BLIND ADAPTIVE INTERFERENCE SUPPRESSION IN DS-CDMA COMMUNICATIONS WITH IMPULSIVE NOISE *

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ABSTRACT

In many wireless systems where multiuser detection techniques may be applied, the ambient channel noise is known through experimental measurements to be decidedly non-Gaussian, due largely to impulsive phenomena. The performance of many multiuser detectors can degrade substantially in the presence of such impulsive ambient noise. In this paper, a blind adaptive robust multiuser detection technique is developed for combating both multiple-access interference and impulsive noise in CDMA communication systems. This technique is nonlinear in nature and it is based on the signal subspace tracking method and the M-estimation method for robust regression. It is seen that the proposed technique offers significant performance gain over linear adaptive multiuser detectors in impulsive noise, with little attendant increase in computational complexity.

1. INTRODUCTION

Recent years have seen a significant interest in advanced signal processing techniques for enhancing the performance of non-orthogonal signaling schemes for multiple-access communications. These techniques generally fall under the category of multiuser detection [7], which refers to optimum or near-optimum demodulation in such situations. By and large, the study of this problem has focused on the situation in which the ambient noise is additive white Gaussian noise (AWGN). As increasingly practical techniques for multiuser detection become available, such as adaptive and blind adaptive multiuser detection methods [2], the situation in which practical multiple-access channels will be ambient-noise limited can be realistically envisioned.

In many physical channels, such as urban and indoor radio channels and underwater acoustic channels, the ambient noise is known through experimental measurements to be decidedly non-Gaussian, due to the impulsive nature of the man-made electromagnetic interference and a great deal of natural noise as well. In view of the lack of realism of an AWGN model for ambient noise arising in many practical channels in which multiuser detection techniques may be applied, natural questions arise concerning the applicability, robustness and performance of multiuser detection techniques for non-Gaussian multiple-access channels. Although performance indices such as mean-square-error (MSE) and signal-to-interference-plus-noise ratio (SINR) for linear multiuser detectors are not affected by the distribution of the noise (only the spectrum matters), the more crucial bit-error rate can depend heavily on the shape of the noise distribution. In particular, impulsive noise can severely degrade the error probability for a given level of ambient noise variance. In the context of multiple-access capability, this implies that fewer users can be supported with conventional detection in an impulsive channel than in a Gaussian channel. However, since non-Gaussian noise can, in fact, be beneficial to system performance if properly treated, the problem of joint mitigation of structured interference and non-Gaussian ambient noise is of interest [5]. A recent study [6] has shown that the performance gains afforded by maximum likelihood (ML) multiuser detection in impulsive noise can be substantial when compared to optimum multiuser detection based on a Gaussian noise assumption. However, the computational complexity of ML detection is quite high, and therefore effective near-optimal multiuser detection techniques in non-Gaussian noise are needed. In this paper, we consider the problem of blind adaptive multiuser detecion in direct-sequence code-division multiple-access (DS-CDMA) channels with non-Gaussian ambient noise.

2. SYSTEM MODEL

Consider a baseband digital DS-CDMA network operating with a coherent BPSK modulation format. The waveform received by a given terminal in such a network can be modeled as, for $-\infty < t < \infty$,

$$r(t) = \sum_{k=1}^{K} A_k \sum_{i=0}^{M-1} b_k(i) s_k(t - iT - \tau_k) + n(t), \tag{1}$$

where M is the number of data symbols per user in the data frame of interest; T is the symbol interval; $n(t)$ is the ambient channel noise; and A_k, τ_k, $\{b_k(i);\ i = 0, 1, \cdots, M-1\}$, and $\{s_k(t);\ 0 \le t \le T\}$, denote, respectively, the received amplitude, delay, symbol stream, and normalized signaling waveform of the k-th user. It is assumed that $s_k(t)$ is supported only on the interval $[0, T]$ and has unit energy, and that $\{b_k(i)\}$ is a collection of independent equiprobable ± 1 random variables. For the direct-sequence spread-spectrum (DS-SS) multiple access format, the user signaling wave-

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forms are of the form

$$s_k(t) = \sum_{j=0}^{N-1} \beta_j^k \, \varphi(t - jT_c), \quad t \in [0, T], \tag{2}$$

where N is the processing gain; $\left(\beta_0^k, \beta_1^k, \cdots, \beta_{N-1}^k\right)$ is a signature sequence of ± 1's assigned to the k-th user; and φ is a normalized chip waveform of duration T_c, where $NT_c = T$.

