Channel Identification with Doppler and Time Shifts Using Mixed Training Signals*

Xiang-Gen Xia
Department of Electrical and Computer Engineering
University of Delaware, Newark, DE 18716. (xxia@ee.udel.edu)

Abstract

In this paper, we present a method to identify channels with both Doppler and time shifts using mixed training signals. The training signals we use consist of two parts, where one part is a constant and the other part is a conventional training signal, such as a pseudo-random signal. These two parts may be separated in either the time or the frequency domain. We provide a necessary and sufficient condition on the channel identifiability in terms of the time and Doppler shifts when the mixed training signals are used. It can be shown that the condition holds almost surely in most cases of interests in practice. Some numerical examples are also presented.

1 Introduction

Doppler and time shifts (or delays or spread) usually occur in wireless mobile digital communication systems with high speed transmission, which often causes problems of channel impairments. Due to the Doppler shifts of moving vehicles, the channel is usually modeled as a time variant linear system and is not as well studied as a time-invariant linear channel is. There have been a tremendous amount of researches on time-invariant linear system identification with both blind and non-blind (using training signals) fashions. This is, however, not equally the case for time-variant linear system identification. Some researches on this topic have appeared, such as [1-5], and increasing attentions have being paid mainly because of the need of wireless digital high speed data communications.

In this paper, we focus on the problem of the channel identification in the presence of both Doppler and time shifts by using training signals. Specifically, the following channel model studied in [1] is used. Let $x(t)$ and $y(t)$ be transmitted and received signals, respectively. Then

$$y(t) = \sum_{k=1}^{N_p} \alpha_k x(t-\tau_k) e^{j\omega_k t} + n(t), \tag{1.1}$$

where α_k, τ_k, and ω_k are the amplitude, the time shift, and the Doppler shift of the kth multipath component in the channel, respectively, and N_p is the number of the total multipath components, and $n(t)$ is the channel additive noise. The Doppler shifts $\omega_k \approx 2v\omega_c/c$ with the carrier frequency ω_c, the velocity v of the moving object, and the velocity c of light. The channel identification here is to estimate the unknown parameters $\{(\alpha_k, \tau_k, \omega_k),\ 1 \leq k \leq N_p\}$ through the knowledge of the transmitted and the received signals $x(t)$ and $y(t)$ in (1.1).

In this paper, we propose to use mixed training signals in the above channel identification, which have two parts separated either in the time domain or in the frequency domain. One part of the training signal is a constant and the other part is a pseudo-random signal or other type of linear time-invariant (LTI) channel identification training signals, such as chirps [6] in a low SNR environment. The constant part is used to identify the Doppler shifts ω_k and the other part is used to identify the time shifts τ_k. The corresponding multipath amplitudes α_k are identified using both parts. The synchronization between the detected Doppler and time shifts is also done by using the both parts. Note that not all channels in (1.1) can be identified with this approach. A necessary and sufficient condition in terms of the Doppler and time shifts on the channel identifiability is given. It turns out that almost all channels (1.1) are identifiable with the approach proposed in this paper in most cases of practical interests. The identifiability is built upon the concept of cycles introduced for the Doppler and time shifts.

2 Channel/Mixed Training Signal Analysis and Identifiability

Let us first analyze the received signal $y(t)$ in (1.1). To analyze the identifiability, for convenience we assume the additive noise $n(t)$ in the model (1.1) does not appear, i.e., $n(t) = 0$. Suppose the transmitted training signal $x(t)$ is a constant, say 1. Then (1.1) becomes

$$y(t) = \sum_{k=1}^{N_p} \alpha_k e^{j\omega_k t}. \tag{2.1}$$

If all Doppler shifts ω_k, $k = 1, 2, ..., N_p$, are distinct, then, by taking a certain discrete Fourier transform for a segment of the received signal $y(t)$, all Doppler shifts ω_k and multipath amplitude coefficients α_k may be detected. If there are duplications of the Doppler shifts ω_k, all the (distinct) Doppler shifts can still be detected with the above

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method but not all the coefficients α_k. For instance, assume $\omega_1 = \omega_2$ and it is not equal to other ω_k. In this case, equation (2.1) becomes

$$y(t) = (\alpha_1 + \alpha_2) e^{j\omega_1 t} + \sum_{k=3}^{N_p} \alpha_k e^{j\omega_k t}. \qquad (2.2)$$

In this case, only the sum $\alpha_1 + \alpha_2$ of the two coefficients α_1 and α_2 can be detected, which is not enough to detect their individual values α_1 and α_2. However, the Doppler frequencies $\{\omega_1, \omega_2, ..., \omega_{N_p}\}$ are still detectable.

