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ABSTRACT

DC coefficient restoration scheme is a technique which can be used to increase the compression ability of transform image coding by not transmitting the DC coefficients but estimating them from the transmitted AC component. In the last decade, the minimum edge difference criterion is used in the scheme. However the criterion fails at the locations where the discontinuities are along the block boundaries and therefore results in observable blocking effect around these locations or higher bit rate. In this paper, we propose a new criterion using the maximum a posterior (MAP) estimation technique which preserves the discontinuities during the DC coefficients restoration and solves the blocking effects in the restored images.

1. INTRODUCTION

The idea of DC coefficient restoration in transform image coding has been proposed and studied in the last decade. The idea was first proposed by Cham and Clarke in [1] and then was used in the DC coefficients prediction in [2]. Recently it is improved by using the global estimation scheme in company with block selection [3]. Promising results can be found in [3, 4, 5]. Efficient numerical algorithm has been investigated in [6].

Until then, the DC coefficient restoration scheme is based on the criterion called minimum edge difference (MED) which has been proposed in [1]. The rationale of the MED criterion is that the variations of the pixel values along the block boundaries in most of the natural images are smooth. Thus it gives a degree of freedom to estimate the DC coefficients from the AC component of the blocks. However, it is obvious that the MED criterion cannot be satisfied at all boundaries. It usually fails at the boundaries where there are edges. The block selection scheme proposed in [3] is a remedy method which uses additional bits to represent the locations and the actual values of the DC coefficients of the blocks adjacent to these boundaries. Thus if there is another improved criterion such that it can resolve the failure at these locations, then the additional bits required by the remedy scheme can be reduced.

In this paper, we propose another criterion using the maximum a posterior (MAP) estimation technique. The Huber minimax function is used in the MAP estimation which preserves the discontinuity of the pixel values across the high activity areas and the edge locations. Moreover, we found that the MED criterion is a limited case of the newly proposed criterion.

In the next section, we shall give a short review about the DC coefficient restoration scheme and the MED criterion. The third

section will give the details of the newly proposed criterion. Simulation results will be presented in section four. The conclusions are given at the end of the paper.

2. DC COEFFICIENT RESTORATION AND THE MED CRITERION

Suppose an original image having the size $N_1n \times N_2n$ is divided into $N_1 \times N_2$ blocks each block having $n \times n$ pixels. Let $\mathbf{X}_{i,j}$ be the $n \times n$ square matrix representing the pixel values of the (i, j)th block where $1 \le i \le N_1$ and $1 \le j \le N_2$. In transform coding, each block is transformed into the transform domain $\mathbf{Y}_{i,j}$ by the two-dimensional separable unitary transform as

$$\mathbf{Y}_{i,j} = \mathbf{T}\mathbf{X}_{i,j}\mathbf{T}^t,\tag{1}$$

where **T** is the $n \times n$ unitary transform matrix such as the DCT. Let $a_{i,j}$ be the DC coefficient of the (i, j)th block, which can be expressed as

$$a_{i,j} = \frac{1}{n} \sum_{p=1}^{n} \sum_{q=1}^{n} x_{i,j}(p,q),$$
(2)

where $x_{i,j}(p,q)$ is the (p,q)th element of the matrix $\mathbf{X}_{i,j}$, $1 \leq p, q \leq n$.

Let $\mathbf{U}_{i,j}$ be the $n \times n$ matrix representing the pixel values of the (i, j)th block whose DC level is zero. The matrix $\mathbf{U}_{i,j}$ is called the AC component of the (i, j)th block. Let $u_{i,j}(p, q)$ be the (p, q)th element of $\mathbf{U}_{i,j}$. Thus

$$x_{i,j}(p,q) = u_{i,j}(p,q) + \frac{a_{i,j}}{n}.$$
(3)

Define the edge difference vector $\mathbf{d}_{1,i,j}$ between the adjacent blocks $\mathbf{X}_{i,j-1}$ and $\mathbf{X}_{i,j}$ as

$$\mathbf{d}_{1,i,j} = \begin{bmatrix} \xi_{1,i,j}(1) \\ \xi_{1,i,j}(2) \\ \vdots \\ \xi_{1,i,j}(n) \end{bmatrix} + \frac{a_{i,j-1} - a_{i,j}}{n} \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}, \quad (4)$$

where

$$\xi_{1,i,j}(k) = u_{i,j-1}(k,n) - u_{i,j}(k,1).$$
(5)

