A RECENT SUBSPACE-BASED APPROACH TO JOINT DETECTION AND TIME-DELAY ESTIMATION IN DS-CDMA SYSTEMS

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ABSTRACT

This paper presents a novel method for jointly estimating the time-delay parameters and detecting the transmitted symbols in a DS-CDMA system. A short training sequeunce is used to get an initial estimate of the time-delay, which is consequently used to detect the symbols. The method then iterates while exploiting signal structure, to improve the performance. Simulation results are presented to compare the algorithm with the decorrelating criterion and the matched-filter receiver in terms of bit-error rate, and with the MUSIC algorithm and the sliding correlator in terms of the variance of the time-delay estimates.

1. INTRODUCTION

Code Division Multiple Access (CDMA) has a number of potential advantages compared to TDMA/FDMA (cell-reuse factor, graceful degradation, speech monitoring, soft handover, multipath resistance), and it is predicted by many to be the method of choice for employment of future mobile cellular communication systems. This fact has contributed strongly to the huge interest for research results concerning CDMA over the last decade. A significant factor that limits the theoretical system capacity in such systems is the near-far effect. A number of receivers that are near-far resistant [1, 2, 3], usually assume some knowledge of the time-delay among respective users in order to suppress the MAI (Multi Access Interference), or at least synchronization to the desired user [3, 4]. More recently, a number of publications has focused on methods that provide estimates of channel parameters (such as time-delay, phase, channel gain), where the challenge again is robustness in environments with very dissimilar power levels among the subscribers [5, 6].

This paper focuses on a method of jointly detecting the transmitted symbols and estimating the time delay parameters. In summary, it is a single-user method, operating on one signal at a time while treating the remaining signals as noise. The criterion used to estimate the respective time-delays can be interpreted as a subspace approach, but without some of the drawbacks of previous methods [7, 8], such as the need to perform an eigendecomposition as well as knowledge of the model order.

2. REFINED SIGNAL MODEL

The model to be described considers a K-user asynchronous DS-CDMA system operating in a fading environment. The code waveforms are assumed to be unit-amplitude, rectangular and periodic, with chip-duration $T_c = T/L$, where T is the symbol period and L is an integer. The symbols belong to some complex alphabet $\mathbf{\Omega}$. The k'th user's baseband signal is $s_k(t) = \sum_{-\infty}^{\infty} d_k(n)c_k(t-nT)$, with $c_k(t)$ being the code waveform of user k. The total received signal is the superposition of all K signals, and can be expressed as

$$r(t) = \sum_{k=1}^{K}\sum_{r=1}^{R_k} \beta_{k,r} s_k(t-\tau_{k,r})\cos(\omega_c t + \theta_{k,r}) + n(t). \quad (1)$$

where $\beta_{k,r}$ is the complex channel gains for each of the R_k paths of user k. The path delays $\tau_{k,r} \in [0,T)$, $r = 1,\ldots,R_k$ and $\theta_{k,r}$ is a random phase.

The received signal is downconverted to in-phase and quadrature components, followed by an integrate-and-dump stage with integration time $T_i = T_c/Q$, i.e. Q is the oversampling factor. Now, for symbol interval n, QL samples of this complex baseband sequence is collected in a vector,

$$\mathbf{r}(n) = [r(nQL+1), \quad \ldots \quad, r(nQL+QL)]$$

and after some straightforward manipulation, one can express the k'th user's contribution to the signal as

$$\mathbf{r}_k(n) = \begin{bmatrix}\mathbf{h}_{k1}(\tau_{k,1}) & \mathbf{h}_{k2}(\tau_{k,1})\ldots\mathbf{h}_{k1}(\tau_{k,R_k}) & \mathbf{h}_{k2}(\tau_{k,R_k})\end{bmatrix}$$
$$\times \begin{bmatrix}\beta_{k,1} & 0 \\ 0 & \beta_{k,1} \\ \vdots & \vdots \\ \beta_{k,R_1} & 0 \\ 0 & \beta_{k,R_1}\end{bmatrix} \begin{bmatrix} d_k(n-1) \\ d_k(n)\end{bmatrix} + \mathbf{n}(n)$$
$$= \mathbf{H}_k\mathbf{B}_k\mathbf{z}_k(n) + \mathbf{n}(n).$$