For the sake of simplicity of discussion, we first restrict our attention to the synchronous case of model (1), in which $\tau_1 = \tau_2 = \cdots = \tau_K = 0$. This does not incur any loss of generality, as the robust multiuser detection technique developed for synchronous system can be readily applied to the asynchronous channels with properly windowed received signal. For the synchronous case of model (1), to demodulate the i-th symbols of the K users, $\{b_k(i)\}_{k=1}^K$, it is sufficient to consider the received signal during the i-th signaling interval, i.e.,

$$r(t) = \sum_{k=1}^{K} A_k \, b_k(i) \, s_k(t - iT) + n(t), \quad t \in [iT, (i+1)T). \tag{3}$$

At the receiver, the received signal $r(t)$ is first filtered by a chip-matched filter and then sampled at the chip rate. The resulting discrete-time signal model is given by

$$\underline{r}(i) = \sum_{k=1}^{K} A_k b_k(i) \, \underline{s}_k + \underline{n}(i), \tag{4}$$

where $\underline{s}_k \stackrel{\triangle}{=} [s_0^k \ \cdots \ s_{N-1}^k]^T = \frac{1}{\sqrt{N}}[\beta_0^k \ \cdots \ \beta_{N-1}^k]^T$, is the normalized signature sequence of the k-th user, and $\underline{n}(i) \stackrel{\triangle}{=} [n_0(i) \ \cdots \ n_{N-1}(i)]^T$ is the channel ambient noise sample vector at the i-th symbol interval. It is assumed that the sequence of noise samples $\{n_j(i)\}$ is a sequence of independent and identically distributed (i.i.d.) random variables with a non-Gaussian distribution.

In this paper, we adopt the commonly used two-term Gaussian mixture model for the additive noise samples $\{n_j(i)\}$. The probability density function (pdf) of this noise model has the form

$$f = (1 - \epsilon) \mathcal{N}\left(0, \nu^2\right) + \epsilon \mathcal{N}\left(0, \kappa\nu^2\right), \tag{5}$$

with $\nu > 0$, $0 \le \epsilon \le 1$, and $\kappa \ge 1$. Here the $\mathcal{N}\left(0, \nu^2\right)$ term represents the nominal background noise, and the $\mathcal{N}\left(0, \kappa\nu^2\right)$ term represents the impulsive component, with ϵ representing the probability that impulses occur. It is usually of interest to study the effects of variation in the shape of a distribution on the performance of the system, by varying the parameters ϵ and κ with fixed total noise variance

$$\sigma^2 \stackrel{\triangle}{=} (1 - \epsilon)\, \nu^2 + \epsilon\, \kappa\, \nu^2. \tag{6}$$

This model serves as an approximation to the more fundamental Middleton Class A noise model [4], and has been used extensively to model physical noise arising in radar and acoustic channels.

3. ROBUST MULTIUSER DETECTION VIA M-REGRESSION

3.1. Robust Multiuser Detector

Consider the synchronous signal model (4). For simplicity we drop the symbol index i and denote $\theta_k \stackrel{\triangle}{=} A_k b_k$. Then (4) can be rewritten as

$$r_j = \sum_{k=1}^{K} s_j^k \, \theta_k + n_j, \qquad j = 1, \cdots, N, \tag{7}$$

We consider the problem of estimating the K unknown parameters $\theta_1, \theta_2, \cdots, \theta_K$ from the N observations $r_1, r_2, \cdots, r_N$ in (7). Given the estimate $\hat{\theta}_k$, the data bits are then determined according to $\hat{b}_k = \text{sgn}(\hat{\theta}_k)$.

Denote $\underline{\theta} \stackrel{\triangle}{=} [\theta_1 \, \theta_2 \, \cdots \, \theta_K]^T$. Here we propose using the class of M-estimators due to Huber [3] for robust estimation of $\underline{\theta}$ in non-Gaussian noise. In this approach, $\underline{\theta}$ is chosen to minimize a sum of a function, ρ, of the residuals,

$$\hat{\underline{\theta}} = \arg \min_{\underline{\theta} \in \mathbb{R}^K} \sum_{j=1}^{N} \rho \left(r_j - \sum_{k=1}^{K} s_j^k \, \theta_k \right). \tag{8}$$