Similarly the time shifts τ_k can be detected in the frequency domain of (1.1) as follows. Taking the Fourier transforms of (1.1) we have

$$Y(e^{j\omega}) = \sum_{k=1}^{N_p} \alpha_k X(\omega - \omega_k) e^{i\tau_k \omega}, \qquad (2.3)$$

where $Y(e^{j\omega})$ and $X(e^{j\omega})$ are the Fourier transforms of $y(t)$ and $x(t)$, respectively. Suppose $X(e^{j\omega})$ is a constant, say 1. Then

$$Y(e^{j\omega}) = \sum_{k=1}^{N_p} \alpha_k e^{i\tau_k \omega}. \qquad (2.4)$$

If all time shifts τ_k, $k = 1, 2, ..., N_p$, are distinct, all these time shifts τ_k and the coefficients α_k can be detected by taking an inverse discrete Fourier transform of (2.4). Similar to the previous time domain analysis, it is not possible to detect all the coefficients α_k when not all the time shifts τ_k are distinct. Furthermore, when for all index k, either ω_k has no repetitions or τ_k has no repetitions, the corresponding coefficients α_k can be detected by the above approach.

When both ω_k, $1 \le k \le N_p$, and τ_k, $1 \le k \le N_p$, are distinct, the order of the coefficients α_k also gives the order for both ω_k, $1 \le k \le N_p$, and τ_k, $1 \le k \le N_p$. When either ω_k, $1 \le k \le N_p$, or τ_k, $1 \le k \le N_p$, has repetitions, two sets of coefficients α_k can be solved from (2.1) and (2.4), and the orders for ω_k and τ_k can also be determined from the matching of the corresponding DFT and IDFT coefficients α_k.

Based on the above analyses, let us consider a training signal $x(t)$ that has two parts either separated in the time domain or in the frequency domain. Without loss of generality we only consider the time domain separation.

When $x(t)$ has two parts separated in the time domain, it has the following form:

$$x(t) = \begin{cases} x_0, & T_0 < t < T_1, \\ x_1(t), & T_1 < t < T_2, \end{cases} \qquad (2.5)$$

where x_0 is a nonzero constant and $x_1(t)$ is a conventional pseudo-random signal or the delta pulse, i.e., its Fourier transform $X_1(e^{j\omega})$ is a constant (flat). In the detection, these two parts are processed separately.

For simplicity we assume that the two part information is available at the same time interval, for example, $[0, T]$, and

$$y_1(t) = \sum_{k=1}^{N_p} \alpha_k e^{j\omega_k t} x_0, \ \ t \in [0, T], \qquad (2.6)$$

and

$$y_2(t) = \sum_{k=1}^{N_p} \alpha_k e^{j\omega_k t} x_1(t - \tau_k), \ \ t \in [0, T]. \qquad (2.7)$$

The goal here is to identify the unknown parameters $\{(\alpha_k, \tau_k, \omega_k), 1 \le k \le N_p\}$ from the above equations (2.6) and (2.7). In the following, we also assume that the sampling interval length of the received signals $y_1(t)$ and $y_2(t)$ is small enough so that all the Doppler shifts ω_k in (2.1) and the time shifts τ_k in (2.4) can be detected by using the discrete Fourier transform (DFT) and the inverse discrete Fourier transform (IDFT) as discussed above.

By the above discussions, we have the following sufficient condition for the identifiability.

Theorem 1: Let $x(t)$ be a training signal with the two parts as described above. Let I and J be any two integer sets such that they do not intersect, i.e., $I \cap J = \phi$, and their union $I \cup J = \{1, 2, ..., N_p\}$. If all the Doppler shifts ω_k for $k \in I$ are distinct and $\omega_i \ne \omega_j$ for $i \in I$ and $j \in J$, and all the time shifts τ_k for $k \in J$ are distinct and $\tau_i \ne \tau_j$ for $i \in I$ and $j \in J$, then the unknown parameters $\{(\alpha_k, \tau_k, \omega_k), 1 \le k \le N_p\}$ are detectable by applying the DFT in the time domain and the IDFT in the frequency domain to the two parts of the received data corresponding to the two parts of the training signal, respectively.

A proof was given in [7].

