A NEW STEREO MATCHING ALGORITHM BASED ON BAYESIAN MODEL

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ABSTRACT

In this paper, we derive the general formula of Bayesian model for stereo matching algorithm and implement it with simplified probabilistic models. The probabilistic models are independence property and similarity between the neighborhood disparities in the configuration. The formula is the generalization of Bayesian model of stereo matching, and can be implemented into the some different forms corresponding to the probabilistic models in the disparity neighborhood system or configuration. We propose a new probabilistic model in order to simplify the joint probability distribution of disparities in the configuration. According to the experimental results, we can conclude that the derived formula generalize the Bayesian model for stereo matching algorithm, and the simplified probabilistic models are reasonable and approximate the pure joint probability distribution very well. Compared with the conventional method of Bayesian model and sum of squared difference(SSD) algorithm, furthermore, our proposed algorithm outperforms the other ones.

1. INTRODUCTION

Stereo matching is to estimate the disparities between stereo images, which are generated from slightly different viewpoints respectively. Stereo matching generates the three dimensional scene structure, disparity map, from the two dimensional stereo images set[5]. Fig.1 shows the concept of stereo image and disparity from two different viewpoints. An object point is projected on the same rectified plane through two cameras. The two points, P_L and P_R, are the projected points of the object point on the rectified plane. The disparity is the difference between distances of two projected points from two focusing points, F_L and F_R respectively. The Fig.1 satisfies the epipolar geometry[4] so that we consider only the one directional disparity.

In order to estimate the disparities between stereo images, various algorithms have been proposed. The sum of squared differences(SSD) algorithm searches for the disparity based on the region. This algorithm calculates the squared differences of intensity between stereo images and aggregates them in the region. Based on the summed differences, it finds the disparity of the minimum summed difference. This algorithm is simple to implement and has advantages to apply the various adaptive post processings in order to reduce the errorneous disparities. However, this algorithm is very dependent on the size of region and suffers from the blurring of the boundary in the disparity map. The gradient based algorithm[6][7] is based on the idea that the same intensity or color in an image may look different corresponding to the visual angles or positions. Due to this misunderstading of human visual system and camera imaging system, the measure based on the difference of intensity or color differences may be incorrect at some angles or positions of viewpoints. This algorithm proposes the measure of stereo matching based on gradient quantities. This algorithm chooses the disparity with most similar gradient of intensity in the stereo images. In this algorithm, it is dufficult to find the correct disparity in the regions whose intensity varies uniformly. The diffusion based algorithm[8][9] is based on the diffusion equation of energy function. That is to say, the derivative of energy function in time domain is equivalent to the Laplacian of the energy function in the spatial domain. With this diffusion equation, the energy function is diffused iteratively and the disparity of minimum energy is selected. And, there are many algorithms to combine with the above indivisual algorithms in order to solve the disadvantages of each algorithm adaptively.

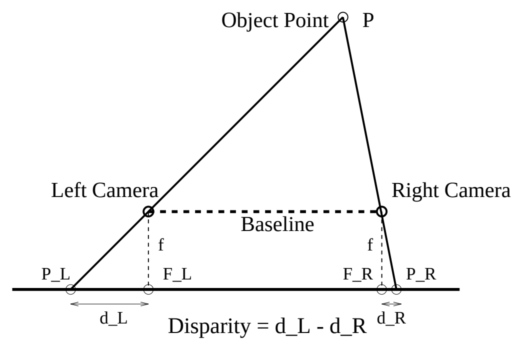

Figure 1: Stereo image and disparity

In this paper, we consider the Bayesian model for stereo matching. This algorithm is based on the Markov Ran-

dom Field(MRF) theory[2][3] and some probabilistic models. Given stereo images set, this algorithm diffuses the energy space iteratively with probabilistic models and searches for the disparity with maximum probability or minimum energy in the converged energy space. Also, a new probability distribution, Gibb's distribution, is introduced to evaluate the probability from error energy, since it is difficult to evaluate the probability directly. This stereo matching algorithm is similar to the diffusion based algorithm in that both the algorithms diffuse the energy spaces and search the disparity with minimum energy. However, the difference is that the diffusion based algorithm diffuses the energy space with diffusion equation of mechanics and this Bayesian model does with Markov and probabilistic models. Scharstein[9][10] has proposed this algorithm and formulated it with probabilistic independence. Since his probabilistic model was not the exact independence between marginal probabilities but the independence of averaged probabilities, however, his formulation was not accurate and was true for specific probabilistic models. We will formulate generally and compare the experimental results of the proposed algorithm with Scharstein's one.
This paper consists of five sections. We survey the basic theory and derive the general formula of Bayesian model for stereo matching in the section 2. In the section 3, a proposed algorithm is described based on the simplified probabilistic models. The experimental results are shown in the section 4, and finally, we conclude the proposed algorithm and experimental results in the section 5.