It will be assumed that $\mathbf{n}(n)$ is white Gaussian with variance σ^2. The columns of $\mathbf{H}_k$ are functions of the time-delays and the code waveforms

$$\mathbf{h}_{11}(\tau) = \left(1 - \frac{\delta}{T_i}\right) ds(\mathbf{c}_1, p - QL)\frac{\delta}{T_i} ds(\mathbf{c}_1, p+1-QL)$$

$$\mathbf{h}_{12}(\tau) = \left(1 - \frac{\delta}{T_i}\right) ds(\mathbf{c}_1, p) \frac{\delta}{T_i} ds(\mathbf{c}_1, p+1)$$

$$\mathbf{c}_1 = [c_1(1) \quad c_1(2) \ldots c_1(QL)]^T.$$

The time delay $\tau = pT_i + \delta$, is such that p is an integer and $\delta \in [0, T_i)$. $ds(\cdot, \cdot)$ is the down-shift operator, acting on an arbitrary N-vector as

$$ds((a_1 \quad a_2 \ldots a_N)^T, q) = \begin{cases} [0 \quad \ldots \quad 0 \quad a_1 \quad a_2 \quad \ldots \quad a_{N-q}] & \text{if } q \geq 0, \\ [a_{1-q} \quad a_{2-q} \quad \ldots \quad a_N \quad 0 \quad \ldots \quad 0] & \text{if } q \leq 0. \end{cases} \quad (2)$$

A model for the complete signal is the simply the sum of all K users and an additive noise term, given by

$$\mathbf{r}(n) = \sum_{k=1}^{K} \mathbf{r}_k(n) = \mathbf{HBz}(n) + \mathbf{n}(n) \quad \in \mathbb{C}^{QL} \quad (3)$$

$$\begin{aligned} \mathbf{H} &= [\mathbf{H}_1 \quad \mathbf{H}_2 \quad \ldots \quad \mathbf{H}_K] \quad \in \mathbb{R}^{QL \times 2R} && \text{(4a)} \\ \mathbf{B} &= \text{diag}\left(\mathbf{B}_1 \quad \mathbf{B}_2 \quad \ldots \quad \mathbf{B}_K\right) \quad \in \mathbb{C}^{2R \times 2K} && \text{(4b)} \\ \mathbf{z}(n) &= [\mathbf{z}_1^T(n) \quad \mathbf{z}_2^T(m) \quad \ldots \quad \mathbf{z}_K^T(n)]^T \quad \in \mathbb{C}^{2K} && \text{(4c)} \end{aligned}$$

The total number of paths is denoted $R = \sum_{k=1}^{K} R_k$.

3. ALGORITHM

The aim is to get an initial estimate of the time-delay for the user of interest (say user 1) using a short training sequence, and then use this estimate to obtain an estimate of the matrix $\mathbf{z}_1$ in (2), which will subsequently be used to refine the estimate of $\boldsymbol{\tau}_1$. The criterion function to be minimized to estimate $\boldsymbol{\tau}_1$ closely follows the approach taken in [9], where the problem is direction-of-arrival estimation.

3.1. The time-delay estimation criterion

In short, the idea is to work on one signal at a time (the signal-of-interest, SOI), and treat the other signals as interference. In order to formulate a likelihood function, this intereference together with the background noise is assumed to be Gaussian. Although this is clearly a false assumption, it has been made in order to simplify the minimization of the resulting criterion to an R_k-dimensional space. A true ML criterion is simple to formulate, but requires minimization over a an R-dimensional space, which is prohibitively complex. Start by rewriting (3) to

$$\begin{aligned} \mathbf{r}(m) &= \mathbf{H}_1\mathbf{B}_1\mathbf{z}_1(m) + \sum_{k=2}^{K} \mathbf{H}_k\mathbf{B}_k\mathbf{z}_k(m) + \mathbf{n}(m) \\ &= \mathbf{H}_1\mathbf{B}_1\mathbf{z}_1(m) + \mathbf{j}(m) \end{aligned}$$

