BLIND FREQUENCY OFFSET AND DELAY ESTIMATION OF LINEARLY MODULATED SIGNALS USING SECOND ORDER CYCLIC STATISTICS

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ABSTRACT

A blind (non-data aided), open loop, joint frequency offset and delay estimation algorithm for a linearly modulated signal in additive stationary noise is developed by exploiting the cyclostationarity of the signal. By considering the sample cyclic autocorrelation function of the received signal and the probability distribution of the estimation error, a general linear model representation of the problem is obtained from which the parameters are estimated using a Bayesian framework. The algorithm is then extended to a multiple signals of interest scenario. The algorithm is simulated for both single and multiple BPSK signals.

1. INTRODUCTION

Frequency offset and delay estimation is typically required in the reception of digital communication signals. The frequency of the incoming signal can differ from that of the local oscillator frequency due to propagation, Doppler effects and mismatch between transmitter and receiver oscillators; and the channel delays the transmitted signal. Data aided and non-data aided and open-loop and closed-loop solutions have been proposed for such an estimation task [4]. In this paper, we exploit the cyclostationary characteristic of communication signals to design a non-data aided, open loop, joint frequency offset-delay estimation algorithm for linearly modulated signals corrupted by additive stationary noise.

Second order cyclic statistics were used in [5] to estimate the frequency and delay of a known signal. A two-dimensional cost function based on the mean squared error between the expected and estimated cyclic autocorrelation function was minimised to obtain the estimates. Recently in [2], a cyclic statistics based frequency-delay estimation algorithm was proposed for flat fading channels. This algorithm was an averaged estimator, requiring the calculation of cyclic autocorrelations at two cyclic frequencies and did not consider the estimation error. It will give a good estimate in a high SNR scenario and with a long symbol sequence. Second order [6] and fourth order [3] nonlinearities have been also used for joint frequency-delay estimation by generating periodic components containing the synchronisation parameters. Our algorithm is based on the cyclic autocorrelation vector at one cyclic frequency and considers the probability distribution of the estimation error, leading to a general linear model representation from which we obtain frequency and delay estimates using a Bayesian approach. Also, we extend the algorithm to a multiple signal scenario, an issue not addressed in the above mentioned papers.

Consider a received signal:

$$r(t) = As(t-\tau)e^{j2\pi f_o t} + v(t) \tag{1}$$

where A is the unknown constant amplitude of the signal, τ is the unknown delay which we assume is less than a symbol period ($\tau < T$), f_o is the unknown frequency offset, $v(t)$ is additive stationary noise and $s(t)$ is given by:

$$s(t) = \sum_k b(k)p(t-kT) \tag{2}$$

with $b(k)$ being the information symbols and $p(t)$ being the signaling pulse of duration T, which has a finite second order cyclic moment. (2) is a linear periodically time varying (LPTV) system and hence $s(t)$ is second order cyclostationary.

Our aim is to estimate the frequency offset f_o and the delay τ from the received signal without the aid of the data symbols.

2. ESTIMATION ALGORITHM

We oversample the received signal (1), i.e., $T_s = T/P$, giving the discrete time data:

$$r(n) = As(n-\epsilon)e^{j2\pi f_o n} + v(n) \tag{3}$$

To obtain the cyclic autocorrelation we need to know the set of cyclic frequencies $\{\alpha\}$. The set of cyclic frequencies can be obtained *a priori* using the statistical test based on the Neyman-Pearson criteria as proposed in [1]. In the case of linearly modulated signals the cyclic frequencies occur at multiples of the symbol rate $1/T$. We also make the following assumptions:

(A1) The symbols $b(k)$ are zero-mean, stationary, uncorrelated and uniformly distributed over a finite alphabet.

(A2) The additive noise $v(n)$ is a WSS process.

(A3) The joint moments of $r(n)$ are absolutely summable.

