ON ESTIMATING THE QUALITY OF NOISY IMAGES

Z. Zhang and Rick. S. Blum

EECS Department, Lehigh University Bethlehem, PA 18015 rblum@eecs.lehigh.edu

ABSTRACT

Some new techniques are proposed for estimating the quality of a noisy image of a natural scene. Analytical justifications are given which explain why these techniques work. Experimental results are provided which indicate that the techniques work well in practice. These techniques need only the images to be evaluated and do not use detailed information about the formation of the image. The focus is on the case where the image is only corrupted by additive Gaussian noise, which is independent from pixel to pixel, but some cases with blurring are also considered. These results should be useful in the process of fusing several images to obtain a higher quality image. Quality measures of this type are needed for fusion, but they have not received much attention to date. In this research, a mixture model is used in conjugation with the expectation-maximization (EM) algorithm to model edge images. This approach yields an accurate representation which should also be useful in other image processing research.

1. INTRODUCTION

A method for estimating the quality of an image is considered here. Our method involves forming histograms of an edge intensity image. The characteristics of the histogram give information about the amount of noise added to the image. The edge intensity image is obtained from a Canny edge detection operation [1] by first convolving the image with a one dimensional mask to find the edge contributions in both the horizontal and vertical directions. For analysis purposes, we model the histograms of the edge images produced by the horizontal and vertical convolutions using a mixture model [2], where the terms in the mixture are zero-mean Gaussian probability density functions (pdfs).

The edge intensity image is formed by setting each pixel to the square root of the sum of the squares of the corresponding pixels in the horizontal and vertical images. This leads to the histogram of the overall intensity being modeled by a mixture of Rayleigh pdfs. The only parameters of the Rayleigh mixture model correspond to the variances and probabilities of the mixture terms in the original Gaussian mixture model. Approximate maximum likelihood estimates of these parameters are easily obtained directly from the overall edge intensity image using the EM algorithm [2, 3]. By studying the effects of noise on these parameters, we motivate techniques to estimate the amount of noise and image quality. For earlier works on noise estimation, the reader is referred to Olsen [4], who compared the performance of several previously

suggested methods. The methods we suggest appear to outperform the methods in [4].

2. HISTOGRAM MODELING

Now we consider, in detail, how we model the histograms used in our algorithm. Consider a noisy image I for which

$$I(h, v) = f(h, v) + n(h, v)$$
(1)

where I is the observed image, f is the ideal image, n is the noise and (h, v) denotes a particular pixel. Let I'_X and I'_Y represent the images which result from convolving the image I with a one dimensional mask, as specified in Canny's algorithm, to find the edge contributions in both the horizontal(x) and vertical(y) directions. Thus I'_X and I'_Y are obtained by linear time-invariant filtering of I. Now let us assume that I'_X and I'_Y can be considered random variables with a joint pdf which can be modeled as a mixture of Gaussian pdfs as

$$f_{I'_X,I'_Y}(x,y) = \sum_{i=1}^M \omega_i \frac{1}{2\pi\sigma_i^2} \exp\left(-\frac{x^2 + y^2}{2\sigma_i^2}\right)$$
(2)

In (2) ω_i is the probability that a random sample comes from the i^{th} term in the mixture so that

$$\sum_{i=1}^{M} \omega_i = 1. \tag{3}$$

For convenience assume the σ_i^2 are ordered so that $\sigma_1^2 < \sigma_2^2 < \ldots < \sigma_M^2$. The Gaussian mixture model is very powerful and results exist [5], which imply that this sort of model can accommodate any zero-mean, circularly symmetric pdf that is monotonically decreasing in $x^2 + y^2$ as long as enough terms are used in the mixture model. In fact, many studies have shown only 3 or 4 terms are needed to provide a good match to most practical pdfs [6, 7]. We found the histograms of I'_X and I'_Y to be monotonically decreasing for the images we analyzed. Now consider the edge intensity image $\|\nabla I\| = \sqrt{I'_X^2 + I'_Y^2}$. Under the above assumptions the pdf of a pixel from $\|\nabla I\|$ can be calculated by using (2) along with the established mapping theory for pdfs [8] to be

$$f_{\|\nabla I\|}(r) = \sum_{i=1}^{M} \omega_i \frac{r}{\sigma_i^2} e^{-\frac{r^2}{2\sigma_i^2}}$$
(4)

which is a mixture of Rayleigh pdfs. In practice, we consider the histogram of $||\nabla I||$ instead of the pdf in (4). Under the appropriate conditions [8] the histogram provides an good estimate of the

This work was supported by the ONR/DOD MURI program under contract N00014-95-1-0601.

pdf. Thus we assume the histogram of $\|\nabla I\|$ can be modeled as in (4) and we have found this to be the case in all the examples we considered. The parameters, $\sigma_i, \omega_i, i = 1, \ldots, M$, in (4) are found using the EM algorithm. A typical example is shown in Fig.1.