The negative log-likelihood function, based on modeling $\mathbf{j}(m)$ as complex Gaussian with covariance $\mathcal{R}_{jj}$, is

$$\ell(\tau_1, \mathbf{B}_1, \mathcal{R}_{jj}) = \log|\mathcal{R}_{jj}| + \text{Tr}\{\mathcal{R}_{jj}^{-1}\Psi_1(\tau_1, \mathbf{B}_1)\} \quad (5)$$

$$\Psi_1(\tau_1, \mathbf{B}_1) \triangleq \frac{1}{N} \sum_{n=1}^{N} \|\mathbf{r}(n) - \mathbf{H}_1\mathbf{B}_1\mathbf{z}_1(n)\|^2.$$

This expression can now be manipulated to give a criterion in $\boldsymbol{\tau}_1$, of dimension equal to the number of paths for the desired user R_1. The details of the derivation can be found in [9], and yields the following

$$f(\boldsymbol{\tau}_1) = \Delta_{11} + \Delta_{22} + \Delta_{11}\Delta_{22} - |\Delta_{12}|^2 \quad (6)$$

where

$$\boldsymbol{\Delta} = \hat{\mathcal{R}}_{zz}^{-\frac{1}{2}} \hat{\mathcal{R}}_{rz}^{*} \hat{\mathbf{W}}^{-\frac{1}{2}} \hat{\mathbf{\Pi}}^{\perp} \hat{\mathbf{W}}^{-\frac{1}{2}} \hat{\mathcal{R}}_{rz} \hat{\mathcal{R}}_{zz}^{-\frac{1}{2}} = \begin{bmatrix} \Delta_{11} & \Delta_{12} \\ \Delta_{12}^* & \Delta_{22} \end{bmatrix}$$

$$\begin{aligned} \hat{\mathbf{\Pi}}^{\perp}(\boldsymbol{\tau}) &= \mathbf{I} - \hat{\mathbf{W}}^{-\frac{1}{2}}\mathbf{H}_1(\mathbf{H}_1^*\hat{\mathbf{W}}^{-1}\mathbf{H}_1)^{-1}\mathbf{H}_1^*\hat{\mathbf{W}}^{-\frac{1}{2}} \\ \hat{\mathbf{W}} &= \hat{\mathcal{R}}_{rr} - \hat{\mathcal{R}}_{rz}\hat{\mathcal{R}}_{zz}^{-1}\hat{\mathcal{R}}_{rz}^* \end{aligned}$$

and $\mathbf{H}_1 = \mathbf{H}(\hat{\boldsymbol{\tau}}_1)$. Once $\hat{\boldsymbol{\tau}}_1$ has been computed, the complex channel gains can be estimated from the expression

$$\hat{\mathbf{B}}_1 = (\mathbf{H}_1^*\hat{\mathbf{W}}^{-1}\mathbf{H}_1)^{-1}\mathbf{H}_1^*\hat{\mathbf{W}}^{-1}\hat{\mathcal{R}}_{rz}\hat{\mathcal{R}}_{zz}^{-1} \quad (7)$$

3.2. The complete detection-estimation algorithm

Assume N vector samples of the signal $\mathbf{r}(n)$ (equation 3) is collected in a matrix $\mathbf{r}_N$ of dimension $L \times N$.

The idea is now to first obtain an initial estimate of the time-delay for the desired signal using a short training sequence. Following this, the matrix $\mathbf{z}_1$ is estimated and used to refine the estimate $\hat{\boldsymbol{\tau}}_1$. This process is repeated in an iterative fashion until convergence, followed by data demodulation ($\hat{\mathbf{z}}_1 \longrightarrow \hat{\mathbf{d}}_{1,N}$). Treating the demodulated data as the *true* transmitted sequence, a final estimate of $\boldsymbol{\tau}_1$ can be obtained. Specifically, the method operates as follows;

