A LEAST-SQUARES INTERPRETATION OF THE SINGLE-STAGE MAXIMIZATION CRITERION FOR MULTICHANNEL BLIND DECONVOLUTION

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ABSTRACT

In order to attain multichannel blind deconvolution of linear time-invariant nonminimum-phase dynamic systems, Inouye and Habe proposed in 1995 a single-stage maximization criterion. The criterion function is the sum of squared forth-order cumulants of the equalizer outputs, and the coefficients of the equalizer are determined at once. On the other hand, one of possible approaches for multichannel blind deconvolution is to construct an equalizer based on the system identified by higher-order cumulant-matching. In this paper, it is shown that the single-stage maximization criterion is equivalent to a least-squares fourth-order cumulant-matching criterion after multichannel pre-whitening of channel outputs. This result provides us with an important interpretation of the single-stage maximization criterion.

1. INTRODUCTION

The problem of multichannel blind deconvolution of linear time-invariant (LTI) dynamic systems has received increasing attention in the past few years beginning with [1]. It arises in a wide variety of applications; in array processing for wideband sources under multipath propagation, in speech and image enhancement, in digital communication. See [1]–[6] and references therein.

In 1995, Inouye and Habe [1] proposed a multistage maximization criterion and a single-state maximization criterion to attain multichannel blind deconvolution of LTI dynamic systems or dynamically mixing signals. The former criterion can be easily implemented as algorithms [6] because it is source-iterative or channel-iterative, i.e., the sources are extracted at each channel and cancelled by one-by-one. The latter criterion is yet difficult to implement as algorithms [6][3], because the sources are extracted at once.

In this paper, we prove that the single-stage maximization criterion for attaining multichannel blind deconvolution is equivalent to a least-squares fourth-order cumulant-matching criterion, provided that multichannel whitening of channel outputs is performed ahead. This is an extension of the result by Wax and Anu [7] in the multichannel static case to the multichannel dynamic case. This equivalence gives us an alternative interpretation of the single-stage maximization criterion.

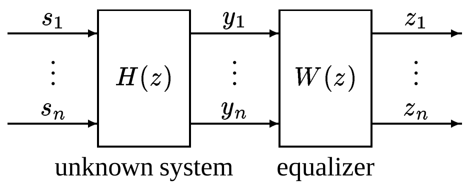

Figure 1: The unknown system and an equalizer

2. MULTICHANNEL BLIND DECONVOLUTION

Let us consider a multichannel linear tine-invariant (LTI) and generally non-causal system described by

$$y(t) = \sum_{k=-\infty}^{\infty} h(k)s(t-k), \tag{1}$$

where $y(t)$ is an n-column output vector, $s(t)$ is an n-column input vector, and $\{h(k)\}$ is an $n \times n$ matrix impulse response sequence. To retrieve the input signal, we process the output signals by an $n \times n$ LTI equalizer $W(z)$ given by

$$W(z) = \sum_{k=-\infty}^{\infty} w(k)z^{-k}. \tag{2}$$

Let us denote the ith component of a vector s by s_i and express the equalizer output as $z(t)$. We allow all of the above variables to be complex-valued. Then the cascade connection of the system and the equalizer is illustrated in the schematic diagram in Fig. 1.

The goal of the blind deconvolution problem is to construct an equalizer recovering the original input signal only from the corresponding output signal of the system.

In the sequel, let γ_x denote the fourth-order cumulant of a complex-valued scalar random variable x, defined as

$$\gamma_x := \mathrm{cum}\{x, x^*, x, x^*\}, \tag{3}$$

where superscript $*$ stands for the complex conjugation of a complex number. Then we make the following basic assumptions on the systems and signals involved.

(A1) The system $H(z)$ is unknown. It is stable in the sense that

$$\sum_{k=-\infty}^{\infty} ||h(k)|| < \infty, \tag{4}$$

where $||\cdot||$ denotes the Euclidean matrix norm. This implies that it has no zero on the unit circle.

(A2) The transfer function $H(z)$ of the unknown system is of full rank on the unit circle $|z| = 1$.

(A3) The input sequence $\{s(t)\}$ is a zero-mean and non-Gaussian random vector process, whose component processes $\{s_i(t)\}$ for $i = 1, \ldots, n$ are mutually independent. Moreover, each component process $\{s_i(t)\}$ is an independently and identically distributed (i.i.d.) process with nonzero variance and nonzero fourth-order cumulant.

(A4) The equalizer $W(z)$ is stable.