Figure 1: Histogram of $||\nabla I||$ and the Rayleigh mixture approximation. Image: lena.pgm with additive Gaussian noise with standard deviation 5. Model: M = 3, $(\sigma_1, \sigma_2, \sigma_3) = (3.96, 6.78, 21.53)$, $(\omega_1, \omega_2, \omega_3) = (0.54, 0.36, 0.10)$

3. NOISE ESTIMATION

Now let us consider the effects of adding noise. Specifically, what are the changes to $\omega_i, \sigma_i^2, i = 1, \ldots, M$. The i = 1 term in (4) has the parameter σ_1^2 which is the smallest σ_i^2 . Thus this term corresponds to the small fluctuations in the image. The i = M term in (4), with the largest parameter σ_M^2 , corresponds to strong edges. Strong edges contain the majority of the information that one is typically interested in. Suppose we add independent identically distributed (i.i.d.) zero-mean Gaussian noise samples to each pixel in the original image. I'_X and I'_Y are related to the original image by two linear filtering operations which each combine pixels that lie along lines in orthogonal directions. Thus this is approximately equivalent to adding independent noise samples to I'_X and I'_Y with the identical appropriate variance. The new joint pdf of I'_X and I'_Y can be put into the form of (2) by changing σ_i^2 to

$$\sigma_i'^2 = \sigma_i^2 + \sigma^2, \quad i = 1, \dots, M$$
 (5)

where $\sigma_i^{'2}$ includes the additive noise and σ^2 is the variance of the noise added to both I'_X and I'_Y (the noise added in (1) results in additive noise to I'_X and I'_Y). The result in (5) can be justified as follows. The model in (2) says that a random vector (x,y) will be generated from one of several Gaussian distributions. ω_i is the probability that the *i*th Gaussian distribution is used. If Gaussian noise is added to the original image, then (2) still holds if σ_i^2 is replaced by $\sigma_i^{'2}$. The reason is that the sum of two Gaussian random variables is another Gaussian random variable. Now performing the mapping to obtain the pdf of $||\nabla I||$, we find that (4) still holds provided σ_i^2 is replaced by $\sigma_i^{'2}$.

provided σ_i^2 is replaced by $\sigma_i^{'2}$. Since σ_1^2 for an ideal image is usually very small, we can use $\sigma_1^{'2}$ to estimate the variance of additive Gaussian noise. $\sigma_1^{'2}$ is estimated using the EM algorithm. Clearly the accuracy of this estimation depends on σ_1 since $\sigma_1'^2 = \sigma_1^2 + \sigma^2$.



Figure 2: The test images

The 17 8-bit 512 \times 512 gray-level images, shown in Fig. 2, were used in an experiment to test this procedure. The results are given in Table 1 for the case with M = 3 in the mixture model. σ_n is the standard deviation of additive Gaussian noise n(h, v) in (1). $\hat{\sigma}_n$ is the average, over the set of test images, of the estimated value of σ_n . $Std(\hat{\sigma}_n)$ is the standard deviation of the estimated value of σ_n . In estimating σ_n we have used the known relationship between σ^2 in (5) and the variance of n(h, v) in (1). The results show that this method tends to over-estimate σ_n when it is small. This is because the method also takes the small random fluctuation in the background of the image as the noise ($\sigma_1 \neq 0$ for the image without noise). If we subtract the $\hat{\sigma}_n$ estimated with $\sigma_n = 0$ from each $\hat{\sigma}_n$, the noise estimation would be given by $\hat{\sigma}_n'$ in Table 1. $Std(\hat{\sigma}'_n)$ is the standard deviation of the corresponding estimate. While, this more accurate estimation is unachievable, it is instructive since the new estimates are excellent. We note that the practical estimate $\hat{\sigma}_n$ appears to perform better than all the methods discussed in [4]. The comparison is not exact since we use a larger set of images than were used in [4].

To test the robustness of this method, zero-mean uniformly distributed noise was also considered. Table 2 shows the experimental results. We found our technique may still provide good results.