1. Compute the sample correlation matrix
$$\hat{\mathcal{R}}_{rr} = \frac{1}{N} \sum_{n=1}^{N} \mathbf{r}(n)\mathbf{r}^*(n)$$
2. Initial estimate of τ_1 using training sequence
$$\hat{\boldsymbol{\tau}}_1 = \arg\min_{\boldsymbol{\tau}_1} f(\boldsymbol{\tau}_1) \quad (8)$$
3. WLS estimate of $\mathbf{z}_1$
$$\hat{\mathbf{z}}_1 = \arg\min_{\mathbf{z}_1} \|\hat{\mathcal{R}}_{rr}^{-\frac{1}{2}}(\mathbf{r}_N - \mathbf{H}(\hat{\boldsymbol{\tau}}_1)\mathbf{z}_1)\|_F^2 \longrightarrow \text{project to } \boldsymbol{\Omega} \quad (\pm 1) \quad (9)$$
4. Refined time-delay estimate using all data $\mathbf{r}_N$ and $\hat{\mathbf{z}}_1$
$$\hat{\boldsymbol{\tau}}_1 = \arg\min_{\boldsymbol{\tau}_1} f(\boldsymbol{\tau}_1) \quad (10)$$
5. Repeat steps 3-4 until convergence
6. Demodulation; $\hat{\mathbf{z}}_1 \longrightarrow \hat{\mathbf{d}}_{1,N}$, (see below)
7. Final time-delay estimate (if desired) as before, using demodulated bits $\mathbf{d}_{1,N}$

The following comments can be made. First of all, the minimization of (8) is reasonable to carry out using a simple grid search, unless the number of multipath (R_k) is large. Else, other numerical methods (Newton-type) can be considered. Also, one can choose to search over a reduced grid in (10), with the assumption that the initial estimate of $\boldsymbol{\tau}$ from (8) is sufficiently good.

The formulation of the likelihood function (5) requires that $\mathcal{R}_{jj}$ be circularly symmetric. Though a reformulation using a real-valued observation vector is straightforward, it has been empirically observed that the complex formulation (5) performs better.

The demodulation in step (6) of the algorithm is carried out using well-known ideas of decision-feedback equalization. It is evident from the model (2) that to obtain $\hat{d}_1(n)$, one should combine the estimates of $\hat{\mathbf{z}}_1(n)$ and $\hat{\mathbf{z}}_1(n+1)$, as well as subtract the already detected $d_1(n-1)$ (assuming it's correct). So that

$$\hat{d}_1(n) = \operatorname{sgn}\Big(\mathbf{f}^*\mathbf{x}(n)\Big) \tag{11}$$

where $\mathbf{f}$ is a 5-tap filter chosen to minimize $\mathrm{E}\big[(d_1(n) - \mathbf{f}^*\mathbf{x}(n))^2\big]$, and $\mathbf{x}(n) \triangleq \big[\hat{\mathbf{z}}_1^T(n) \quad \hat{\mathbf{z}}_1^T(n+1) \quad \hat{d}_1(n-1)\big]^T$. the derivation is straightforward but omitted due to space limitations.

4. SIMULATION RESULTS

A 5-user system was simulated, using code sequences of length $L = 15$. Two primitive polynomials of order 4 were combined as $h(x) = g_1(x)g_2(x)$, where $g_1(x) = x^4 + x + 1$ and $g_2(x) = x^4 + x^3 + 1$, in order to generate these sequences. No oversampling was used (i.e. $Q = 1$), time-delays were randomly chosen as $\boldsymbol{\tau} = [2.52 \quad 4.31 \quad 6.52 \quad 9.92 \quad 13.24]^T$, and $\angle\beta_k = 0, \quad \forall k$. A scenario with only one path per user on an AWGN channel is considered. Results were averaged over 500 Monte Carlo simulations with 400 bits in each burst.

In figure (1), the proposed method is compared to the decorrelating criterion and the mathed-filter receiver for a near-far ratio of 0 and 10 dB (i.e. $\beta_k/\beta_1 = 0(10) \quad dB \quad \forall k$). A training sequence of 20 bits was used. This should be compared to figure (2), which is for the same scenario, except that the training sequence has been increased to 40 bits. As is well known, the matched-filter receiver's performance deteriorates rapidly in a near-far scenario, whereas the decorrelator is insensitive to this. The plots seem to indicate that the length of the training sequence only plays a role when the intereference powers are much higher than the SOI.