Suppose that ρ has a derivative $\psi = \rho'$, then the solution to (8) satisfies the implicit equation

$$\sum_{j=1}^{N} \psi \left(r_j - \sum_{k=1}^{K} s_j^k \, \theta_k \right) s_j^k = 0, \quad j = 1, \cdots, N, \tag{9}$$

or in vector form

$$\underline{S}^T \, \psi\left(\underline{r} - \underline{S}\,\underline{\theta}\right) = \underline{0}_K, \tag{10}$$

where $\psi(\underline{x}) \stackrel{\triangle}{=} [\psi(x_1), \cdots, \psi(x_K)]^T$ for any $\underline{x} \in \mathbb{R}^K$; $\underline{S} \stackrel{\triangle}{=} [\underline{s}_1 \, \underline{s}_2 \, \cdots \, \underline{s}_K]$; $\underline{r} \stackrel{\triangle}{=} [r_1 \, r_2 \, \cdots \, r_N]^T$, and $\underline{0}_K$ denotes a K-dimensional zero vector. The Huber penalty function and its derivative are given respectively by

$$\rho_H(x) = \begin{cases} \frac{x^2}{2\nu^2}, & \text{for } |x| \le k\nu^2, \\ \frac{k^2\nu^2}{2} - k|x|, & \text{for } |x| > k\nu^2, \end{cases} \tag{11}$$

$$\psi_H(x) = \begin{cases} \frac{x}{\nu^2}, & \text{for } |x| \le k\nu^2, \\ k\,\text{sgn}(x), & \text{for } |x| > k\nu^2, \end{cases} \tag{12}$$

where k, ϵ and ν are connected through

$$\frac{\phi(k\nu)}{k\nu} - Q(k\nu) = \frac{\epsilon}{2(1-\epsilon)}, \tag{13}$$

where $\phi(x) \stackrel{\triangle}{=} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$, and $Q(x) \stackrel{\triangle}{=} \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\frac{x^2}{2}} \mathrm{d}x$. Equation (9) can be solved iteratively by the modified residual method [3].

3.2. Simulation Examples

We demonstrate the performance gains achieved by the robust multiuser detector over the linear decorrelator in impulsive noise. The noise distribution parameters are $\epsilon = 0.01$ and $\kappa = 100$. The bit error rate versus SNR for the two detectors is plotted in Figure 1. Also shown in this figure is the performance of an "approximate" minimax decorrelating detector, in which the parameter k is taken as $k = \frac{1.5}{\sigma}$, and the step size parameter μ is set as $\mu = \sigma^2$. The reason for studying such an approximate robust detector is that in practice, it is unlikely that the exact parameters ϵ and ν in the noise model (5) are known to the receiver. However, the total noise variance σ^2 can be estimated from the received signal (as discussed in the next section). Hence if we could set some simple rule for choosing the parameters k [as opposed to calculating it exactly from equation (13)] and μ, then this approximate robust detector is much easier to implement than the exact one. It is seen from Figure 1 that the robust decorrelating multiuser detector offers significant performance gains over the linear decorrelating detector. Moreover this performance gain increases as the SNR increases. Another important observation is that the performance of the robust multiuser detector is insensitive to the parameters ϵ and κ in the noise model, which is evidenced by the fact that the performance of the approximate robust detector is very close to that of the exact robust detector.

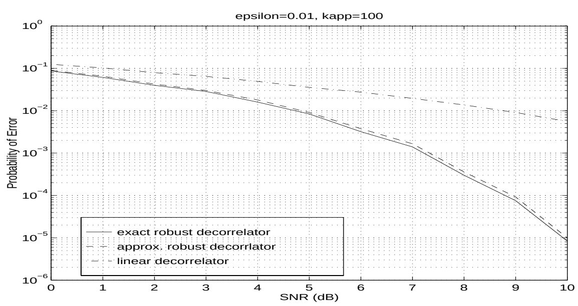

Figure 1. Probability of error versus signal-to-noise ratio (SNR) for user 1 for the exact robust detector, an approximate robust detector and the linear decorrelating detector.

In the next example we consider the performance of the robust decorrelator in Gaussian noise. Shown in Figure 2 are the bit error rate curves for the robust decorrelator and the linear decorrelator. It is seen that there is only a very slight performance degradation by the robust decorrelator in Gassian channels, relative to the linear decorrelator, which is the optimal decorrelating detector in Gaussian noise.

4. BLIND ADAPTIVE ROBUST MULTIUSER DETECTOR

Throughout this paper, we have assumed that the signature waveforms of all users are known to the receiver in order to implement the robust multiuser detectors. One remarkable feature of the linear multiuser detectors is that there exist *blind* techniques that can be used to adapt these detectors, which allow one to use a linear multiuser detector for a given user with no knowledge beyond that required for implementation of the conventional matched-filter detector for that user.

