WAVELET BASED ANALYSIS OF ROTATIONAL MOTION IN DIGITAL IMAGE SEQUENCES. *

Mingqi Kong † *Jean-Pierre Leduc* * *Bijoy K. Ghosh* † *Jon Corbett* * *Victor M. Wickerhauser* *

Washington University in Saint Louis
*Department of Mathematics
†Department of Systems Science and Mathematics
One Brookings Drive, P.O. Box 1146
Saint Louis, MO 63130

ABSTRACT

This paper addresses the problem of estimating, analyzing and tracking objects moving with spatio-temporal rotational motion (spin or orbit). It is assumed that the digital signals of interest are acquired from a camera and structured as digital image sequences. The trajectories in the signal are two-dimensional spatial projections in time of motion taking place in a three-dimensional space. The purpose of this work is to focus on the rotational motion i.e. estimate the angular velocity. In natural scenes, rotational motion usually composes with translational or accelerated motion on a trajectory. This paper shows that trajectory parameters and rotational motion can be efficiently estimated and tracked either simultaneously or separately. The final goal of this work is to provide selective reconstructions of moving objects of interest. This paper constructs new continuous wavelet transforms that can be tuned to both translational and rotational motion. The parameters of analysis that are taken into account in these rotational wavelet transforms are space and time position, velocity, spatial scale, angular orientation and angular velocity. The continuous wavelet functions are finally discretized for signal processing. The link between rotational motion, symmetry and critical sampling is also presented. Applications are presented with tracking and estimation.

1. INTRODUCTION

In this paper, we present new spatio-temporal continuous wavelet transforms that are tuned to rotational and translational motion. These wavelets extend our previous work done on the Galilean wavelet transforms that are tuned to the velocity. The analysis that we propose in this paper is performed according to criteria based on spatial and temporal position, angular orientation, velocity, angular velocity and spatial scale. The definition of a rotational motion depends upon the axis around which rotation takes place. If the axis is the center of gravity of the object, the motion refers to a *spin* (ball, whirl). If the objet revolves around an external axis, the motion refers to an *orbit*. This paper will mainly focus on spinning and translation in digital image sequences i.e. two-dimensional space and time signals acquired by a camera or a planar sensor. Some principles about sampling and symmetries will be clearly evidenced in this paper from Fourier spectra since the purpose of these continuous wavelets is to be discretized to analyze digital signals acquired by imaging sensors or radars.

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The approach of motion filtering considered in this paper differs fundamentally from other techniques that have been proposed in the literature such as those based on optical flow, pel-recursive, block matching and Bayesian models. The continuous wavelet transform provides motion estimations that are robust not only against image noise and blurr but also against motion noise (i.e. jitter) [3], [6], [7]. Moreover, as a result of both the spatio-temporal filtering and the interpolation wavelet properties, the wavelet technique can resolve temporary occlusion problems. It has been demonstrated that the wavelet transform behaves as a matched filter and performs minimum-mean-square error estimation of the motion parameters [7] [6].

The study of the rotational motion is part of the harmonic oscillator. The study of the translational or linear motion belongs to the Galilean wavelet. Translational motion composes with the rotational motion to put the harmonic oscillator on a trajectory (i.e. spinning on a trajectory). Fourier analysis shows that both motions keep distinct signatures and that trajectories can be built independently from the spinning motion whose estimation can be addressed afterwards. A tracking algorithm is eventually presented that exploits a Lie algebra for damped harmonic oscillator, a Kalman filter, the Galilean and the rotational wavelet transforms separately.

2. ROTATIONAL MOTION AND SYMMETRIES

The next five sections are devoted to describe the spatio-temporal transformation for the rotational and the translational motion respectively. The aim is first to show intuitively that the spectrum of those transformations are significantly apart. Consequently, translational motion $\vec{v}$ is measurable independently from the rotational motion θ. Several fundamental consequencies can be deduced from these observations in terms of wavelet and signal processing.