2. FORMULATION

In the stereo matching to estimate the disparity map between the stereo images, the starting point is to maximize the conditional probability of disparity given stereo images.

$$\begin{aligned} &maximize \quad p(\mathbf{D}|\,\{\mathbf{I}\}), \\ &where, \quad \{\mathbf{I}\} = \{\mathbf{I}_R,\ \mathbf{I}_K,\ K=1,2..\}. \end{aligned} \tag{1}$$

In *eq.* (1), the symbol $\{\mathbf{I}\}$ is the stereo image set and $\mathbf{I}_R$ is the reference image to compare with the other images in the stereo image set. The symbol $\mathbf{D}$ is the disparity map of the stereo image set and describes the three dimensional structure in the stereo images. Before maximizing the conditional probability, however, the main problem of stereo matching is how to calculate the conditional probability given only stereo image set. We need a new measure to transform an available measure into probability measure. The Gibb's distribution is the probability distribution whose random variable is related to the energy dimension,

$$p(x) \;=\; C\,\exp\left\{-\frac{E(x)}{T}\right\}. \tag{2}$$

In the Gibb's distribution, the symbol $E(x)$ is the energy and can be obtained from some energy functions of x. The Gibb's distribution is a kind of function to transform the energy into probability[3]. By transforming the conditional probability using Bayesian rule of probability theory and Gibb's distribution, we obtain an energy relation,

$$E(\mathbf{D}|\,\{\mathbf{I}\}) \propto E(\{\mathbf{I}\}\,|\mathbf{D}) + E(\mathbf{D}). \tag{3}$$

In *eq.* (3), the first term means the error energy due to the intensity differneces between stereo images given disparity map, and the second term is the error energy based on disparity distribution. The second energy, $E(\mathbf{D})$, increases as the difference between adjacent disparities increases. This disparity based energy has an important role in regularization of disparity ditribution in the disparity map.

Now, we consider the pointwise energy $E(d_{i,j})$. According to Markov Random Field(MRF) theory[3], it is proved that it is possible to estimate the disparity of a position, if all of the joint probability distributions between neighborhood disparities are known in advance. In order to calculate the disparity based energy measure, we have a basis on MRF theory. By the probability theory, some manipolations and Gibb's distribution, we obtain the general formula of Bayesian model of stereo matching,

$$\begin{aligned} E(d_{i,j}) \;&=\; E_0(d_{i,j}) \\ &-\;\log\sum_{\mathcal{N}\in\mathcal{S}}\left\{p(\mathbf{d}_\mathcal{N})\prod_{d_n\in\mathcal{N}}\exp\left\{-\rho_d(d_{i,j}-d_n)\right\}\right\}, \end{aligned} \tag{4}$$

where, the first energy term in the right side of *eq.* (3) is redefined as $E_0(d_{i,j})$, and the $\mathcal{S}$ means by the super set of sets which consists of all configurations of neighboring disparities with respect to $d_{i,j}$. Also, $\mathcal{N}$ is the configuration of neighboring disparities, and $\mathbf{d}_\mathcal{N}$ consists of the disparities in the configuration $\mathcal{N}$.

3. PROPOSED ALGORITHM

In this section, we propose a stereo matching algorithm based on *eq.* (4) and probabilistic models, such as independence of probability and similarity between disparities in the configuration. In *eq.* (4), it is necessary to calculate the joint probability of disparities in the configuration, $p(\mathbf{d}_\mathcal{N})$. However, it is difficult to obtain the exact joint probability distribution in real cases because of too much computing power. We assume that the disparities in the configuration are independent one another. The joint probability $p(\mathbf{d}_\mathcal{N})$ can be rewritten as the product of marginal probabilities of each disparity in the configuration.