The cyclic autocorrelation of $r(n)$ at the m^{th} lag and at cyclic frequency $\alpha = k/T; k = 0, 1, ..$ can be written as:

$$R_r^\alpha(m) = A^2 e^{-j2\pi\alpha\epsilon} R_s^\alpha(m) e^{-j2\pi f_o m} + R_v^\alpha(m) \quad (4)$$

where $R_s^\alpha(m)$ is the cyclic autocorrelation of $s(n)$ and $R_v^\alpha(m)$ is the cyclic autocorrelation of $v(n)$ at cyclic frequency α.

Considering $\alpha = 1/T$, since $R_v^\alpha(m) = 0$ for $\alpha \neq 0$, we can write:

$$R_r^\alpha(m) = A^2 e^{-j2\pi\alpha\epsilon} R_s^\alpha(m) e^{-j2\pi f_o m} \quad (5)$$

In practice, we would only have a finite number of samples and $R_r^\alpha(m)$ has to be estimated from these samples:

$$\hat{R}_r^\alpha(m) = \frac{1}{N}\sum_{n=0}^{N-1} r(n) r^*(n+m) e^{-j2\pi\alpha n} \quad (6)$$

It can be shown [1] that under assumption (A3), $\hat{R}_r^\alpha(m)$ converges in the mean square sense:

$$R_r^\alpha(m) = \lim_{N\to\infty} E\{\hat{R}_r^\alpha(m)\} \quad (7)$$

and that $[\hat{R}_r^\alpha(m) - R_r^\alpha(m)]$ is asymptotically complex normal.

Hence for $\alpha = 1/T$ we can write:

$$\hat{\mathbf{R}}_\mathbf{r}^\alpha(\mathbf{m}) = A^2 e^{-j2\pi\alpha\epsilon} \mathbf{R}_\mathbf{s}^\alpha(\mathbf{m}) \cdot e^{-j2\pi f_o \mathbf{m}} + \mathbf{e}(\mathbf{m});$$
$$\mathbf{m} = [0, 1, .., M]' \quad (8)$$

where $\cdot$ denotes component-wise multiplication, $\mathbf{R}_\mathbf{s}^\alpha(\mathbf{m})$ is a vector of calculated cyclic autocorrelation of $\mathbf{s(n)}$, $\hat{\mathbf{R}}_\mathbf{r}^\alpha(\mathbf{m})$ is a vector of cyclic autocorrelation estimates from the samples of $\mathbf{r(n)}$ and $\mathbf{e(m)}$ is the estimation error. M is the maximum lag for which $\mathbf{R}_\mathbf{s}^\alpha(\mathbf{m})$ has a non-zero value. M should be large enough to *shape* the likelihood function. $\mathbf{R}_\mathbf{s}^\alpha(\mathbf{m})$ can be calculated beforehand knowing the pulse function $p(t)$ and stored in memory. $\hat{\mathbf{R}}_\mathbf{r}^\alpha(\mathbf{m})$ is calculated on-line using frequency shifted versions of the received signal.

Separating the real and imaginary components:

$$\begin{aligned}\begin{pmatrix} Re\{\mathbf{y}\} \\ Im\{\mathbf{y}\}\end{pmatrix} &= \begin{pmatrix} Re\{\mathbf{g}\} & Im\{\mathbf{g}\} \\ Im\{\mathbf{g}\} & -Re\{\mathbf{g}\}\end{pmatrix}\begin{pmatrix} Re\{h\} \\ Im\{h\}\end{pmatrix} \quad (9) \\ &+ \begin{pmatrix} Re\{\mathbf{e}\} \\ Im\{\mathbf{e}\}\end{pmatrix} \\ \mathbf{d} &= \mathbf{Gb} + \mathbf{n}\end{aligned}$$

where

$$h = A^2 e^{-j2\pi\alpha\epsilon} \quad (10)$$

$$\mathbf{y} = [R_r^\alpha(0), R_r^\alpha(1), .., R_r^\alpha(M)]' \quad (11)$$

$$\mathbf{g} = \begin{pmatrix} R_s^\alpha(0) \\ R_s^\alpha(1) e^{-j2\pi f_o} \\ .. \\ .. \\ R_s^\alpha(M) e^{-j2\pi f_o M}\end{pmatrix} \quad (12)$$

$$\mathbf{e} = [e(0), e(1), .., e(M)]' \quad (13)$$

The likelihood function is:

$$p(\mathbf{d}|\mathbf{b}, f_o, \mathbf{C}, I) =$$
$$(2\pi)^{-N/2} |\mathbf{C}^{-1}| \exp[-(\mathbf{d} - \mathbf{Gb})' \mathbf{C}^{-1} (\mathbf{d} - \mathbf{Gb})] \quad (14)$$

where $\mathbf{C}$ is the covariance matrix.