To study the relations of symmetry between space and time domain and Fourier domain, let us consider a square-shape signal spinning with uniform angular velocity θ about its center of gravity. The still square shape is defined as

$$\begin{aligned} \chi_c(\vec{x},t) &= 1 \quad -1 \leq x(1), x(2) \leq +1 \\ &= 0 \quad \text{otherwise} \end{aligned} \tag{1}$$

The rotational transformation reads as

$$R_{\theta t} = \begin{pmatrix} \cos\theta t & -\sin\theta t \\ \sin\theta t & \cos\theta t \end{pmatrix} \tag{2}$$

The Fourier transform of the rotating square is then

$$\hat{\tilde{\chi}}_c(\vec{k},\omega) = \int_{\mathbf{R}^2\times\mathbf{R}} dt d^2\vec{x}\ \chi_c\left[R_{\theta t}\vec{x}, t\right]\ e^{-i(\vec{k}.\vec{x}\ +\ \omega t)} \tag{3}$$

$$= \int_{\mathbf{R}} dt\ e^{-i\omega t}\ \frac{\sin\left[k_1\cos\theta t + k_2\sin\theta t\right]}{k_1\cos\theta t + k_2\sin\theta t} \quad (4)$$

$$\times \frac{\sin\left[k_2\cos\theta t - k_1\sin\theta t\right]}{k_2\cos\theta t - k_1\sin\theta t} \quad (5)$$

where we use $\tilde{}$ and $\hat{}$ for moving signal and Fourier transform respectively. ω and $\vec{k}$ are the temporal and the spatial frequencies. If t tends to zero, the Fourier transform is that one of the still square with $\frac{\sin k}{k}$ functions. To provide a simple representation of the spectrum, let us section according to $k_2 = 0$ and expand into series both functions $\sin[x] \approx x - \frac{x}{3!}$, the Fourier transform simplifies as

$$\begin{aligned} \hat{\tilde{\chi}}_c(k_1,\omega) \quad \approx \quad & C\ \int_{\mathbf{R}} dt\ e^{-\omega t}\ \left(1 - k_1^3 + \frac{k_1^6}{8}\right) \\ & -C\ \int_{\mathbf{R}} dt\ e^{-\omega t}\ \frac{k_1^6}{8}\cos(4\theta)t \end{aligned} \quad (6)$$

where C is a calculation constant. This shows the existence of one component in the plane $\omega = 0$ (in the left term) and two components equally distant from the k_1-axis (in the right term). The distance of these components with the plane $\omega = 0$ is $4 \times \theta$. If a square is rotating with an angular velocity of $\theta = 45^o$ per image (i.e. between two consecutive image), the spectrum reaches the Shannon sampling bound beyond which the spectrum is undergoing aliasing (Figures 3 4 5). The spin of any symmetrical polygon of 2n edges will then reach the bound at $\frac{\pi}{2n}$.

3. ROTATIONAL MOTION

The transformation of the rotational motion that has been stated in Equation 2 is part of the harmonic oscillator which belongs to the Weyl-Heisenberg group [5]. The group element and the matrix representation are defined as

$$g \;=\; \{\theta_0, \theta, \tau_1\} \;\equiv\; \begin{pmatrix} 1 & \theta & \theta_0 \\ 0 & 1 & \tau_1 \\ 0 & 0 & 1 \end{pmatrix} \quad (7)$$

where θ_0, θ and $\tau_1 \in \mathbf{R}$ stand for initial angular orientation, angular velocity and time translation respectively. The law of composition and the inverse element are defined by the corresponding matrix multiplication. The group representation T in the temporal Hilbert space of the signal reads

$$\left[T(g)\Psi\right](t) \;=\; e^{im\theta_0}\ e^{i\theta t}\ \Psi\left(t - \tau_1\right) \quad (8)$$

where $m \in \mathbf{R}$. This representation is unitary, irreducible and but not square integrable. Sectioning the group with θ_0, i.e. loosing the initial angular position, yields a square integrable representation then wavelet transforms with representation

$$\left[T(g)\Psi\right](t) \;=\; e^{i\theta t}\ \Psi\left(t - \tau_1\right) \quad (9)$$

These wavelet transforms are tuned to the angular velocity of interest θt. The harmonic oscillator will be now put into motion on a trajectory.

