ON EXISTENCE OF FIR PRINCIPAL COMPONENT FILTER BANKS‡

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ABSTRACT

In this paper we have two interesting results. One is of theoretical interest and the other practical. The theoretical result is that the optimum FIR orthonormal filter bank of a fixed **finite** degree that maximizes the coding gain does not always contain an optimum compaction filter. In other words, in general, there does not exist a principal component filter bank (PCFB) of a given nonzero degree. This is sharply in contrast to the cases of transfom coders and ideal subband coders where the existence of PCFB's are assured by their very construction. The practical result of the paper is that constraining the filter corresponding to the largest subband variance to be a compaction filter does not result in a significant loss of performance for practical input signals. Since there exist very efficient methods to design FIR compaction filters and since the best completion of the filter bank given the first filter is trivially done by a KLT, we see that this is an exteremely efficient method despite the fact that it is suboptimum.

1. INTRODUCTION

There have been important recent developments in the design of optimal subband coders. With unconstrained filters, the optimal orthonormal filter bank problem has been solved [6, 8, 10] and the biorthogonal case has been understood quite well [1, 11]. If the filters are order-constrained, then optimal filter banks are not known except for the special case of two channel filter banks and for a restricted class of second order input statistics [3].

Consider an $M-$channel FIR orthonormal uniform filter bank shown in Fig. 1. Using the well-known polyphase representation [9], we can draw this as in Fig. 2 where $\mathbf{E}(z)$ is the polyphase matrix. This can be factorized as $\mathbf{E}(z) = \mathbf{U}\mathbf{V}_1(z)\mathbf{V}_2(z)\ldots\mathbf{V}_\mu(z)$ (see Sec. 2 for details). Moulin et al. [5] proposed the following algorithm for the optimization of the filter bank for maximum coding gain:
1) Design the first filter $H_0(z)$ to be a compaction filter [3].
2) Factorize the polyphase vector $\mathbf{e}_0^\dagger(z)$ of $H_0(z)$ as $\mathbf{e}_0^\dagger(z) = \mathbf{w}_0^\dagger\mathbf{W}_1(z)\mathbf{W}_2(z)\ldots\mathbf{W}_\mu(z)$ (See Sec. 2 for details). Let $\mathbf{V}_n(z) = \mathbf{W}_n(z),\ n = 1,\ldots,\mu$.
3) Choose $\mathbf{U}$ to be the KLT for its input vector. The first row of the KLT is necessarily $\mathbf{w}_0^\dagger$. If not one can increase the compaction gain violating the optimal compaction property of $H_0(z)$.

The authors of [5] use the argument that if one designs a principal component filter bank (PCFB) (see Sec. 4 for definition), then it maximizes the coding gain. The first filter of a PCFB has to be a compaction filter. Hence the above algorithm should be optimum. They assume implicitly that a PCFB exists. If the ideal filters are allowed, then a PCFB does exist and it maximizes the coding gain [6, 10]. Similarly if the filter orders are less than the number of channels, then the KLT achieves the maximum coding gain and it is a PCFB. We show in this paper that in the intermediate case, there does not always exist a PCFB. Hence the above algorithm is in general suboptimum. Nevertheless, as we show by some examples in Sec. 5, the suboptimality is not significant for practical signals. Since the design of FIR compaction filters is well studied [3, 4, 7] and there exist very efficient algorithms like the **window method** in [3], we see that the above method is very efficient for the design of signal-adapted FIR orthonormal filter banks.