We assume $\mathbf{C} = \sigma^2 \mathbf{I}$ for computational simplicity and thus the likelihood can be written as:

$$p(\mathbf{d}|\mathbf{b}, f_o, \sigma, I) =$$
$$(2\pi\sigma^2)^{-N/2} \exp[-\frac{(\mathbf{d} - \mathbf{Gb})'(\mathbf{d} - \mathbf{Gb})}{2\sigma^2}] \quad (15)$$

Using Bayes' theorem with uniform priors for f_o and $\mathbf{b}$ and Jeffrey's prior for σ, the posterior probability density is given by:

$$p(\mathbf{b}, f_o, \sigma|\mathbf{d}, I) =$$
$$(2\pi\sigma^2)^{-N/2} \exp[-\frac{(\mathbf{d} - \mathbf{Gb})'(\mathbf{d} - \mathbf{Gb})}{2\sigma^2}] \frac{1}{\sigma} \quad (16)$$

Since (16) is a quadratic function in the linear parameter $\mathbf{b}$, maximising the exponential with respect to $\mathbf{b}$ gives the LSE estimate for $\mathbf{b}$ as:

$$\hat{\mathbf{b}} = \begin{pmatrix} A^2\cos(2\pi\alpha\epsilon) \\ A^2\sin(2\pi\alpha\epsilon)\end{pmatrix} = (\mathbf{G}'\mathbf{G})^{-1}\mathbf{G}'\mathbf{d} \quad (17)$$

The estimate for the delay is obtained as:

$$\hat{\epsilon} = \frac{\arctan(\frac{b(2)}{b(1)})}{2\pi\alpha} \quad (18)$$

where arctan is the four quadrant tangent. Substituting (17) in (16) and integrating out the nuisance parameters gives:

$$p(f_o|\mathbf{d}, I) =$$
$$\frac{(\mathbf{d}'\mathbf{d} - \mathbf{d}'\mathbf{G}(\mathbf{G}'\mathbf{G})^{-1}\mathbf{G}'\mathbf{d})^{-(M-2)/2}}{\sqrt{\det(\mathbf{G}'\mathbf{G})}} \quad (19)$$

The Maximum A Posteriori (MAP) estimate for f_o is obtained by locating the maximum of the posterior pdf (19) using an one-dimensional optimisation scheme.

$$\hat{f}_o = \max_{f_o}[p(f_o|\mathbf{d}, I)] \quad (20)$$

3. MULTIPLE SIGNALS

The algorithm presented in the previous section is insensitive to the presence of any interfering signals provided the interfering signals do not have the same cyclic frequency as the signal of interest (i.e., in the case of multiple linearly modulated signals this corresponds to different pulse rates) or if they are uncorrelated with respect to the signal of interest. However, if we are interested in estimating the frequencies and delays of multiple signals of distinct pulse rates, then we can run the above algorithm in parallel with α corresponding to the set of distinct cyclic frequencies. As far as each signal is concerned the other signals are interferers which do not affect the cyclic autocorrelation function. Note that this interference suppression property is not available in the conventional correlation domain.

In this section we consider a more interesting scenario, where we have multiple signals with the same pulse rate, i.e., the same cyclic frequency.

Consider the received signal model:

$$r(t) = \sum_{l=1}^{L} A_l s(t-\tau_l) e^{j2\pi f_{ol} t} + v(t) \tag{21}$$

where L is the number of signals known *a-priori*, A_l, τ_l, f_{ol} are respectively the unknown constant amplitude, unknown delay and unknown frequency of the l^{th} signal, $v(t)$ is additive stationary noise and $s(t)$ is a second order cyclostationary signal.