4. TRANSLATIONAL MOTION

The spectrum of the translational or linear motion also occupies a particular part of the Fourier domain [3], it is located in a plane orthogonal to the velocity vector $(\vec{v}, 1)$ of equation $\omega + \vec{k}\vec{v} = 0$. The wavelet tuned to translational motion are also located in this plane, they have been described in [3], [6], [7]. A simplified version is considered here that takes into account spatial scale $a \in \mathbf{R}^+\backslash\{0\}$, spatial translation $\vec{b} \in \mathbf{R}^2$, temporal translation τ_2, spatial orientation $r_{\theta_0} \in SO(2)$ and velocity $\vec{v} \in \mathbf{R}^2$. The group element reads

$$g \;=\; \{\vec{b}, \tau_2, \vec{v}, a, r_{\theta_0}\} \;\equiv\; \begin{pmatrix} a\ r_{\theta_0} & \vec{v} & \vec{b} \\ 0 & 1 & \tau_2 \\ 0 & 0 & 1 \end{pmatrix} \quad (10)$$

The group representation in the spatio-temporal Hilbert space of the signal reads [7]

$$\left[T(g)\hat{\Psi}\right](\vec{k}, \omega) \;=\; a^{\frac{1}{2}}\ e^{i(\omega\tau_2\ +\ \vec{k}\vec{b})}\hat{\Psi}\left(ar_{-\theta_0}\vec{k}, \omega + \vec{k}\vec{v}\right) \quad (11)$$

This representation is unitary, irreducible and square integrable at the expense of considering the group extension (to be interpretated as the uncertainty between velocity and translation [7]). This generates the family of the Galilean wavelets that are tuned to all the local parameters of the motion. The parameter of the local spatial orientation that is provided in r_{θ_0} allows estimating any local value of θ at any time; this solves the problem opened in previous section after sectioning with θ_0.

5. COMPOSING BOTH TRANSFORMATIONS

To derive the group that takes into account the harmonic oscillator on the trajectory, we need to compose both transformations i.e.

$$\begin{aligned} & \begin{pmatrix} a\ r_{\theta_0}r(\theta t) & \vec{v} & \vec{b} \\ 0 & 1 & \tau \\ 0 & 0 & 1 \end{pmatrix} \\ = & \begin{pmatrix} a\ r_{\theta_0} & \vec{v} & \vec{b} \\ 0 & 1 & \tau \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} r(\theta t) & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \end{aligned} \quad (12)$$

This provides the law of composition of a new group with representation in space and time domain

$$\left[T(g)\Psi\right](\vec{x}, t) \;=\; a^{-\frac{1}{2}}\ \Psi\left(ar_{\theta_0}r(\theta t)\vec{x} - \vec{v}t - \vec{b}, t - \tau\right) \quad (13)$$

These representations are still unitary, irreducible and square integrable. These are the spatio-temporal rotational continuous wavelet transforms fitting to our analysis purpose.

