BLIND IMAGE RESTORATION USING LOCAL BOUND CONSTRAINTS

Kaaren May, Tania Stathaki*, and Aggelos K. Katsaggelos***

*Signal Processing and Digital Systems Section
Imperial College of Science, Technology, and Medicine
London SW7 2BT, U.K.

**Department of Electrical and Computer Engineering
Northwestern University
Evanston, IL 60208, U.S.A.

ABSTRACT

A new method of incorporating local image characteristics into blind image restoration is proposed. The local variance of the degraded image is used as a measure of spatial activity, from which individual pixel bounds are determined. A parameter defined by the user controls the degree of smoothing. The local bounds define the solution more precisely than smoothness constraints on the image (including those that are spatially-adaptive), reducing the number of possible solutions and leading to a faster rate of convergence. Experimental results demonstrate the potential of this method as an alternative/supplement to smoothing constraints in blind image restoration.

1. INTRODUCTION

During image formation and recording, blurring may occur due to relative motion between the object and camera, wrong focus, and atmospheric turbulence. Noise originating in the formation process, the transmission medium, or the recording process may further degrade the image.

Image degradation can be modelled as

$$y = Dx + n, \tag{1}$$

where x, y, and n represent the lexicographically-ordered original and degraded images, and additive noise. The matrix D here represents a space-invariant linear distortion.

The goal of blind image restoration is to simultaneously estimate the blur and the original image, based on partial knowledge of their characteristics. The primary difficulty is lack of sufficient information, as the problem admits a possibly infinite number of solutions in the absence of sufficient constraints on the blur and image. The question is how to develop a set of constraints which adequately characterize these variables.

In classical image restoration, where the blur is explicitly known, smoothness constraints on the image have been used to regularize the ill-posed restoration problem [1]. Piecewise smoothness of both the image and the blur has been applied to blind restoration as a means of more accurately specifying the set of admissible solutions [2]. However, convergence to local minima may still result [2].

Recently, it has been shown that the introduction of spatially-adaptive bounds can improve the quality of the restoration in the non-blind case [3]. Based on these results, a similar set of constraints has been developed for blind restoration, which further define the problem and lead to increased convergence rates. Overall, the quality of the restoration, measured as the improvement in signal-to-noise ratio, is much improved over the solution obtained through the use of conventional smoothing operators.

The organization of this paper is as follows. In Section 2, a mathematical formulation for blind image restoration is given. The procedure for determining the pixel bounds is then presented in Section 3. Section 4 describes the implementation of the algorithm, which is used to generate the experimental results in Section 5. The results are discussed, and areas for further research are proposed, in Section 6.

2. BLIND IMAGE RESTORATION

Blind deconvolution can be formulated as minimization of the following cost function:

$$J(\hat{x}, \hat{d}) = ||y - \hat{D}\hat{x}||^2 + \alpha ||C\hat{x}||^2, \tag{2}$$

subject to the constraints:

$$\begin{cases} \hat{d}(i,j) \geq 0, & i,j \in S_D \\ \hat{d}(i,j) = 0, & \text{otherwise,} \end{cases} \tag{3}$$

$$\sum_{i,j \in S_D} \hat{d}(i,j) = 1, \tag{4}$$

and

$$\begin{cases} \hat{x}(m,n) \geq 0, & m,n \in S_X \\ \hat{x}(m,n) = 0, & \text{otherwise.} \end{cases} \tag{5}$$

In equation (2), $\hat{x}$ and $\hat{d}$ are the image and point-spread function (PSF) estimates, C is a high-pass operator, and the regularization parameter α controls the trade-off between fidelity to the data and smoothness of the solution. It is assumed that the PSF and image supports, S_D and S_X, are known. However, the assumption that S_D is known exactly can be relaxed, and the algorithm is then implemented by beginning with a conservatively large estimate of S_D, and then gradually pruning the ROS during successive updates of the PSF [1, Chapter 6]. Knowledge of the image ROS eliminates the trivial solution $\hat{x} = y$ and $\hat{d} = \delta(i,j)$; an alternative is to impose a piecewise smoothness constraint on the PSF [2]. The positivity constraints on the PSF and image stem from the assumption that the original and degraded images are formed from radiant energy, which is unsigned. Conservation of energy during the blurring process leads to constraint (4).