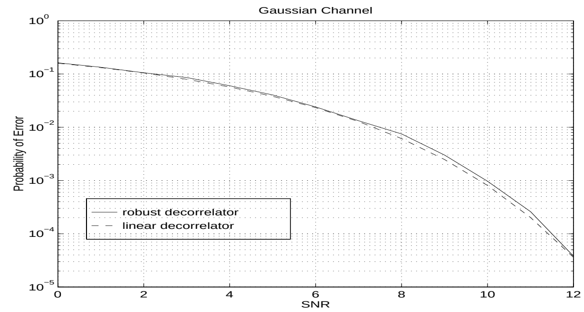

Figure 2. Probability of error versus signal-to-noise ratio (SNR) for user 1 for the robust decorrelating detector and the linear decorrelating detector, in a synchronous CDMA channel with Gaussian noise. $N = 31, K = 6$. The powers of the interferers are 10dB above the power of user 1.

There are two major approaches to blind adaptive multiuser detection. In the first approach the received signal is passed through a linear filter, which is chosen to minimize, within a constraint, the mean-square value of its output [1]. Adaptation algorithms such as least-mean-squares (LMS) or recursive-least-squares (RLS) can be applied for updating the filter weights. Ideally the adaptation will lead the filter converge to the linear MMSE multiuser detector, irrespective of the noise distribution. Therefore this approach can not be used to adapt the robust multiuser detector, which is nonlinear in nature.

Another approach to blind multiuser detection is the subspace-based method proposed in [8], through which both the linear decorrelating detector and the linear MMSE detector can be obtained blindly. As will be discussed next, this approach is more fruitful in leading to a blind adaptive robust multiuser detection method.

4.1. Subspace Concept

Denote $\underline{A} \triangleq \mathrm{diag}(A_1^2, \cdots, A_K^2)$. Since the data bits of K users $\{b_k(i)\}$ are independent ± 1 random variables, and they are independent of the noise samples $\{\underline{n}(i)\}$, the autocorrelation matrix of the received signal $\underline{r}(i)$ in (4) is then given by

$$\underline{C} \;\triangleq\; E\left\{\underline{r}(i)\,\underline{r}(i)^T\right\} \;=\; \underline{S}\,\underline{A}\,\underline{S}^T + \sigma^2 \underline{I}_N. \qquad (14)$$

By performing an eigendecomposition of the matrix $\underline{C}$, we can write

$$\underline{C} \;=\; \underline{U}\,\underline{\Lambda}\,\underline{U}^T \;=\; [\underline{U}_s\;\underline{U}_n]\begin{bmatrix} \underline{\Lambda}_s & \\ & \underline{\Lambda}_n \end{bmatrix}\begin{bmatrix} \underline{U}_s^T \\ \underline{U}_n^T \end{bmatrix}, \qquad (15)$$

where $\underline{U} = [\underline{U}_s\;\underline{U}_n]$, $\underline{\Lambda} = \mathrm{diag}(\underline{\Lambda}_s, \underline{\Lambda}_n)$; $\underline{\Lambda}_s = \mathrm{diag}(\lambda_1, \cdots, \lambda_K)$ contains the K largest eigenvalues of $\underline{C}$ in descending order and $\underline{U}_s = [\underline{u}_1 \;\cdots\; \underline{u}_K]$ contains the corresponding orthonormal eigenvectors; $\underline{\Lambda}_n = \sigma^2 \underline{I}_{N-K}$ and $\underline{U}_n = [\underline{u}_{K+1} \;\cdots\; \underline{u}_N]$ contains the $N-K$ orthonormal eigenvectors that correspond to the eigenvalue σ^2. It is easy to see that $\mathrm{range}\,(\underline{S}) = \mathrm{range}\,(\underline{U}_s)$. The range space of $\underline{U}_s$ is called the *signal subspace* and its orthogonal complement, the *noise subspace*, is spanned by $\underline{U}_n$. The following result is instrumental to developing the subspace-based blind robust multiuser detector.

Proposition 1 *Suppose that*

$$\sum_{k=1}^{K} \theta_k \, \underline{s}_k \;=\; \sum_{j=1}^{K} \zeta_j \, \underline{u}_j, \qquad \theta_k \in \mathcal{R}, \quad \zeta_j \in \mathcal{R}. \quad (16)$$

Then we have

$$\theta_k \;=\; \alpha_k \sum_{j=1}^{K} \frac{\underline{u}_j^T \underline{s}_k}{\lambda_j - \sigma^2} \zeta_j, \qquad k = 1, \cdots, K, \quad (17)$$

where α_k is a positive constant.