Similarly the edge difference vector $\mathbf{d}_{2,i,j}$ between the adjacent blocks $\mathbf{X}_{i-1,j}$ and $\mathbf{X}_{i,j}$ is defined as

$$\mathbf{d}_{2,i,j} = \begin{bmatrix} \xi_{2,i,j}(1) \\ \xi_{2,i,j}(2) \\ \vdots \\ \xi_{2,i,j}(n) \end{bmatrix} + \frac{a_{i-1,j} - a_{i,j}}{n} \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}, \quad (6)$$

where

$$\xi_{2,i,j}(k) = u_{i-1,j}(n,k) - u_{i,j}(1,k).$$
(7)

An illustration of $\mathbf{d}_{1,i,j}$ and $\mathbf{d}_{2,i,j}$ can be found in figure 1.

The MED model suggests that the value of $a_{i,j}$ tends to minimize

$$\|\mathbf{d}_{1,i,j}\|^{2} + \|\mathbf{d}_{2,i,j}\|^{2} + \|\mathbf{d}_{1,i,j+1}\|^{2} + \|\mathbf{d}_{2,i+1,j}\|^{2}.$$
 (8)

Tse and Cham [3] proposed a global estimation scheme which estimates all the DC coefficients of the blocks by minimizing the sum of the energy of all the possible edge difference vectors of the image, which is given by

$$\Sigma_g = \sum_{i=1}^{N_1} \sum_{j=2}^{N_2} \|\mathbf{d}_{1,i,j}\|^2 + \sum_{i=2}^{N_1} \sum_{j=1}^{N_2} \|\mathbf{d}_{2,i,j}\|^2.$$
(9)

The minimization of Σ_g is equivalent to solve N_1N_2 linear equations simultaneously. The solution can be achieved by using the iteration method suggested in [6].



Figure 1: Definitions of $\mathbf{d}_{1,i,j}$ and $\mathbf{d}_{2,i,j}$.

3. DC COEFFICIENT RESTORATION USING THE MAP ESTIMATION

The DC coefficient restoration problem can be written as an inverse problem similar to the image restoration as [4]

$$\mathbf{u} = \mathbf{H}\mathbf{x},\tag{10}$$

where **x** is the original pixel values arranged in the lexicographical order. **H** is the operator which removes the DC coefficients of the blocks dividing the image **x** and gives the block AC coefficients **u**. The operator **H** is a singular, idempotent symmetric square matrix with the rank $N_1N_2(n^2-1)$. It has $N_1N_2(n^2-1)$ nonzero eigenvalues and all are equal to 1. Moreover all the singular values

of \mathbf{H} are equal to 1. Thus the condition number of \mathbf{H} is 1. As a result, the inverse problem (10) is not ill-posed, and is similar to the noise-free interpolation problem.

Using the MAP technique, we can estimate the image $\hat{\mathbf{x}}$ by

$$\hat{\mathbf{x}} = \arg \max_{\mathbf{a}} L(\mathbf{x}(\mathbf{a})|\mathbf{u}), \tag{11}$$

where $L(\cdot)$ is the log-likelihood function and **a** is the vector containing the DC coefficients to be estimated. The MAP estimate of the image can be computed by

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{a}} \{ -\log P(\mathbf{u}|\mathbf{x}(\mathbf{a})) - \log P(\mathbf{x}(\mathbf{a})) \}, \quad (12)$$

where

$$P(\mathbf{u}|\mathbf{x}(\mathbf{a})) = \begin{cases} 1 & \mathbf{u} = \mathbf{H}\mathbf{x}(\mathbf{a}), \\ 0 & \text{otherwise.} \end{cases}$$
(13)

We model $P(\mathbf{x}(\mathbf{a}))$ as the Gibbs distribution given by

$$P(\mathbf{x}(\mathbf{a})) = \frac{1}{K} \exp\left\{-\sum_{c \in C} V_c(\mathbf{x}(\mathbf{a}))\right\},$$
(14)

where K is the normalization constant. $V_c(\cdot)$ is some function of a local group of points c called clique, and C denotes the set of all cliques. We are only interested in using the pixel values along the block boundaries to estimate the DC coefficients. The C is chosen to be the pixels along the block boundaries.