The above optimization problem is often solved by using gradient-based methods to minimize alternately with respect to the PSF and the image. Constraints (3)–(5) are implemented by projecting the current solution onto the specified bounds, and normalising $\hat{d}$, after a specified number of iterations. However, since the problem is inherently nonlinear, convergence to local minima can occur. This is often dealt with by trying to obtain good initial estimates of the image and the PSF. Although some authors have examined the use of nonlinear techniques such as simulated annealing [4], these methods tend to be very time-consuming. An alternative is to develop more precise bounds which guide the steepest descent algorithms to the neighbourhood of a good solution.

3. DEVELOPMENT OF LOCAL CONSTRAINTS

The bounds for the image are defined as:

$$b_l(m,n) = \begin{cases} \max\left(\overline{x}(m,n) - \beta\dfrac{\sigma^2(m,n)}{\sigma^2_{\max}}, 0\right), & m,n \in S_X \\ 0, & \text{otherwise} \end{cases} \tag{6}$$

$$b_u(m,n) = \begin{cases} \overline{x}(m,n) + \beta\dfrac{\sigma^2(m,n)}{\sigma^2_{\max}}, & m,n \in S_X \\ 0, & \text{otherwise,} \end{cases} \tag{7}$$

where $\overline{x}(m,n)$ and $\sigma^2(m,n)$ are the local mean and variance at pixel m,n of the degraded image measured over a 3×3 window, $\sigma^2_{\max}$ is the maximum local variance over the entire image, and β is a constant controlling the tightness of the bounds.

The projection operator expressing local smoothness is then defined as:

$$P_X(\hat{x}(m,n)) = \begin{cases} b_l(m,n), & \hat{x}(m,n) < b_l(m,n) \\ b_u(m,n), & \hat{x}(m,n) > b_u(m,n) \\ \hat{x}(m,n), & \text{otherwise.} \end{cases} \tag{8}$$

Similarly, for $\hat{d}$ we write

$$P_D(\hat{d}(i,j)) = \begin{cases} \hat{d}(i,j), & \hat{d}(i,j) \geq 0, \quad i,j \in S_D \\ 0, & \text{otherwise.} \end{cases} \tag{9}$$

In the absence of noise, the original and degraded images only differ in the vicinity of the edges, where the variance is high. Therefore, the bounds for these pixels should be relatively large. In uniform areas of low variance, the original and degraded images are very close, and tighter bounds can be used, resulting in more smoothing. This is in agreement with the noise masking property in areas of high spatial activity of the human visual system [3], [5].

4. IMPLEMENTATION OF THE BLIND RESTORATION ALGORITHM

The ideas presented in Section 3 are incorporated into the following algorithm.

1. Determine the bounds $b_l(m,n)$ and $b_u(m,n)$ from the degraded image. Set the initial image estimate to $\hat{x}_0 = P_X y$. Initialize the iteration numbers $l, k = 0$.
2. Minimize with respect to the PSF:
   - Solve for the PSF parameters.
   - Set $\hat{d} = P_D \hat{d}$.
   - Normalize the solution so that $\sum \hat{d}(i,j) = 1$.
3. Minimize with respect to the image:

   $$\hat{x}_{k+1} = P_X(\hat{x}_k + \hat{D}^T y - (\hat{D}^T\hat{D} + \alpha C^T C)\hat{x}_k).$$

   Set $k = k+1$. If

   $$\frac{||\hat{x}_{k+1} - \hat{x}_k||^2}{||\hat{x}_k||^2} \leq 10^{-6},$$

   then repeat step 3.

4. Increment l. If

$$\frac{||\hat{y}_{l+1} - \hat{y}_l||^2}{||y||^2} \leq 10^{-6},$$

then go to step 2.

