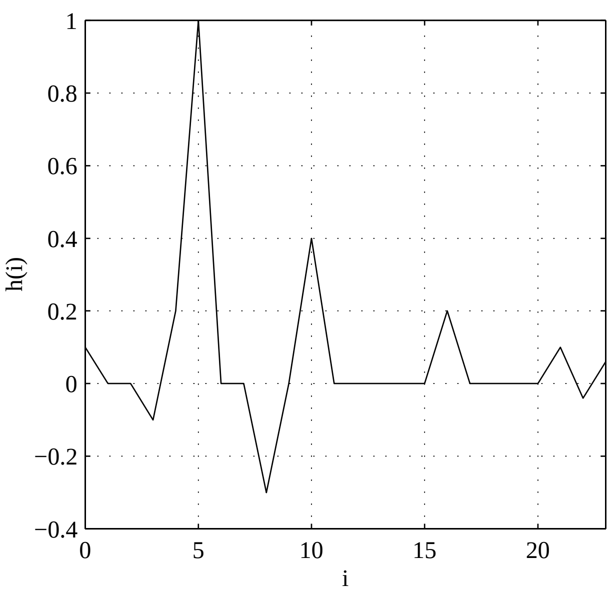

Figure 2: Impulse response of the simulated channel.

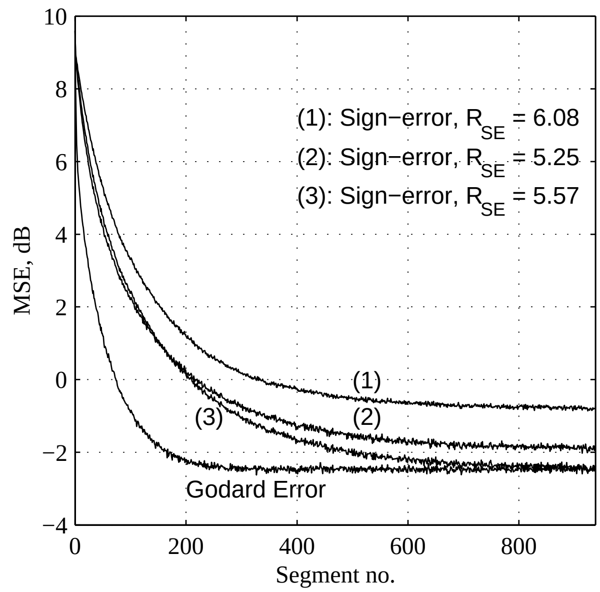

Figure 3: Convergence characteristics for the Godard and sign-error algorithms. For Godard's algorithm, $\mu = 10^{-7}$. For the sign-error algorithms, $\mu = 5\text{x}10^{-6}$.

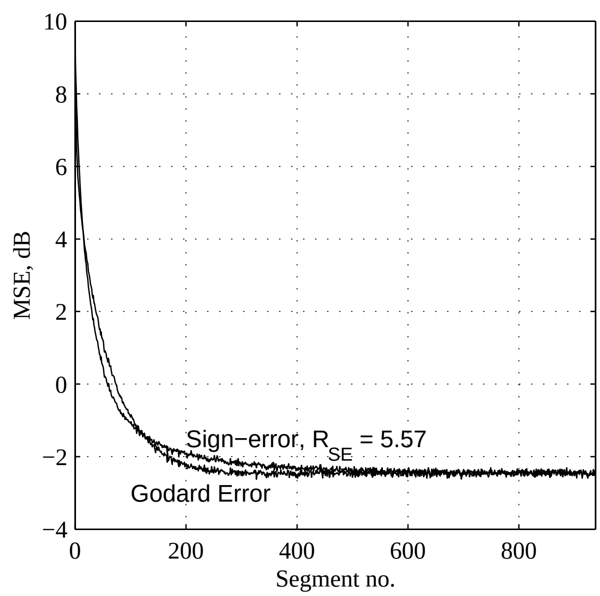

Figure 4: Improved convergence for the sign-error algorithm. $\mu = 2\text{x}10^{-5}$ for the first 100 segments, and $\mu = 5\text{x}10^{-6}$ thereafter. For Godard's algorithm, $\mu = 10^{-7}$ throughout.