For the blind deconvolution of the unknown system, we can use only the outputs. Thus there are inherent ambiguities in the multichannel blind deconvolution problem and hence the multichannel blind deconvolution problem is formulated as follows [1] : Find an equalizer $W(z)$ so that the transfer function of the combined system takes the form of

$$W(z)H(z) = \Lambda(z)DP, \tag{5}$$

where $\Lambda(z)$ is a diagonal matrix with diagonal entries z^{-l_i} for $i = 1, \ldots, n$ with l_i being an integer, D is an $n \times n$ constant diagonal matrix, and P is an $n \times n$ permutation matrix.

In order to eliminate the magnitude ambiguity D, we may assume at the outset that the variance of each component $s_i(t)$ of the input signal is one by dividing $s_i(t)$ by the square root of the variance.

3. A SINGLE-STAGE MAXIMIZATION CRITERION

To solve the blind deconvolution problem, Inouye and Habe proposed the following criterion [1]: Maximize $\sum_{i=1}^{n} |\gamma_{z_i}|^2$

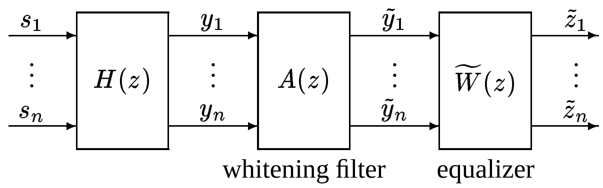

Figure 2: The cascade system of an unknown system followed by a whitening filter and an equalizer

subject to $E\{z(t+k)z^\dagger(t)\} = I\delta(k)$ for $k = 0, \pm 1, \ldots,$ where $z(t)$ is the output of the equalizer, $E\{\cdot\}$ and $\dagger$ are respectively expectation and complex conjugate transposition, I denotes the $n \times n$ identity matrix, and $\delta(k)$ stands for the Kronecker delta. This criterion is referred to as the single-stage maximization criterion, because it determines all of the coefficients of the equalizer at once. It is shown in [6] that, if there is no additive noise, then this criterion gives an exact solution to multichannel blind deconvolution problem.

To simplify the constrains in the criterion, we first whiten the output signal $y(t)$. This whitening procedure corresponds to the one for the blind deconvolution of single-input and single-output (SISO) dynamic systems by Shalvi and Weinstein [8] and also the one for the blind deconvolution of instantaneous mixtures by Comon [9].

The cascade connection of the unknown system followed by a whitening filter is illustrated in the schematic diagram in Fig. 2, where $A(z)$ is a whitening filter of $y(t)$ and $\tilde{y}(t)$ is its output given by

$$\tilde{y}(t) = \sum_{k=0}^{\infty} a(k)y(t-k). \tag{6}$$

It is noted that the cascade system $\widetilde{H}(z) = A(z)H(z)$ satisfies the paraunitary condition

$$\widetilde{H}(e^{j\omega})\widetilde{H}^\dagger(e^{j\omega}) = I, \qquad \omega \in [-\pi, \pi). \tag{7}$$

Next, we have to find an equalizer, denoted by $\widetilde{W}(z)$, for the cascade system $\widetilde{H}(z)$. We can see $\widetilde{W}(z)$ is also paraunitary since $E\{z(t+k)z^\dagger(t)\} = I\delta(k)$. Thus the criterion amounts to

$$\arg \max_{\{\tilde{w}(k)\}\in\mathcal{P}} \sum_{i=1}^{n} |\gamma_{\tilde{z}_i}|^2, \tag{8}$$

where $\mathcal{P}$ is the set of $n \times n$ matrix sequences whose z transforms are paraunitary and $\tilde{z}(t)$ is the output of the equalizer $\widetilde{W}(z)$ given by

$$\tilde{z}(t) = \sum_{k=-\infty}^{\infty} \tilde{w}(k)\tilde{y}(t-k). \tag{9}$$

4. AN ALTERNATIVE INTERPRETATION OF THE SINGLE-STAGE MAXIMIZATION CRITERION

In this section, we show that the single-stage maximization criterion is equivalent to a least-squares cumulant-matching criterion if the pre-whitening procedure is performed.