2. LESS KNOWN FACTS ABOUT FIR ORTHONORMAL FILTER BANKS

Consider the FIR filter bank shown in Fig. 1 and its polyphase decomposition in Fig. 2 where $\mathbf{E}(z) = \sum_{n=0}^{K}\mathbf{E}_n z^{-n}$. Here $\mathbf{E}_n$'s are $M\times M$ constant matrices with $\mathbf{E}_K \neq \mathbf{0}$. The number K is called the **order** of $\mathbf{E}(z)$. Let $\mathbf{E}(z)$ be orthonormal or paraunitary. That is, $\mathbf{E}^\dagger(e^{j\omega})\mathbf{E}(e^{j\omega}) = \mathbf{I},\ \forall\omega$. Let μ denote the **degree** of $\mathbf{E}(z)$ which is the minimum number of delay elements for its implementation. In general we have $\mu \geq K$. It is well known that $\mathbf{E}(z)$ can be factored as [9]

$$\mathbf{E}(z) = \mathbf{U}\mathbf{V}_1(z)\mathbf{V}_2(z)\ldots\mathbf{V}_\mu(z) \tag{1}$$

where $\mathbf{U}$ is unitary, $\mathbf{V}_n(z) = \mathbf{I} - \mathbf{v}_n\mathbf{v}_n^\dagger + z^{-1}\mathbf{v}_n\mathbf{v}_n^\dagger,\ n = 1,\ldots,\mu$, and $\mathbf{v}_n$'s have unit norm (see Fig. 3). Conversely any $\mathbf{E}(z)$ of the form (1) is FIR orthonormal of degree μ as long as $\mathbf{U}$ is unitary and $\mathbf{v}_n$'s have unit norm.

If we are interested in only one filter, say $H_0(z)$ of the filter bank as in the case of compaction problem, we need to consider only the corresponding row $\mathbf{e}_0^\dagger(z)$ of $\mathbf{E}(z)$ whose elements are the polyphase components of $H_0(z)$. Let ν be the degree of $\mathbf{e}_0^\dagger(z)$. Then similar to (1) we can write

$$\mathbf{e}_0^\dagger(z) = \mathbf{w}_0^\dagger\mathbf{W}_1(z)\mathbf{W}_2(z)\ldots\mathbf{W}_\nu(z) \tag{2}$$

where $\mathbf{W}_n(z) = \mathbf{I} - \mathbf{w}_n\mathbf{w}_n^\dagger + z^{-1}\mathbf{w}_n\mathbf{w}_n^\dagger,\ n = 1,\ldots,\nu$, and the vectors $\mathbf{w}_n,\ n = 0,\ldots,\nu$ have unit norm [9].

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2.1. Uniqueness of the factorizations

The factorization of $M \times M$ polyphase matrix $\mathbf{E}(z)$ of a given degree μ is in general not unique and $K \leq \mu$. The factorization of $\mathbf{e}_0^\dagger(z)$ of degree ν is on the other hand unique. The order of $\mathbf{e}_0^\dagger(z)$ is equal to its degree ν [9]. This implies the following: $\mathbf{w}_n^\dagger \mathbf{w}_{n+1} \neq 0, n = 0, \ldots, \nu - 1$.

Fact. If in the factorization of $\mathbf{E}(z)$ in (1), the vectors $\mathbf{v}_n$ turn out to be such that $\mathbf{v}_n^\dagger \mathbf{v}_{n+1} \neq 0, n = 1, \ldots, \mu - 1$, then we have $K = \mu$ and the factorization is unique. Otherwise, $K < \mu$ and the factorization is not unique.

Proof. From (1), we can write the highest possible coefficient $\mathbf{E}_\mu = \mathbf{U}\mathbf{v}_1\mathbf{v}_1^\dagger\mathbf{v}_2\mathbf{v}_2^\dagger \ldots \mathbf{v}_\mu\mathbf{v}_\mu^\dagger$. Since $\mathbf{v}_n^\dagger\mathbf{v}_{n+1} \neq 0$, and since $\mathbf{U}$ is nonsingular, we conclude that $\mathbf{E}_\mu \neq 0$ and therefore $K = \mu$. The ith row of $\mathbf{E}(z)$ is $\mathbf{e}_i^\dagger(z) = \mathbf{u}_i^\dagger \mathbf{V}_1(z)\mathbf{V}_2(z)\ldots\mathbf{V}_\mu(z)$. There exists at least one index i, say $i = 0$, such that $\mathbf{u}_0^\dagger \mathbf{v}_1 \neq 0$ (otherwise $\mathbf{U}$ has to be singular). Hence the degree of $\mathbf{e}_0^\dagger(z)$ is μ. This implies that $\mathbf{V}_n(z)$'s are unique. Since $\mathbf{U} = \mathbf{E}(1)$ is unique, we conclude that the factorization (1) is indeed unique. If on the other hand we have $\mathbf{v}_n^\dagger\mathbf{v}_{n+1} = 0$ for some n, then it can be shown that $\mathbf{E}_0 = (\mathbf{I} - \mathbf{v}_1\mathbf{v}_1^\dagger)(\mathbf{I} - \mathbf{v}_2\mathbf{v}_2^\dagger)\ldots(\mathbf{I} - \mathbf{v}_\mu\mathbf{v}_\mu^\dagger)$ has rank less than $M - 1$. This implies that the rank reduction [9] can start with more than one possible vector $\mathbf{v}_\mu$, implying that the factorization is not unique. ■