The above result leads to a subspace-based blind robust multiuser detection technique as follows. From the received data $\{\underline{r}(i)\}$, we can estimate the signal subspace components, i.e., $\underline{\Lambda}_s$, $\underline{U}_s$, σ. The received signal $\underline{r}$ can be expressed as

$$\underline{r} \;=\; \underline{S}\,\underline{\theta} + \underline{n} \;=\; \underline{U}_s \, \underline{\zeta} + \underline{n}, \quad (18)$$

where $\underline{\zeta} \triangleq [\zeta_1, \cdots, \zeta_K]^T$. Now instead of robustly estimating the parameters $\underline{\theta}$ using the known signature waveforms $\underline{S}$ of all users, as is done in the previous section, we can robustly estimate the parameters $\underline{\zeta}$ using the estimated signal subspace coordinates $\underline{U}_s$. Finally, we compute the parameter θ_k of the user of interest using (17). Notice that in this way, to demodulate the k-th user's data bit $b_k(i)$, the only prior knowledge required at the receiver is the signature waveform of this user, thus the term *blind* robust multiuser detector. Notice also that since the columns of $\underline{U}_s$ are orthonormal, the modified residual method for updating the robust estimate of $\underline{\zeta}$ is given by

$$\underline{z}^l \;\triangleq\; \psi\left(\underline{r} - \underline{U}_s \, \underline{\zeta}^l\right), \quad (19)$$

$$\underline{\zeta}^{l+1} \;=\; \underline{\zeta}^l + \frac{1}{\mu} \underline{U}_s^T \, \underline{z}^l. \quad (20)$$

The computationally efficient sequential eigendecomposition (subspace tracking) algorithms can be employed for adaptively updating the estimated signal subspace components. At the i-th symbol interval, after receiving the i-th data vector $\underline{r}(i)$, the signal subspace components are updated by the a subspace tracking algorithm. Then the robust procedure (19), (20) and (17) is invoked to demodulate the k-th user's data bit $b_k(i)$.

4.2. Simulation Examples

As before we consider the synchronous system with $K = 6$ users and spreading gain $N = 31$. The noise distribution parameters are $\epsilon = 0.01$ and $\kappa = 100$. The powers of all interferers are 10dB above user 1. The performance of the blind adaptive robust multiuser detector based on subspace tracking is shown in Figure 3, where the PASTd algorithm from [9] is used for tracking the signal subspace parameters. It is seen from this figure that as in the nonadaptive case, the robust multiuser detector offers significant performance gain over the linear multiuser detector in impulsive noise. Furthermore, by employing the subspace tracking technique the blind robust multiuser detector has a practical computational complexity and incurs no delay in data demodulation.

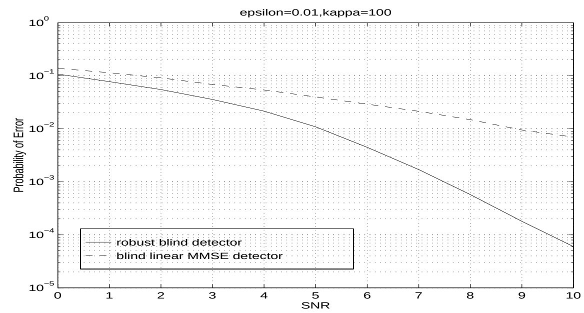

Figure 3. Probability of error versus signal-to-noise ratio (SNR) for user 1 for the blind robust detector and the blind linear detector, using subspace tracking, in a synchronous CDMA channel with non-Gaussian noise. $N = 31, K = 6$. The powers of the interferers are 10dB above the power of user 1.

5. CONCLUSIONS

In many practical wireless channels in which multiuser detection techniques may be applied, the ambient noise is likely to have an impulsive component that gives rise to larger tail probabilities than is predicted by the Gaussian model. We have proposed a robust multiuser detection technique that is seen to significantly outperform the linear multiuser detectors in non-Gaussian ambient noise, in terms of data detection error probability. This technique is based on the M-estimation method for robust regression. We have also developed a subspace-based blind adaptive technique for implementing the robust multiuser detectors, which requires only the signature waveform of the user of interest in order to robustly demodulate that user's data.

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