4. QUALITY ESTIMATION

The above method can provide an approximate estimation of the variance of additive Gaussian noise. A method to directly estimate the quality, for example signal-to-noise ratio (SNR), of an image is based on considering the quantity

$$Q = \int_{2\mu}^{\infty} f_{||\nabla I||}(r) dr \tag{6}$$

σ_n	$\hat{\sigma}_n$	$Std(\hat{\sigma}_n)$	$\hat{\sigma}_n'$	$Std(\hat{\sigma}_{n}^{'})$
0	4.50	2.64	0	0
2	5.03	2.43	1.98	0.26
5	6.79	1.94	4.77	0.29
10	11.00	1.38	9.79	0.28
15	15.59	1.03	14.73	0.36
20	20.21	0.87	19.54	0.65
25	24.85	0.79	24.31	0.55
30	29.55	0.89	29.10	0.68
40	39.12	0.77	38.78	0.53
50	48.74	0.84	48.47	0.69

Table 1: The average and the standard deviation of the additive Gaussian noise estimation

σ_n	$\hat{\sigma}_n$	$Std(\hat{\sigma}_n)$	$\hat{\sigma}_n'$	$Std(\hat{\sigma}_{n}^{'})$
0	4.59	2.60	0	0
2	5.11	2.42	2.05	0.20
5	6.98	1.86	4.95	0.28
10	11.49	1.11	10.28	0.31
15	16.61	0.79	15.78	0.39
20	21.78	0.75	21.15	0.46
25	26.68	1.09	26.18	0.84
30	31.82	1.01	31.39	0.76
40	42.07	1.04	41.74	0.80
50	51.99	1.07	51.73	0.92

Table 2: The average and the standard deviation of the additive uniform noise estimation

where μ is the mean of $\|\nabla I\|$. In practice, we again approximate Q using histograms and we calculate μ by averaging over the image. We can show that if $f_{\|\nabla I\|}$ is described by a nontrivial mixture as in (4), Q will decrease when i.i.d. Gaussian noise is added. Thus the value of Q for the noisy image is always smaller than the value of Q for the image without noise. The one exception is if the signal is Gaussian distributed itself (the requirement is really that the signal can be modeled with an M = 1 term model in (2)). In this case the image does not have a structure which is different from the Gaussian noise so it is reasonable that this case will cause problems. For the Gaussian signal case, the Q will have the minimum value $e^{-\pi}$ which will not change when we add i.i.d. Gaussian noise to the image. This is easy to demonstrate by using the relationship

$$\mu = \sqrt{\frac{\pi}{2}}\omega_1 \sigma_1 \tag{7}$$

which holds for M = 1 in (4) as given in equation (5-48), p. 112 of the 2nd edition of [9]. Upon inserting (7) into (6) and performing a change of variables $u = r/\sigma_1$ we obtain an expression for Q which doesn't depend on σ_1 and which evaluates to $e^{-\pi}$.

We now outline a proof that Q decreases when i.i.d. Gaussian noise is added for a nontrivial case with M = 2. Consider the Gaussian mixture model in (2) with M = 2 and $\sigma_2 = \sigma_1 + \epsilon$. In this case we have (using an extension to (7))

$$Q = \omega_1 e^{\frac{-\pi(\omega_1 \sigma_1 + \omega_2(\sigma_1 + \epsilon))^2}{\sigma_1^2}} + \omega_2 e^{\frac{-\pi(\omega_1 \sigma_1 + \omega_2(\sigma_1 + \epsilon))^2}{(\sigma_1 + \epsilon)^2}}$$
(8)

First we show that a Rayleigh distribution for $f_{\|\nabla I\|}(r)$ leads to the minimum value of Q. Computing the derivative with respect

to ϵ we obtain that

$$\frac{dQ}{d\epsilon}\Big|_{\epsilon=0} = 0 \tag{9}$$

$$\frac{dQ}{d\epsilon} > 0 \quad \text{for} \quad 0 < \epsilon < \infty \tag{10}$$

Thus, $Q(\epsilon)$ is a monotonic increasing function with minimum value at $\epsilon = 0$. Since $Q(\epsilon)$ is a monotonic increasing function, we just need to prove that ϵ will decrease when we add noise. Define Δ^2 such that

$$\Delta^2 = \sigma_2^2 - \sigma_1^2 \tag{11}$$

where σ_1 and σ_2 are the parameters before adding noise. Using (11) we have (assume $\epsilon > 0$)

$$\epsilon = \sigma_2 - \sigma_1 = \frac{\Delta^2}{\sigma_2 + \sigma_1} \tag{12}$$

After adding zero-mean i.i.d. Gaussian noise with variance σ^2 , the parameters of the modified Rayleigh mixture model become $\sigma_1^{'2} = \sigma_1^2 + \sigma^2$ and $\sigma_2^{'2} = \sigma_2^2 + \sigma^2$ as shown above. Thus, the new difference

$$\epsilon' = \sigma'_2 - \sigma'_1 = \frac{\Delta^2}{\sigma'_2 + \sigma'_1}$$
 (13)

must be smaller since Δ^2 is unchanged and $\sigma'_2 + \sigma'_1$ has increased. Now since Q is monotonic increasing in ϵ it must also decrease. While the proof given here is for M = 2, a similar proof can be given for M > 2.