$$p(\mathbf{d}_\mathcal{N}) \;=\; \prod_{d_n\in\mathcal{N}} p(d_n). \tag{5}$$

In addition, we should choose the configuration and its elements, d_n, so as to maximize the joint probability $p(\mathbf{d}_\mathcal{N})$. We assume that the disparity varies or distributes continuousely with higher probability than it does abruptly. As is the same case of the intensity distribution in the image, the disparity varies smoothly except for the boundaries of objects. Based on this assumption, we estimate all the disparities in the configuration be equivalent. That is to say, a configuration consists of only one valued disparity set. We call this similarity model *plain configuration model.*

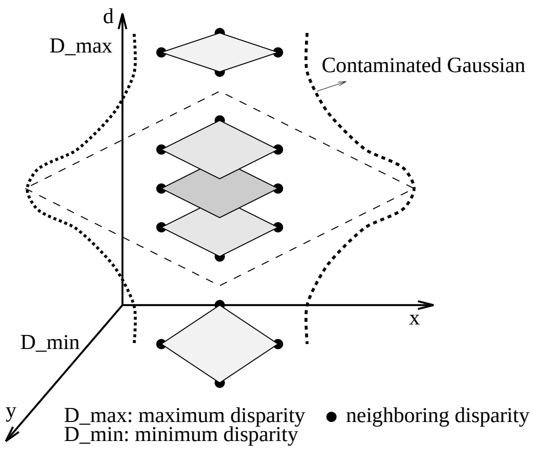

Figure 2: Plain Configuration Model and Probabilistic Diffusion

This model is reasonable estimation since we have no any information of the distribution of disparity, and disparities in the configuration have strong correlations between them. Moreover, this model is consistent with the disparity based regularization measure which makes the distribution of disparity smooth and continuous. Fig.2 shows the concept of the plain configuration model and probabilistic diffusion in the energy space. Four disparities with the same value construct a configuration and they are assumed to be independent one another. With the above probabilistic models, the general formula is changed as follows,

$$\begin{aligned} E(d_{i,j}) &= E_0(d_{i,j}) \\ &- \log \sum_{\mathcal{N}\in\mathcal{S}} \left\{ \prod_{d_n\in\mathcal{N}} \exp\left\{-\rho_d(d_{i,j}-d_n)\right\} p(d_n) \right\}. \end{aligned} \tag{6}$$

4. EXPERIMENTAL RESULTS

We had experiments with *eq.* (6) and 2 stereo images sets, which consist of 5 images generated from 5 different viewpoints. The reference image is always the center image. In order to evaluate the performance, we implemented the proposed algorithm with the same configuration and all the same parameters of the Scharstein's implementation and compared the disparity maps generated from the proposed one with Scharstein's maps and those of SSD method. The SSD method was processed with 7x7 window. Fig.3 shows the comparison of disparity maps for *random dot* generated from three algorithms respectively. As we can see in Fig.3, the disparity map from the proposed algorithm is superior to that from Scharstein's algorithm. The disparity boundary in the Fig.3 (d) is more accurate, and the false disparities are fewer than that in Fig.3 (b). In four figures, the left and right areas result from the modulo operation in the implemetation program, and they are meaningless regions in the disparity maps. Fig.3 (c) is the disparity map generated from the general formula, *eq.* (4), in which the *plain configuration model* is not used. However, the disparity map from general formula is not better than that from the proposed algorithm. This result can be explained in the view of the maximal joint probability of disparities in the configuration. In *eq.* (4), the joint probability should be chosen as largely as possible to make the energy be minimized. And, the joint probability of *plain configuration* is usually larger than the pure joint probability of all configurations, if the distribution of disparity is generally continuous. Fig.4 shows the comparison of disparity maps for *face* image. As is the same result of the *random dot* image, the proposed algorithm outperforms Scharstein's one.

5. CONCLUSIONS

In this paper, we derived the general formula of Bayesian model for stereo matching and implemented the proposed algorithm based on the simplified probabilistic models such as independence of probability and similarity between disparities in the configuration. According to the experimental results, the performance of the proposed algorithm outperformed the conventional algorithm, in that the boundaries of disparity were clearer and errorneous disparities were reduced very effectively. These results mean that the derived formula is adequate and the probabilistic models are reasonable and sufficient to estimate the joint probability distribution of disparities in the configuration. Also, We can conclude that the derived formula is the general form and can be changed into the some different forms based on the reasonable probabilistic models.