Let us define the fourth-order cumulant of an n-vector process $\{x(t)\}$ as in [10] and [11] as follows: The fourth-order cumulant $c_x(m_1, m_2, m_3)$ for a fixed integers m_1, m_2, m_3 is an n^4-column vector whose $\{(k_1-1)n^3 + (k_2-1)n^2 + (k_3-1)n + k_4\}$th component is $cum\{x_{k_1}(t), x^*_{k_2}(t+m_1), x_{k_3}(t+m_2), x^*_{k_4}(t+m_3)\}$, where $k_i = 1, \ldots, n$ for $i = 1, 2, 3, 4$. Then, from (1), we can derive a key relation of the fourth-order cumulants of LTI systems [10] [11]:

$$\begin{aligned} & c_y(m_1, m_2, m_3) \\ &= \sum_{k_1,k_2,k_3,k_4} [h(k_1) \otimes h^*(k_2) \otimes h(k_3) \otimes h^*(k_4)] \qquad (10) \\ & \times c_s(m_1 + k_1 - k_2, m_2 + k_1 - k_3, m_3 + k_1 - k_4) \\ & \quad \text{for } m_1, m_2, m_3 = 0, \pm 1, \ldots, \end{aligned}$$

where $\otimes$ denotes the Kronecker product. Under the assumption **(A3)**, the above equation reduces to

$$\begin{aligned} c_y(m_1, m_2, m_3) &= \sum_i [h(i) \otimes h^*(i + m_1) \\ & \otimes h(i + m_2) \otimes h^*(i + m_3)] c_s(0, 0, 0). \qquad (11) \end{aligned}$$

For notational simplicity, we express $c_s(0, 0, 0)$ as

$$c_s(0, 0, 0) = U\gamma_s \qquad (12)$$

where

$$\gamma_s = (\gamma_{s_1}, \gamma_{s_2}, \ldots, \gamma_{s_n})^T \qquad (13)$$

and U is an $n^4 \times n$ matrix with elements u_{ij}'s defined by

$$u_{ij} = \begin{cases} 1 & \text{, if } i = (j-1)(n^3 + n^2 + n) + j \\ & \quad \text{for } j = 1, 2, \ldots, n, \\ 0 & \text{, otherwise.} \end{cases} \qquad (14)$$

From the above definition, U satisfies

$$U^\dagger U = I. \qquad (15)$$

Now let us consider a least-squares cumulant-matching problem: Given the values of $c_y(m_1, m_2, m_3)$'s of the output of the unknown system, determine an $n \times n$ matrix impulse response $\{g(k)\}$ and the fourth-order cumulant γ_s of the input signal $s(t)$ by

$$\min_{\{g(k)\}, \gamma_s} \sum_{m_1,m_2,m_3=-\infty}^{\infty} ||c_y(m_1, m_2, m_3) - c_x(m_1, m_2, m_3)||^2 \qquad (16)$$

subject to

$$E\{x(t+k)x^\dagger(t)\} = E\{y(t+k)y^\dagger(t)\}, \qquad (17)$$

where

$$x(t) = \sum_{k=-\infty}^{\infty} g(k)s(t-k). \qquad (18)$$

As in the previous section, we utilize the whitened output $\tilde{y}(t)$ in (6) instead of $y(t)$ in (1). Then the constrains reduce to

$$\begin{aligned} E\{x(t+k)x^\dagger(t)\} &= E\{\tilde{y}(t+k)\tilde{y}^\dagger(t)\} \\ &= I\delta(k). \qquad (19) \end{aligned}$$

This means that $\{g(k)\}$ is in $\mathcal{P}$ so that

$$\sum_\tau g(\tau + k)g^\dagger(\tau) = I\delta(k). \qquad (20)$$

Thus the criterion results in

$$\min_{\{g(k)\} \in \mathcal{P}, \gamma_s} \sum ||c_{\tilde{y}}(m_1, m_2, m_3) - c_x(m_1, m_2, m_3)||^2. \qquad (21)$$

Note the following properties of the Kronecker products [10] [11]:

$$(A \otimes B)^\dagger = A^\dagger \otimes B^\dagger, \qquad (22)$$

and

$$(A \otimes B) \cdot (C \otimes D) = AC \otimes BD, \qquad (23)$$

where A, B, C, and D are of appropriate size of dimensions for defining the matrix multiplications. Using these properties, from (11) with (18), we have