We oversample the received signal (21), i.e., $T_s = T/P$, giving the discrete time data:

$$r(n) = \sum_{l=1}^{L} A_l s(n-\epsilon_l) e^{j2\pi f_{ol} n} + v(n) \tag{22}$$

We assume that (A4) the frequency offsets $f_{ol} < 1/T$, which is a realistic assumption. This allows us to write the cyclic autocorrelation function of $r(n)$ at the m^{th} lag and at cyclic frequency $\alpha = 1/T$ as:

$$\hat{\mathbf{R}}_{\mathbf{r}}^{\alpha}(\mathbf{m}) = \sum_{l=1}^{L} A_l^2 e^{-j2\pi\alpha\epsilon_l} \mathbf{R}_{\mathbf{s}}^{\alpha}(\mathbf{m}) \cdot e^{-j2\pi f_{ol}\mathbf{m}} + \mathbf{e}(\mathbf{m}); \quad m = 0, 1, .., M \tag{23}$$

where $\mathbf{R}_{\mathbf{s}}^{\alpha}(\mathbf{m})$ is the known cyclic autocorrelation of $\mathbf{s}(\mathbf{n})$, $\hat{\mathbf{R}}_{\mathbf{r}}^{\alpha}(\mathbf{m})$ is a vector of cyclic autocorrelation estimates from the received signal $\mathbf{r}(\mathbf{n})$ and $\mathbf{e}(\mathbf{m})$ is the estimation error. The cross correlation terms between the signals are zero under assumption (A4):

$$\mathbf{R}_{\mathbf{s}}{}^{(\alpha+f_{oa}-f_{ob})}(\mathbf{m}) = \mathbf{0}; \quad a \neq b \tag{24}$$

Strictly we only require:

$$f_{oa} - f_{ob} \neq k/T; \quad a \neq b, \quad k = 0, 1, 2, .. \tag{25}$$

Separating the real and imaginary components we obtain a general linear model representation $\mathbf{d} = \mathbf{G}\mathbf{b} + \mathbf{n}$, where $\mathbf{b}$ is a vector of length $2L$ and $\mathbf{G}$ is a $M \times 2L$ matrix. Using a Bayesian approach as before, the estimate for the delay of the l^{th} signal is obtained as:

$$\hat{\epsilon}_l = \frac{\arctan\left(\frac{b(2l)}{b(2l-1)}\right)}{2\pi\alpha} \tag{26}$$

where arctan is the four quadrant tangent.

The frequency estimates are obtained by maximising the posterior density:

$$p(f_o|\mathbf{d}, I) = \frac{\left(\mathbf{d'd} - \mathbf{d'G}(\mathbf{G'G})^{-1}\mathbf{G'd}\right)^{-(M-2L)/2}}{\sqrt{\det(\mathbf{G'G})}} \tag{27}$$

This is a L-dimensional function and the Maximum-A-Posteriori estimate of the L frequencies is given by the global maximum of the posterior density. Performing a large multi-dimensional search is very costly and we need to resort to computationally efficient search schemes.

The special structure of the posterior density (27) can be exploited to split up the L-dimensional search into a series of one-dimensional searches. For the case of two signals, we observe that the posterior density has a ridge at $f_1 = f_{o1}$ running parallel to the f_2 axis and a ridge at $f_2 = f_{o2}$ running parallel to the f_1 axis. Thus the maximum can be located by first searching along one axis, $f_1 = 0$ or $f_2 = 0$, to find the ridge and then a search along the ridge to find the maximum. This is similar to the Fast Maximum Likelihood method in [7]. When we have more than two signals an iterative algorithm as in [7] can be used build up the model matrix.

4. SIMULATIONS

We simulated our algorithms, both single signal and multiple signal cases, using BPSK signals with uncorrelated, equally probable ± 1 symbols. The symbols were pulse shaped with a raised cosine pulse ($\alpha = 0.5$), truncated to $8P+1$ taps and delayed by $4P$. The transmitted sequence was corrupted with additive white Gaussian noise (AWGN). The received sequence was oversampled, $P = 10$, and the frequency and delay were estimated. All the results were averaged over 100 trials.