5. EXPERIMENTAL RESULTS

A number of experiments have been carried out with various images, blurs, and noise levels. The original 256×256 "Lena" image, superimposed on a black background, was degraded by a 5×5 blur and by additive white Gaussian noise. The results for both a Gaussian and a uniform blur in noise levels of 20 and 30 dB have been reported in Tables 1 and 2. During the restoration, the PSF was assumed to be separable and symmetric, in which case the Levenberg-Marquardt method was used to perform the minimization in step 2. In all simulations, the regularization parameter α was set to 0, in order to illustrate the effect of the pixel bounds alone. As a measure of the quality of the restoration, the improvement in SNR (dB) was used:

$$\Delta_{SNR} = 10 \log_{10} \frac{||y - x||^2}{||\hat{x} - x||^2}.$$

The quality of the PSF estimate was measured by:

$$\epsilon_d = \frac{||d - \hat{d}||}{||d||}.$$

In the tables, the number of outer iterations l is given, along with k, the total number of updates of the image estimate before convergence of the algorithm.

Tables 1 and 2 list the results for various values of the parameter β. They are compared to the case where no local constraints are used, indicated by N/A in the tables.

From Tables 1 and 2, it can be seen that, as with the regularization parameter α of equation (2), the quality of the restoration is highly dependent on the parameter β. Generally, the more severe the distortion, the smaller β should be, although a general method of estimating β has not yet been determined. It can also be seen that as the bounds are tightened, the number of iterations needed to reach a solution is decreased.

Table 1: 5 x 5 Gaussian PSF ($\sigma^2 = 1$)

30 dB SNR					
β	l	k	Δ_{SNR} (dB)	ϵ_d	$\|\|y - \hat{y}\|\|^2$
1.0	3	17	2.6415	0.3385	4.0799
2.0	4	23	3.2813	0.2787	2.7522
4.0	5	26	3.6459	0.2505	2.1687
8.0	5	28	3.4829	0.2590	1.8799
N/A	5	31	2.7665	0.2848	1.5562
20 dB SNR					
β	l	k	Δ_{SNR} (dB)	ϵ_d	$\|\|y - \hat{y}\|\|^2$
0.25	4	18	1.6229	0.3195	31.4631
0.5	6	27	2.0120	0.4288	24.3703
1.0	8	26	1.4422	0.9115	20.2720
N/A	9	49	-2.2229	0.9814	4.0059

Table 2: 5 x 5 uniform PSF

30 dB SNR					
β	l	k	Δ_{SNR} (dB)	ϵ_d	$\|\|y - \hat{y}\|\|^2$
0.25	4	25	2.0723	0.5566	11.9835
0.5	6	49	2.4902	0.3039	6.7101
1.0	6	46	1.9116	0.3227	4.7029
2.0	6	45	1.4944	0.3513	4.2663
N/A	5	47	0.5773	0.4403	3.9930
20 dB SNR					
β	l	k	Δ_{SNR} (dB)	ϵ_d	$\|\|y - \hat{y}\|\|^2$
0.25	7	38	2.1884	0.6076	31.4628
0.5	8	39	1.9079	1.3778	26.6830
1.0	8	43	0.9204	1.8636	20.2869
N/A	8	97	-3.6749	1.8684	8.3157

In Figures 3 and 4, there is noticeable oversmoothing in areas of high detail, which may be because the bounds were taken from the degraded image. Therefore, it would be worthwhile to investigate how the use of iterative bounds affects the performance of the algorithm.

It can, however, be seen that the introduction of local bounds results in a significant increase in the accuracy of the PSF and image estimates, particularly for severe degradations. In combination with conventional smoothing operators, the bounds can help to further define the problem and to improve the convergence properties of existing algorithms.

6. CONCLUSIONS

In this paper, a method of implementing local pixel bounds in blind image restoration was proposed. These bounds were determined from the local mean and variance of the degraded image. The bounds can be used to further improve the performance of blind restoration algorithms which incorporate smoothness constraints on the image and/or blur. Areas for further research include a general method of determining the parameter β, and the use of iterative bounds.

Figure 1: Blind restoration of image degraded by Gaussian PSF, 30 dB SNR, $\beta = 4.0$, $\Delta SNR = 3.6459$

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Figure 2: Blind restoration of image degraded by Gaussian PSF, 20 dB SNR, $\beta = 0.5$, $\Delta SNR = 2.0120$

Figure 3: Blind restoration of image degraded by uniform PSF, 30 dB SNR, $\beta = 0.5$, $\Delta SNR = 2.4902$

Figure 4: Blind restoration of image degraded by uniform PSF, 20 dB SNR, $\beta = 0.25$, $\Delta SNR = 2.1884$