6. ROTATIONAL WAVELET TRANSFORM

Let the signal subject to analysis be $S(\vec{x}, t)$ and be defined in the Hilbert space $L^2(\mathbf{R}^2 \times \mathbf{R}, d^2\vec{x}dt)$. The wavelet transform $W[S; g_t]$, with $g_t = \{\vec{b}, \tau, \vec{v}, a, \theta_0, \theta\}$ is defined as an inner product

$$\begin{aligned} W[s; g_t] \quad &= \quad c_\Psi^{-1/2}\ < \Psi_{g_t} \mid S > \\ &= \quad c_\Psi^{-1/2} \int_{\mathbf{R}^2\times\mathbf{R}} d^2\vec{x}dt\bar{\Psi}_{g_t}\left(\vec{x}, t\right) S\left(\vec{x}, t\right) \end{aligned}$$

where the overbar $\bar{}$ stands for the complex conjugate. The wavelet, Ψ, is a *mother wavelet*. It must satisfy the condition of admissibility calculated from square integrability [7]

$$c_\Psi \;\approx\; (2\pi)^3 \int_{\mathbf{R}^2\times\mathbf{R}} d^2\vec{k}\ d\omega\ \frac{|\widehat{\Psi}(\vec{k}, \omega)|^2}{|\vec{k}|^2} \;<\; \infty\ .$$

7. TRACKING OF ROTATIONAL MOTION

In this section, we develop an algorithm combining Kalman filter, rotational and translational wavelet transforms, and a dynamical structure (a Lie algebra) for the damped, harmonically driven, harmonic oscillator. According to the previous sections, the estimation and the tracking may be done globally on all the parameters or split into two parts focusing first on the trajectory and secondly, if there is interest, on the spinning motion. That kind of tracking is obtained by diagonalizing the prediction matrix. Let the state equation of the system be given by

$$\vec{u}(t) \;=\; \{x(t), v(t), \cos[\omega t], \sin[\omega t]\}^T \qquad (14)$$

The evolution is given by the prediction equation

$$\vec{u}(t+\tau) \;=\; e^{t\Omega}\vec{u}(t) \;=\; e^{tA^T}\vec{u}(t) \qquad (15)$$

where Ω is an operator of the Lie algebra (a subalgebra of $gl(4, \mathbf{R})$ and A is a 4×4 matrix obtained from the operator Ω. It can be formulated as

$$A \;=\; \begin{pmatrix} 0 & -\alpha & 0 & 0 \\ 1 & -\beta & 0 & 0 \\ 0 & \gamma & 0 & \omega \\ 0 & 0 & -\omega & 0 \end{pmatrix} \qquad (16)$$

we have also

$$\vec{u}(t+\tau) \;=\; \{x(t+\tau), v(t+\tau), \cos[\omega(t+\tau)], \sin[\omega(t+\tau)]\}^T \qquad (17)$$

To yield an interesting closed form of the prediction equation that splits translational motion from rotational motion, it is desirable to diagonalize the matrix e^{tA^T}. Eventually, the calculations lead to two 2-component relations

$$\begin{pmatrix} x(t+\tau) \\ v(t+\tau) \end{pmatrix} \;=\; V^{-1}D_1V \begin{pmatrix} x(t) \\ v(t) \end{pmatrix} + \left[\left(V^{-1}D_1V\right)C - C\left(U^{-1}\; D_2\; U\right)\right] \begin{pmatrix} \cos(\omega t) \\ \sin(\omega t) \end{pmatrix} \qquad (18)$$

and

$$\begin{pmatrix} \cos[\omega(t+\tau)] \\ \sin[\omega(t+\tau)] \end{pmatrix} \;=\; \left(U^{-1}D_2U\right) \begin{pmatrix} \cos(\omega t) \\ \sin(\omega t) \end{pmatrix} \qquad (19)$$

with $D_1 = diag(e^{\lambda_1 t}, e^{\lambda_2 t})$, $D_2 = diag(e^{i\omega t}, e^{-i\omega t})$ two diagonal matrices, and

$$V \;=\; \begin{pmatrix} \lambda_2 & -1 \\ -\lambda_1 & 1 \end{pmatrix}, \qquad U \;=\; \frac{1}{2}\begin{pmatrix} \lambda_2 & -1 \\ -\lambda_1 & 1 \end{pmatrix} \qquad (20)$$

It is possible to check in 18 that differentiating $\vec{x}(t+\tau)$ yields $\vec{v}(t+\tau)$. The tracking strategy is based on combining *Kalman filters* and *wavelet transform*. The state of the Kalman filter is currently composed of some 14 or all the wavelet parameters ([7] for translational case).