An obvious case in Theorem 1 is as what was mentioned earlier, i.e., either all ω_k, $1 \le k \le N_p$, or all τ_k, $1 \le k \le N_p$, are distinct, which corresponds to the case of $I = \phi$ or $J = \phi$ in Theorem 1. The identifiability problem now arises from the possible duplications of the Doppler shifts ω_k and the time shifts τ_k as discussed in (2.2). In this case, ambiguities might exist in the detected amplitude coefficients α_k. In the following, we want to provide a necessary and sufficient condition on the identifiability of the coefficients α_k in terms of the Doppler shifts ω_k and the time shifts τ_k.

To study the identifiability of the coefficients α_k using the DFT of $y_1(t)$ in (2.6) and the IDFT of the Fourier transform of $y_2(t)$ in (2.7), let us first see an example. Consider the case $N_p = 4$, $\omega_1 = \omega_2 \ne \omega_3 = \omega_4$, and $\tau_1 = \tau_3 \ne \tau_2 = \tau_4$. In this case, using the DFT and IDFT to $y_1(t)$ and $Y_2(e^{j\omega})$ the following summations can be detected, i.e., β_i and γ_i for $i = 1, 2$ can be detected:

$$\alpha_1 + \alpha_2 = \beta_1 \qquad (2.8)$$

$$\alpha_3 + \alpha_4 = \beta_2 \qquad (2.9)$$

$$\alpha_1 + \alpha_3 = \gamma_1 \qquad (2.10)$$

$$\alpha_2 + \alpha_4 = \gamma_2. \qquad (2.11)$$

Clearly, it is not possible to solve for α_i for $i = 1, 2, 3, 4$ from the above equations. The mathematical reason for this is of course that the coefficient matrix of these equations does not have a full rank. There is, however, another intuitive reason as follows. Let us start with the unknown α_1 in (2.8): α_1 is connected to α_2 via (2.8); α_2 is connected to α_4 via (2.11); α_4 is connected to α_3 via (2.9); and finally α_3 is connected back to α_1 via (2.11). One can see that there is a cycle between the unknowns α_i for $i = 1, 2, 3, 4$, as shown in Fig. 1, which causes to the unsolvability of the coefficients α_k.

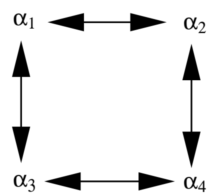

Figure 1: A cycle between α_k.

It was shown in [7] that the above cycle pattern causes the unsolvability is true not only for the above particular example but also for general cases. Notice that the cycle length in Fig. 1 is 4. A general setting of the repetitions of ω_k and τ_k is as follows. Let I_1,..., I_f be a *partition* of the integer set

$$\mathcal{I} \triangleq \{1, 2, ..., N_p\}$$

such that all the Doppler shifts ω_k for $k \in I_l$ for any fixed l are equal, i.e.,

$$\omega_k = \tilde{\omega}_l \quad \text{for all } k \in I_l, \tag{2.12}$$

where "partition" means any two sets I_{l_1} and I_{l_2} for $l_1 \neq l_2$ do not intersect, i.e., $I_{l_1} \cap I_{l_2} = \phi$ for $l_1 \neq l_2$, and the union of all I_l is the integer set $\mathcal{I}$, i.e.,

$$\bigcup_{l=1}^{f} I_l = \mathcal{I},$$

and each set I_l is not empty. Let J_1,..., J_g be another *partition* of the integer set $\mathcal{I}$ such that all the time shifts τ_k for $k \in I_l$ for any fixed l are equal, i.e.,

$$\tau_k = \tilde{\tau}_l \quad \text{for all } k \in J_l. \tag{2.13}$$

Similar to the discussion in (2.2), the following summations can only be detected from the DFT of $y_1(t)$ in (2.6) and the IDFT of the Fourier transform $Y_2(e^{j\omega})$ of $y_2(t)$ in (2.7):

$$\sum_{k \in I_l} \alpha_k = \beta_l, \quad 1 \le l \le f, \tag{2.14}$$

$$\sum_{k \in J_l} \alpha_k = \gamma_l, \quad 1 \le l \le g, \tag{2.15}$$

where β_l and γ_l are the detected values.

Theorem 2: Channel (1.1) with Doppler and time shifts and a mixed training signal as described before is identifiable *if and only if* there is no any cycles as shown in Fig. 1 for the variables in (2.14)-(2.15) with length at least 4.