In order to compute Q using a histogram we just count the number of pixels of $\|\nabla I\|$ greater than 2μ where μ is the average over the image $\|\nabla I\|$. So we actually do not need to use the EM algorithm here. Only the gradient intensity will be used and thus the calculation complexity is very low. In order to validate the utility of the Q measurement, we define QR as

$$QR = 10\log_{10}\frac{Q}{\mathbf{e}^{-\pi}} \tag{14}$$

and compare it with SNR defined as

$$SNR = 10\log_{10}\frac{\sigma_f^2}{\sigma_n^2} \tag{15}$$

where σ_f^2 and σ_n^2 are the sample variances of the signal and noise (the signal is known). We compared (14) and (15) for the images in Fig 2. The results on all the test images are similar to that shown in Fig. 3. The results show that it is possible to use QR to judge the relative quality of two images for fusion.

5. MULTI-SCALE QUALITY MEASUREMENT FOR NOISY IMAGES

In some applications, such as image fusion and multi-scale image processing, the SNRs of a set of images at several different scales are of interest. For example, it may be possible to process an image at a set of different scales. To choose the scale with highest SNR is then an interesting problem. In addition, noise is not the only type of degradation of interest. The analysis introduced here also considers blurring, another degradation which is of interest in many situations. In some cases, such as image fusion, an objective measurement of the overall quality of the image is very useful. We propose using

$$IQ = \sigma_M Q^2 \tag{16}$$



Figure 3: The comparison between QR and SNR measurement

If the image was blurred, σ_M would decrease significantly. At the same time, the Q will increase a little bit due to the decrease of noise level. On the other hand, adding noise will mainly influence Q, while not changing σ_M much.

To evaluate the suitability of this approach, we compared it with the SNR measurement in (15). We use a Gaussian smoothing operation for the blurring process. Suppose the signal in (1) is known. We convolve the image with a Gaussian kernel

$$g(h, v, t) = \frac{1}{2\pi t} e^{\frac{-(h^2 + v^2)}{2t}}$$
(17)

and we compute SNR(t) of the result using (15) at each t.

Fig. 4 shows one of our experimental results. All curves are normalized so that the maximum normalized IQ and normalized SNR is 1. All of the images we tested, see Fig. 2, provided similar curves which show that the proposed IQ measurement fits the SNR measurement quite well. It is possible to generalize (16) to

$$IQ = g_1(\sigma_M)g_2(Q) \tag{18}$$

where g_1 and g_2 are two increasing functions. By selecting g_1 and g_2 differently, the relative importance of noise and blurring can be varied. In fact, it might be possible to find g_1 and g_2 to obtain a better correspondence between IQ and SNR in Fig. 4, but we have not investigated this further.

6. CONCLUSION

In this paper, we have proposed new techniques to blindly estimate the quality of an image of natural scene. A Rayleigh mixture density is used to model the distribution of image edge intensity. The parameters of the Rayleigh mixture density, which are found using the EM algorithm, are used for noise and SNR estimation. Experiments on a set of test images show the techniques work well. Analytical justifications explain why these techniques work. Modeling the distribution of the edge intensity using the mixture model and using the EM algorithm to find the best mixture model works extremely well in the examples considered here. This suggests a new set of tools which we believe can be fruitfully employed in many other image processing applications.



Figure 4: Comparison between IQ measurement and SNR measurement

7. REFERENCES

- [1] J. Canny, "A computational approach to edge detection", IEEE Trans. PAMI, vol. 8, no. 6, pp. 679-98, 1986.
- [2] R.A. Redner, H.F. Walker, "Mixture densities, maximum likelihood and the EM algorithm," *SIAM Review*, vol. 26. pp. 195-239, April 1984.
- [3] A.P. Dempster, N.M. Laird, D.B. Rubin, "Maximum likelihood from incomplete data via the EM algorithm," *J. of the Royal Statistical Soc.*, vol. 39, no. 1, pp. 1-38, 1977.
- [4] S.I. Olsen, "Estimation of noise in images: An evaluation," *CVGIP: Graphic Models and Image processing*, Vol. 55, No. 4, pp. 319-323, Jul. 1993.
- [5] L. A. Liporace, "Maximum likelihood estimation for multivariate observations of markov sources," *IEEE Trans. Information Theory*, vol. IT-28, no. 5, pp. 729-734, Sept. 1982.
- [6] K. Vastola, "Threshold detection in narrow-band non-Gaussian noise," *IEEE Transactions on Communications* Vol. COM-32, No. 2, Feb. 1984.
- [7] P. A. Delancy, "Signal detection in multivariate class-A interference", *IEEE Transactions on Communications*, Vol. COM-43, No. 2/3/4, pp. 365-373, Feb. 1995.
- [8] K.S. Shanmugan and A.M. Breipohl, *Random Signals: Detection, Estimation and Data Analysis*, New York: John Wiley & Sons, 1988.
- [9] A. Papoulus, Probability, Random Variables, and Stochastic Processes, New York: McGraw-Hill, 1984.