The observation equation also exploits the wavelet transform as a motion-based extraction tool tuned to the current exact state parameters. The CWT captures and isolates the selected objects from the scene S to provide a display I,

$$I(\vec{b}, \tau) \;= <\Psi_{g_{opt}}|S+V> \;. \qquad (21)$$

I is the segmented image of the selected object, displayed alone at its correct location; S is the original signal under analysis, $V(\vec{b}, \tau)$ is the noise produced by the optical sensors and g_{opt} is the set of optimal parameters corresponding to the estimation of state parameters which is performed with Morlet wavelets as described in the next section.

8. MORLET WAVELET AND APPLICATIONS

The applications presented in this paper are based on Morlet wavelets. An anisotropic *Morlet wavelet* is admissible as an continuous wavelet in the rotational and translational family; it defines a non-separable filter

$$\Psi(\vec{x}, t) = e^{i\vec{k}_0\vec{X}}\; e^{-\frac{1}{2}<\vec{X}\;|\;C\vec{X}>} - e^{-\frac{1}{2}<\vec{k}_0\;|\;D\vec{k}_0>}\; e^{-\frac{1}{2}<\vec{X}|C\vec{X}>}$$

where $\vec{X} = (\vec{x}, t)^T \in \mathbf{R}^2 \times \mathbf{R}$, C is a positive definite matrix and, $D = C^{-1}$. For $2D + T$ signals, $\left[C = \begin{pmatrix} 1/\epsilon_x & 0 & 0 \\ 0 & 1/\epsilon_y & 0 \\ 0 & 0 & 1/\epsilon_t \end{pmatrix}\right]$ where the ϵ factors introduce anisotropy in the wavelet shape. Figures 3, 4, 5 presents three configurations of rotational wavelets: four symmetrical Morlet wavelets are considered in the space and time domain, and successively put in rotational motion, put into velocity and transformed in the Fourier domain where the inner product with signal is taking place. Figure 6 presents the detection of scale and angular velocity using the square of the modulus of the rotational Morlet wavelet. This estimation is performed by integrating over the whole image sequence the square modulus of wavelet transform at velocity $\vec{v}_0 = (-2.7, -0.1)$ to determine

$$\max\arg_{\theta,a} \int_{\mathbf{R}^2}\int_{\mathbf{R}} d\vec{b}d\tau \;\; |<\Psi_{\vec{v},\theta t,a,\vec{b},\tau}|S>|^2_{\vec{v}=\vec{v}_0} \qquad (22)$$

9. CONCLUSIONS

This paper has presented a new practical and efficient way of estimating and tracking trajectories with new spatio-temporal wavelets. The technique is robust against image noise, motion jitter and temporary occlusions [3], [6], [7]. The motion analysis performed by the wavelet transform may be split into two distinct parts: the first estimates locations and velocities and tracks the trajectory, the second estimates the angular velocity. The object in motion is clearly subject to two distinct motions, a displacement and a spinning. This analysis is supported by the Fourier signatures of these motions. Some further theoretical work about this new wavelet family is going to be presented to demonstrate the link between the classical Lie groups, the rotation in 2D and 3D, the harmonic oscillator, the wavelet representation Kalman filtering and the symmetry properties. The work also generalizes the previous works that have been done on the Galilean wavelets [3], [6], [7]. Further numerical work to be presented will also include selective reconstructions of rotating objects.

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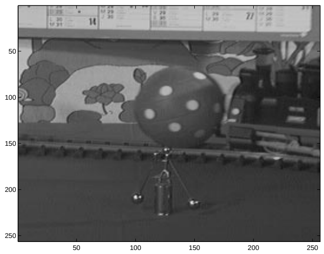

Figure 1. Image 15 in the Caltrain sequence ([256 × 256] × 32 images): analysis of the ball which is spinning (four white spots), and moving on a quasi-horizontal trajectory: deceleration from images 1 to 14 followed by acceleration from images 15 to 32.