A proof is given in [7] by precisely introducing the concept of cycles for variables α_k or the Doppler and time shifts ω_k and τ_k.

3 Probability Analysis of the Identifiability

In this section, we show the probability for the channel identifiability, i.e., for the necessary and sufficient condition in Theorem 2 to hold. The condition is in terms of the Doppler and time shifts ω_k and τ_k. Since in practical digital processing, these Doppler and time shifts are quantized to finite values. For convenience, we assume that there are total M_d possible different values of the Doppler shifts and total M_t possible values for the time shifts. In other words, each ω_k may take one of M_d different values

$$D_{range} = \{v_{d,1}, v_{d,2}, ..., v_{d,M_d}\}, \tag{3.1}$$

and each τ_k may take one of M_t different values

$$T_{range} = \{v_{t,1}, v_{t,2}, ..., v_{t,M_t}\}. \tag{3.2}$$

For example, $D_{range} = \{-50\text{Hz}, -49\text{Hz}, ..., 50\text{Hz}\}$ and $T_{range} = \{0\mu\text{s}, 1\mu\text{s}, ..., 100\mu\text{s}\}$. The two numbers M_d and M_t can be determined when the Doppler spread width f_m and the rms time spread width σ_τ are known for a given channel.

As we mentioned earlier, we have a sufficient condition in Theorem 1 and a necessary and sufficient condition in Theorem 2. These two conditions coincide for $N_p = 1, 2, 3$. Although the probability expression for the sufficient condition in Theorem 1 to hold for a general N_p is complicated, the probabilities for the conditions in Theorems 1-2 to hold were calculated in [7] when $N_p = 4$.

Three probability curves are plotted in Fig. 2, where we set $M_d = M_t$ and the x-axis indicates the variable M_d, which is from 4 to 101. The first curve is for the obvious case in Theorem 1 when all ω_k or all τ_k are distinct. One can clearly see that the probability for the necessary and sufficient condition in Theorem 2 is above the one for sufficient condition in Theorem 1. When the total numbers M_d and M_t of the possible Doppler and time shifts are large relative to the total number N_p of multipath components in a channel, the necessary and sufficient condition in Theorem 2 holds almost surely, i.e., the probability is very close to 1.

Although in the above we only studied the case when $N_p = 4$, for a general N_p the probability for the necessary and sufficient condition in Theorem 2 to hold can be approximated by the following formula, [7].

Probability (the condition in Theorem 2 holds)

$$\approx 1 - \frac{\begin{pmatrix} M_d \\ 2 \end{pmatrix} \begin{pmatrix} N_p \\ 4 \end{pmatrix} \begin{pmatrix} 4 \\ 2 \end{pmatrix} \begin{pmatrix} M_t \\ 2 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix}}{\begin{pmatrix} M_d M_t \\ N_p \end{pmatrix}}, \tag{3.3}$$

which are very close to 1 when M_d and M_t are relatively larger than N_p.

4 Numerical Simulations

In the following simulations, we use $N_p = 4$, and

$$\frac{\omega_k}{2\pi} \in D_{range} = \{-50\text{Hz}, -49\text{Hz}, ..., 50\text{Hz}\}, \quad k = 1, 2, 3, 4,$$

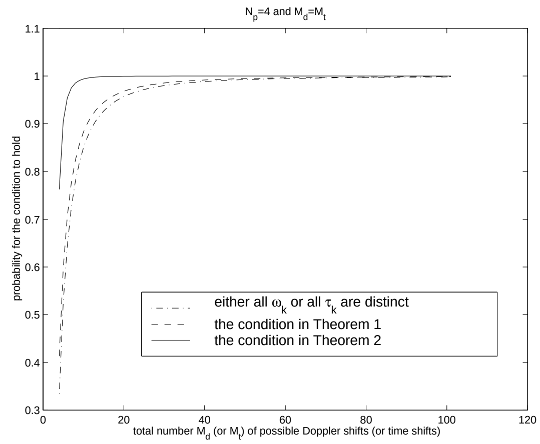

Figure 2: Probabilities for the conditions in Theorems 1-2 to hold when $N_p = 4$.