6. ACKNOWLEDGEMENT

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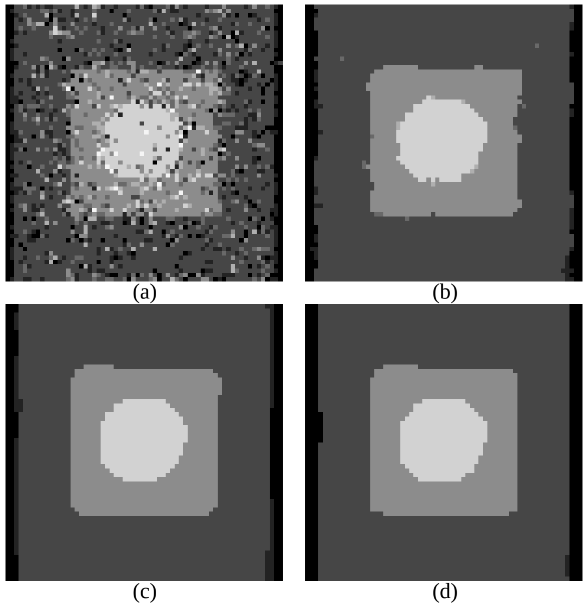

Figure 3: Comparison of disparity map for *random dot*: (a) SSD: 7x7, (b) Scharstein's, (c) General formula, (d) Proposed

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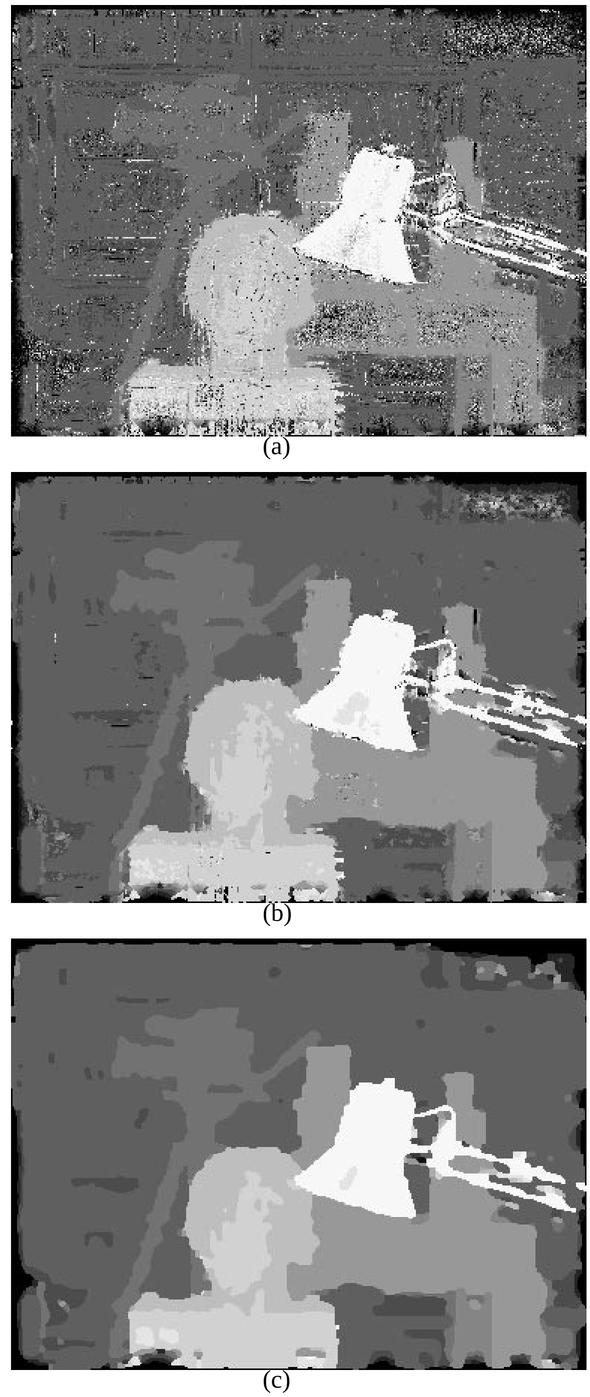

Figure 4: Comparison of disparity map for *face*: (a) SSD: 7x7, (b) Scharstein's, (c) Proposed