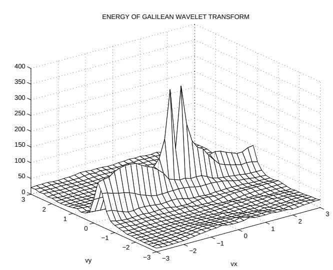

Figure 2. Analysis of the velocities contained in the ball at image 18. The ball is windowed and the square modulus of the Galilean wavelet is integrated on the ball domain at image 18: two signatures are visible, the two symmetric peaks for the rotational motion and a "domed wall" for the accelerated motion.

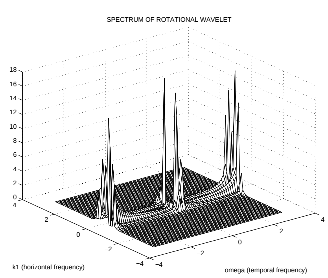

Figure 3. Four symmetrical rotational Morlet wavelet functions close to $\theta t = (\pi/4)t$, and $\vec{v} = \{0, 0\}$: square modulus in plane of the (ω, k_x) axes i.e. $k_y = 0$. This yields critical temporal sampling.

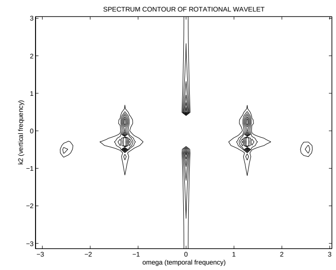

Figure 4. Four symmetrical rotational Morlet wavelet function at $\theta t = (\pi/10)t$, and $\vec{v} = \{0, 0\}$: contours of the square modulus in plane of the (ω, k_y) axes i.e. $k_y = 0$. This yields $4 \times \pi/10$ on the temporal frequency axis.

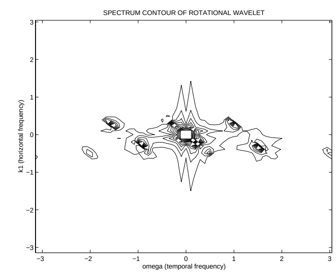

Figure 5. Four symmetrical rotational Morlet wavelet functions at $\theta t = (\pi/8)t$, and $\vec{v} = \{1, 0\}$: square modulus in plane of the (ω, k_x) axes i.e. $k_y = 0$.

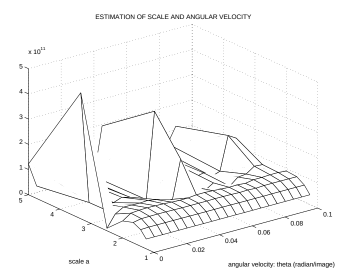

Figure 6. Estimation with rotational wavelets of the angular velocity and the scale of the rotating ball in caltrain sequence. The diagram sketches the square of the modulus of the wavelet transform at $\vec{v} = \{-2.7, 0.1\}$. The component at $\theta = 0$ stands for the non rotating structures. The component at $\theta = 0.045, a = 3.3$ is the actual ball contribution which is observed rotating of $\pi/2$ over 32 images. $\theta = 0.09$ is a harmonic.

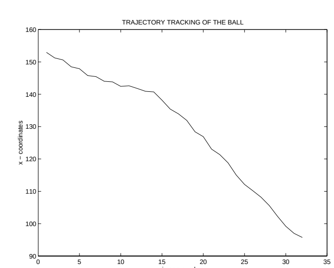

Figure 7. Tracking of the moving ball position in the Caltrain sequence with Galilean wavelets. At image 15, the ball is pushed and keeps constant speed. Coordinates decreases as a result of the motion steering to the left.