Substitute (14) into (12), we can found the restored DC coefficients and the restored image using

$$\hat{\mathbf{x}}(\mathbf{a}) = \arg\min_{\mathbf{x}(\mathbf{a}) \in X} \sum_{c \in C} V_c, \qquad (15)$$

where X is the set that

$$X = \{ \mathbf{x} : \mathbf{u} = \mathbf{H}\mathbf{x} \}.$$
(16)

We apply the Huber minimax function which is defined as

$$\rho(v) = \begin{cases} v^2 & |v| \le T, \\ T^2 + 2T(|v| - T) & |v| > T, \end{cases}$$
(17)

to model the function $V_c(\cdot)$. Huber minimax function is a convex function which can be efficiently computed to find its global minimum. It has the property that it can preserve the discontinuities. The x^2 term gives the smoothing property during the minimization. When |x| > T, the linearly varying cost preserves the discontinuities which is governed by the parameter T. It has been used in image interpolation [7] and image restoration problems [8].

Using the Huber minimax function, we propose

$$\sum_{c \in C} V_c(\mathbf{x}(\mathbf{a})) = \sum_{i=1}^{N_1} \sum_{j=2}^{N_2} \sum_{k=1}^n \rho\left(\xi_{1,i,j}(k) + \frac{a_{i,j-1} - a_{i,j}}{n}\right) \\ + \sum_{i=2}^{N_1} \sum_{j=1}^{N_2} \sum_{k=1}^n \rho\left(\xi_{2,i,j}(k) + \frac{a_{i-1,j} - a_{i,j}}{n}\right).$$
(18)

Since the variables are the DC coefficients $a_{i,j}$, thus the minimization using MAP is

$$\hat{\mathbf{a}} = \arg\min_{\mathbf{a}} \left\{ \sum_{i=1}^{N_1} \sum_{j=2}^{N_2} \sum_{k=1}^n \rho\left(\xi_{1,i,j}(k) + \frac{a_{i,j-1} - a_{i,j}}{n}\right) + \sum_{i=2}^{N_1} \sum_{j=1}^{N_2} \sum_{k=1}^n \rho\left(\xi_{2,i,j}(k) + \frac{a_{i-1,j} - a_{i,j}}{n}\right) \right\}.$$
(19)

It can be easily shown that when $T = \infty$, the minimization (19) is the global estimation using the MED criterion. The minimization (19), however, is a nonlinear problem which can be solved by conjugate gradient minimization technique [9]. If the block selection scheme is applied, the original DC coefficients of the selected blocks become the boundary conditions which are kept to be constant during the minimization.

4. SIMULATION RESULTS

Four images each having the size 512×512 are used in the experiments. The images are divided into 8×8 blocks. Since we would like to demonstrate the effectiveness of the newly proposed criterion against the MED criterion, the original AC coefficients are used in the DC coefficient restoration in the experiment.

Figures 2-5 show the PSNR of the DC coefficients restored images with different T of the four test images. For each graph in figures 2-5, the five sets of curves show the PSNR of the restored image from different amount of selected original DC coefficients, ranging from 0% to 20%. Figures 2-5 show that, with a suitable chosen parameter T, the PSNR of the restored images using the proposed criterion is better than those using the MED criterion.

Figure 6 shows the DC coefficients restored image Lena using the MED criterion and the proposed criterion, respectively, without using any original DC coefficients. Figure 7 gives the magnified region around the hat border from the two restored images in figure 6 respectively. It shows that the blocking effect due to the discontinuity across the hat border observable in the image restored by the MED criterion is absent in that restored by the proposed criterion. This is due to the discontinuity preservation property of the functional used in the proposed scheme. Similar results are found in the other three test images.

From the results, we also found that the improvements of the new criterion from the MED criterion is decreased if the number of original DC coefficients selected is increased. It is because most of the discontinuities are eliminated by the large amount of the block selection.

Quantized AC coefficients were also used to test the new criterion. However we found that the improvements are less significant. One may conclude that more accurate AC coefficients are required to use the new criterion effectively.

5. CONCLUSIONS

A new DC coefficient restoration criterion using the MAP estimation technique is proposed in this paper. The new criterion preserves the discontinuities and helps solving the blocking effect due to the failure of the MED criterion on the block boundaries along the discontinuities.

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Figure 2: PSNR of the restored image Airplane.



Figure 3: PSNR of the restored image Lena.



Figure 4: PSNR of the restored image Peppers.



Figure 5: PSNR of the restored image Sailboat.



(a) MED



(b) Proposed

Figure 6: DC coefficients restored images using the MED and the proposed criteria.



Figure 7: Magnified hat regions of the restored images using the MED and the proposed criteria.