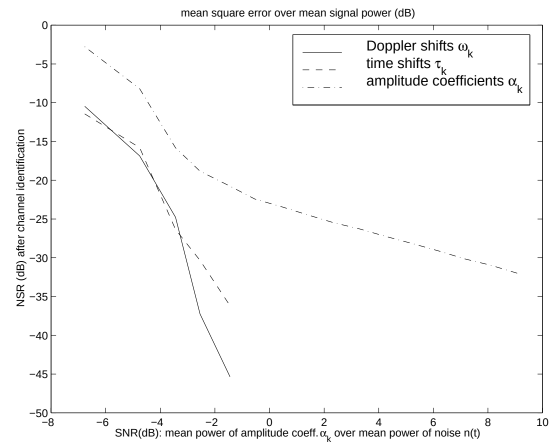

Figure 3: MSE for the detected ω_k, τ_k, and α_k, vs. the SNR.

and

$$\tau_k \in T_{range} = \{0\mu\text{s}, 1\mu\text{s}, ..., 100\mu\text{s}\}, \quad k = 1, 2, 3, 4.$$

The Doppler and time shifts ω_k and τ_k for $k = 1, 2, 3, 4$ are randomly chosen from the above sets D_{range} and T_{range}, respectively, such that all pairs (ω_k, τ_k) for $k = 1, 2, 3, 4$ are distinct. The amplitude coefficients α_k for $k = 1, 2, 3, 4$ are randomly chosen from Gaussian random processes with all possible real values.

For the first piece $y_1(t)$ of data, the sampling rate is chosen as $1/T = 128$, i.e., $-63 \le l \le 64$,

$$y_1[l] = y_1(l/128) = \sum_{k=1}^{N_p} \alpha_k e^{jl\omega_k/128} + n_1(l/128). \quad (4.1)$$

For the second piece $Y_2(\omega)$ of data, the sampling rate is chosen $1/T = 128/(2\pi)$, i.e., $0 \le l \le 127$,

$$Y_2[l] = Y_2(2\pi l/128) = \sum_{k=1}^{N_p} \alpha_k e^{jl2\pi\tau_k/128} + n_2(2\pi l/128). \quad (4.2)$$

Fig. 3 shows the curves of the ratios of the mean square errors (MSE) of the true ω_k, τ_k, and α_k, and their detected values over their mean powers. The x-axis is the ratios of the mean powers of the amplitude coefficients α_k over the variance of the additive noise n_i, $i = 1, 2$, in (4.1)-(4.2). In this Fig. 1, 10000 Monte Carlo simulations are implemented.

5 Conclusions

In this paper, we proposed a channel identification algorithm using a mixed training signal, where the channel has both the Doppler shifts and time shifts. The mixed training signals consist of two parts with one part constant and the other part a conventional training signal, such as a pseudo-random signal. These two parts of signals may be separated either in the time domain or in the frequency domain. A necessary and sufficient condition was given for the channel identifiability based on the mixed training signal approach. A probability analysis for the identifiability was presented. It turns out that almost all channels of practical interests are identifiable. Some numerical examples were presented.

References

[1] M. D. Hahm, Z. I. Mitrovski, and E. L. Titlebaum, "Deconvolution in the presence of Doppler with application to specular multipath parameter estimation," *IEEE Trans. on Signal Processing*, vol. 45, pp.2203-2219, Sept. 1997.
[2] H. A. Cirpan and M. K. Tsatsanis, "Maximum likelihood blind channel estimation in the presence of Doppler shifts," preprint, submitted to *IEEE Trans. on Signal Processing*, Nov. 18, 1996.
[3] H. Liu, G. B. Giannakis, and M. K. Tsatsanis, "Time-varying system identification: a deterministic blind approach using antenna arrays," *Proc. 30th Conf. Info. Sci. Syst. (CISS'96)*, Princeton Univ., Princeton, NJ, March 20-22, 1996.
[4] M. K. Tsatsanis and G. B. Giannakis, "Subspace methods for blind estimation of time-varying FIR channels," *IEEE Trans. on Signal Processing*, 1997, to appear.
[5] M. Stojanovic, J. Catipovic, and J. G. Proakis, "Adaptive multichannel combining and equalization for underwater acoustic communications," *Journal of Acoustical Society of America*, pp.1621-1631, Sept. 1993.
[6] X.-G. Xia, "System identification using chirp signals and time-variant filters in the joint time-frequency domain," *IEEE Trans. on Signal Processing*, vol. 45, pp.2072-2084, Aug. 1997.
[7] X.-G. Xia, "Channel identification in the presence of Doppler using mixed training signals," technical report #97-9-2, Department of Electrical and Computer Engineering, University of Delaware, Sept. 1997.