It is important to keep in mind that the decorrelator requires the knowledge of all the user's time-delays, and that the matched-filter receiver is synchronized to the SOI. Results reported in [10] has shown that the performance of the decorrelator is very sensitive to the quality of the time-delay estimates, especially at high interference levels. Therefore, one can rightly claim that the results shown here are biased in favor of these methods.

The performance criterion for the estimation of τ_1 is the standard deviation of the estimation error, $\mathrm{std}(\hat{\tau}_1 - \tau_1)$.

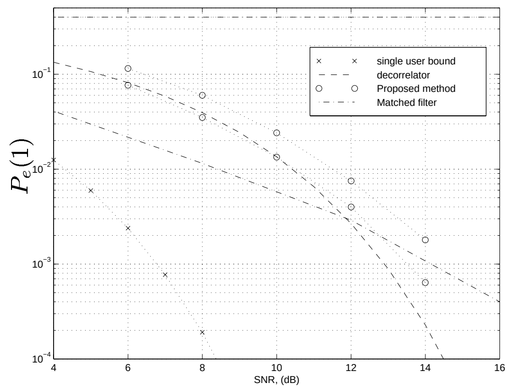

Figure 1: Probability of bit-error vs. SNR for proposed method using a short (20 bits) training sequence, for a near-far ratio of 0 (lower) and 10 dB (upper).

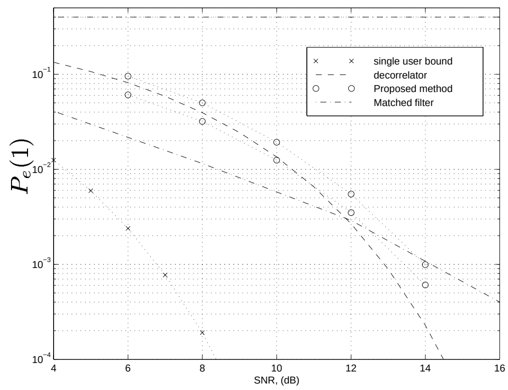

Figure 2: Same as figure (1) using a long (40 bits) training sequence.

The results are shown in figures (3) and (4), for a near-far ratio of 0 and 10 dB, respectively. The proposed method is compared to the MUSIC estimator [8, 7] (which assumes knowledge of the model order, K), and the sliding correlator (assuming the entire burst to be known), as well as the Cramer-Rao bound, which is conditioned on the transmitted symbols [11].

It can be seen that the proposed method has better performance than the MUSIC algorithm as long as the training sequence is sufficienly long ($40/400 = 10\%$ overhead). This is intuitive, since it estimates the symbols and uses this (mostly correct) information to obtain $\hat{\tau}_1$. Also, the sliding correlator is not able to handle those scenarios where the multi-access interference dominates the background noise. Table 1 lists the percentage of outliers, which were defined as those trials where $|\hat{\tau}_1 - \tau_1| \geq 0.5$ chips. These were excluded when computing the standard deviation of $\hat{\tau}_1$ in order to make the comparison to the Cramer-Rao bound meaningful. The numbers show that with 40 training bits, there are no outliers except for low SNR and high interference levels.

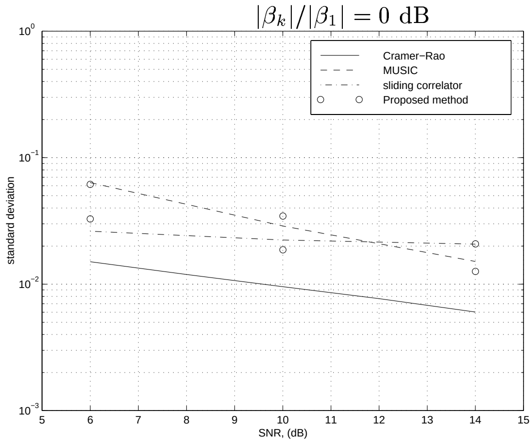

Figure 3: Standard deviation of estimation error in $\hat{\tau}_1$ for a near-far ratio of 0 dB, for short (upper) and long (lower) training sequence, respectively