Case 1 : To compare our algorithm for a single signal scenario with that of [2], a BPSK sequence of 200 symbols was generated and pulse shaped and corrupted with AWGN of varying variance. The frequency offset was fixed at $f_o = 0.1/T$ and the delay at $\tau = 0.4P$. The mean squared error (MSE) of the estimates, normalised to the symbol rate are shown in fig. 1. '$\circ$' marks for our algorithm and '$\triangle$' for that of [2]. We observe a significant improvement in the performance.

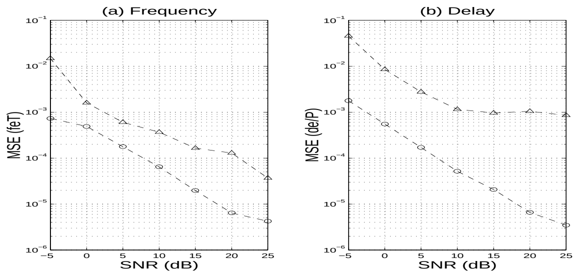

Figure 1: Estimator variance for a single BPSK signal : ours vs [2]

Case 2 : Three BPSK signals, 400 symbols long, with the same pulse rate and with frequency offsets $f_{o1} = 0.2/T$, $f_{o2} = 0.4/T$, $f_{o3} = 0.7/T$ and delays $\tau_1 = 0.3P$, $\tau_2 = 0.5P$, $\tau_3 = 0.7P$ were corrupted with AWGN of increasing variance, and estimates of frequency offsets and delays were obtained using the algorithm that was extended to a multiple signal scenario. The signal amplitudes were 0 dB each. The results are shown in fig. 2.

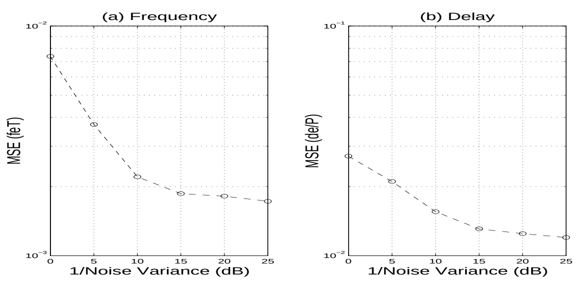

Figure 2: Average estimator variance for three BPSK signals

Case 3 : Two BPSK signals were corrupted with AWGN. The frequency offset of one signal was varied, $f_{o2} = 0.2/T - 0.9/T$, while fixing the other parameters at $f_{o1} = 0.1/T$, $\tau_1 = 0.4P$, $\tau_2 = 0.8P$. Fig.3 shows the estimates for a noise variance of 0.01 with a symbol length of 400.

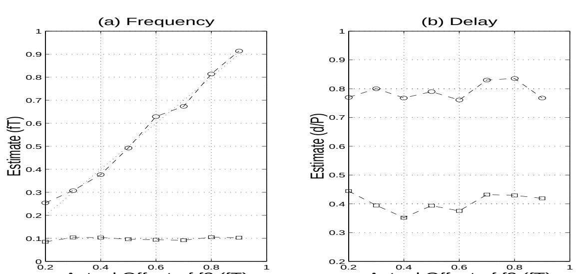

Figure 3: Estimates for two BPSK signals as the frequency separation varies

5. CONCLUSION

We have presented a blind, open-loop, joint frequency offset-delay estimation algorithm for linearly modulated signals in additive stationary noise based on second order cyclic statistics. This algorithm does not depend on the colour or the distribution of the additive noise as long as the noise is a WSS process. By considering the probability distribution of the estimation error, a general linear model representation of the problem is obtained from which consistent estimates are obtained using a Bayesian approach. For a single signal scenario, the delay is estimated as a LSE estimate and the frequency offset via an one dimensional optimisation. For a multiple signal scenario, under a realistic assumption about the frequency offsets, the use of cyclic statistics allow us to eliminate the cross correlation between the signals and the delays are obtained as LSE estimates and the frequency offsets are given by a multi-dimensional optimisation. By utilising the special structure of the posterior probability distribution, the multidimensional search can be reduced to a sequence of one dimensional searches.

6. REFERENCES

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