$$\begin{aligned} & \sum_{m_1,m_2,m_3} ||c_x(m_1, m_2, m_3)||^2 \\ &= \sum_{m_1,m_2,m_3} \sum_{i,j} \Big(\gamma_s^\dagger U^\dagger \\ & \times [g^\dagger(j) \otimes g^T(j + m_1) \otimes g^\dagger(j + m_2) \otimes g^T(j + m_3)]\Big) \\ & \times \Big([g(i) \otimes g^*(i + m_1) \otimes g(i + m_2) \otimes g^*(i + m_3)] U\gamma_s\Big) \\ &= \gamma_s^\dagger U^\dagger \sum_{i,j} \Big(g^\dagger(j)g(i) \otimes \sum_{m_1} g^T(j + m_1)g^*(i + m_1) \\ & \qquad \otimes \sum_{m_2} g^\dagger(j + m_2)g(i + m_2) \\ & \qquad \otimes \sum_{m_3} g^T(j + m_3)g^*(i + m_3) \Big) U\gamma_s \\ &= \gamma_s^\dagger \gamma_s. \qquad (24) \end{aligned}$$

Here we used (20) and (15) in the last equality.

Thus we get

$$\begin{aligned} & \sum_{m_1,m_2,m_3} ||c_{\tilde{y}}(m_1, m_2, m_3) - c_x(m_1, m_2, m_3)||^2 \\ &= \sum ||c_{\tilde{y}}(m_1, m_2, m_3)||^2 - \hat{\gamma}_s^\dagger \gamma_s - \gamma_s^\dagger \hat{\gamma}_s + \gamma_s^\dagger \gamma_s \\ &= \sum ||c_{\tilde{y}}(m_1, m_2, m_3)||^2 + ||\gamma_s - \hat{\gamma}_s||^2 - ||\hat{\gamma}_s||^2, \qquad (25) \end{aligned}$$

where

$$\hat{\gamma}_s := U^\dagger \sum_{m_1,m_2,m_3,i} [g^\dagger(i) \otimes g^T(i+m_1) \otimes g^\dagger(i+m_2) \otimes g^T(i+m_3)] c_{\tilde{y}}(m_1,m_2,m_3). \quad (26)$$

The first equality in (25) comes from (27) below, which is derived from (12) and the formula (11) with (18):

$$\gamma_s^\dagger \hat{\gamma}_s = \sum_{m_1,m_2,m_3} c_x^\dagger(m_1,m_2,m_3) c_{\tilde{y}}(m_1,m_2,m_3). \quad (27)$$

Since the first term in (25) is irrelevant to arguments γ_s and $\{g(k)\}$, the least-squares estimates of γ_s and $\{g(k)\}$ are respectively given by $\hat{\gamma}_s$ in (26) and by

$$\arg \max_{\{g(k)\} \in \mathcal{P}} ||\hat{\gamma}_s||^2. \quad (28)$$

On the other hand, if we set

$$\tilde{z}(t) = \sum_k \tilde{w}(k) \tilde{y}(t-k), \quad (29)$$

then, from (10), the fourth-order cumulant $c_{\tilde{z}}(0,0,0)$ of $\tilde{z}(t)$ is related to the fourth-order cumulants of $\tilde{y}(t)$ such as

$$c_{\tilde{z}}(0,0,0) = \sum_{m_1,m_2,m_3,i} [\tilde{w}(i) \otimes \tilde{w}^*(i-m_1) \otimes \tilde{w}(i-m_2) \otimes \tilde{w}^*(i-m_3)] c_{\tilde{y}}(m_1,m_2,m_3). \quad (30)$$

If we put

$$\tilde{w}(k) = g^\dagger(-k), \quad \text{for } k = 0, \pm 1, \ldots, \quad (31)$$

then, using (26), (30) and the definition of U, we have

$$||\hat{\gamma}_s||^2 = \sum_{i=1}^{n} |\gamma_{\tilde{z}_i}|^2. \quad (32)$$

Therefore, (28) is equivalent to

$$\arg \max_{\{\tilde{w}(k)\} \in \mathcal{P}} \sum_{i=1}^{n} |\gamma_{\tilde{z}_i}|^2, \quad (33)$$

because $\{g(k)\} \in \mathcal{P}$ implies $\{\tilde{w}(k)\} \in \mathcal{P}$.

Comparing (8) and (9) with (33) and (29), we can find that the single-stage maximization criterion is equivalent to the least-squares cumulant-matching criterion.

5. CONCLUSIONS

We have shown that the single-stage maximization criterion for multichannel blind deconvolution is equivalent to the least-squares fourth-order cumulant matching criterion after multichannel pre-whitening of channel outputs. This equivalence provides us a key link between the single-stage maximization criteria and the least-squares cumulant-matching criterion.

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