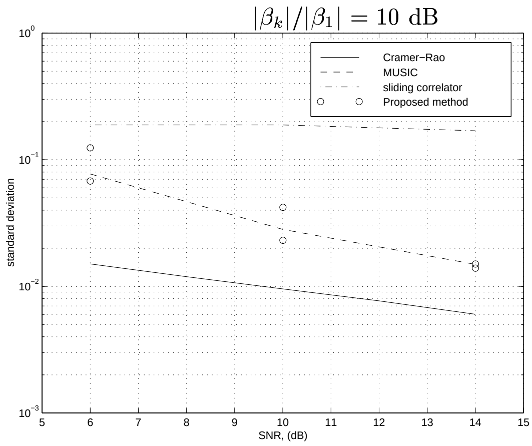

Figure 4: Same as figure (3) with a near-far ratio of 10 dB.

5. CONCLUSION

A novel method which performs joint estimation of time-delay parameters and symbol detection in a DS-CDMA system was presented. The method can be interpreted in a subspace-context, but unlike some previously reported methods, it does not need to perform an eigendecomposition, nor does it need knowledge of the model order. The good performance is obtained by *iterating* between (time-delay) estimation and (symbol) detection, as well as exploiting finite-alphabet signal structure.

	Proposed method				correlator	
N_t	20		40			
$\frac{P_k}{P_1}(dB)$	0	10	0	10	0	10
SNR						
6	23 %	37.8 %	0 %	4.2 %	0 %	10 %
10	5.4%	12.8 %	0 %	0 %	0 %	10 %
14	2 %	3.2 %	0 %	0 %	0 %	10 %

Table 1: percentage of outliers

6. REFERENCES

[1] R. Lupas and S. Verdu. "Linear Multiuser Detectors for Synchronous Code-Division Multiple-Access Channels". *IEEE Transactions on Information Theory*, Vol. 35(1):pp. 123–136, January 1989.

[2] M. Varanasi and B. Aazhang. "Multistage Detection in Asynchronous Code-Division Multiple-Access Communications". *IEEE Transactions on Communications*, Vol.38(4):pp. 509–519, April 1990.

[3] U. Madhow and M.L. Honig. "MMSE Interference Suppression for Direct-Sequence Spread Spectrum CDMA". *IEEE Transactions on Communications*, Vol.42(12):pp.3178–3188, December 1994.

[4] M. Honig, U. Madhow, and S. Verdú. "Blind Adaptive Multiuser Detection". *IEEE Transactions on Information Theory*, Vol.41(4):pp. 944–960, July 1995.

[5] T.J. Lim and L.K. Rasmussen. "Adaptive Symbol and Parameter Estimation in Asynchronous Multiuser CDMA Detectors". *IEEE Transactions on Communications*, Vol.45(2):pp. 213–220, February 1997.

[6] D. Zheng, J. Li, S.L. Miller, and E.G. Ström. "An Efficient Code-Timing Estimator for DS-CDMA Signals". *IEEE Transactions on Signal Processing*, Vol.45(1):pp.82–89, January 1997.

[7] Stephen E. Bensley and Behnaam Aazhang. "Subspace-Based Channel Estimation for Code Division Multiple Access Communication Systems". *IEEE Transactions on Communications*, Vol. 44(8):pp. 1009–1020, August 1996.

[8] E.G. Ström and S. Parkvall. "Joint Parametere Estimation and Detection of DS-CDMA Signals in Fading Channels". In *Proceeding IEEE Global Telecommunications Conference*, pages pp. 1109–1113, 1995.

[9] M. Viberg, P. Stoica, and B. Ottersten. "Maximum Likelihood Array Processing in Spatially Correlated Noise Fields Using Parametrized Signals". *IEEE Transactions on Signal Processing*, April 1997.

[10] S. Parkvall, B. Ottersten, and E.G.Ström. "Sensitivity Analysis of Linear DS-CDMA Detectors to Propagation Delay Estimation Errors". In *Proc. IEEE Global Telecommunications Conf*, pages pp.1872–1876, November 1995.

[11] E.G. Ström and F. Malmsten. "Maximum Likelihood Synchronization of DS-CDMA Signals Trnasmitted over Multipath Channels". In *Submitted to ICC'98*, 1998.