Now, returning to $\mathbf{E}(z)$ of degree μ and its first row $\mathbf{e}_0^\dagger(z)$ of degree ν, assume that $\mu = \nu$. Then by the uniqueness of the representation (2) it follows that $\mathbf{V}_n(z) = \mathbf{W}_n(z), n = 1, \ldots, \mu$, and $\mathbf{w}_0 = \mathbf{u}_0$ where $\mathbf{u}_0^\dagger$ is the first row of $\mathbf{U}$. Therefore all the other filters $H_i(z), i = 1, \ldots, M-1$ can be determined by the remaining $M - 1$ rows of the unitary matrix $\mathbf{U}$. This is the main observation from [12] that is used by Moulin et al. [5] to design signal-adapted FIR orthonormal filter banks.

3. FIR CODING AND COMPACTION PROBLEMS

Let $\mathbf{x}(n)$ in Fig. 2 be WSS with power spectral density (psd) matrix $\mathbf{S}_{\mathbf{xx}}(e^{j\omega})$. Assume $\mathbf{E}(z)$ is FIR orthonormal and let $\mathbf{R}(e^{j\omega}) = \mathbf{E}^\dagger(e^{j\omega})$ in Fig. 2. With high-bit rate assumptions on the quantization noise sources, and with optimal bit allocation, the reconstruction error is $\mathcal{E} = c2^{-2b}\phi^{1/M}$ where [10]

$$\phi = \prod_{i=0}^{M-1} \sigma_{x_i}^2 = \prod_{i=0}^{M-1} \int_{-\pi}^{\pi} \left[\mathbf{E}^\dagger(e^{j\omega})\mathbf{S}_{\mathbf{xx}}(e^{j\omega})\mathbf{E}(e^{j\omega})\right]_{ii} \frac{d\omega}{2\pi} \tag{3}$$

Here $\sigma_{x_i}^2$ is the variance of the ith subband. If $x(n)$ is WSS, the coding gain is $G_{coding} = \frac{\sigma_x^2}{(\prod_{i=0}^{M-1}\sigma_{x_i}^2)^{1/M}} = \frac{\sigma_x^2}{\phi^{1/M}}$. By the orthonormality, $\sigma_x^2 = \frac{1}{M}\sum_{i=0}^{M-1}\sigma_{x_i}^2$. Let $\mathcal{O}_\mu$ denote the class of $M \times M$ FIR orthonormal polyphase matrices with degree less than or equal to μ. The **coding problem** is the following:

$$\min_{\mathbf{E}(z)\in\mathcal{O}_\mu} \prod_{i=0}^{M-1} \int_{-\pi}^{\pi} \left[\mathbf{E}^\dagger(e^{j\omega})\mathbf{S}_{\mathbf{xx}}(e^{j\omega})\mathbf{E}(e^{j\omega})\right]_{ii} \frac{d\omega}{2\pi} \tag{4}$$

The energy compaction problem, on the other hand, is concerned with making one of the subband variances of an orthonormal filter bank as large as possible. If the original signal is WSS, then the compaction gain is defined as $G_{comp} = \frac{\max_i(\sigma_{x_i}^2)}{\sigma_x^2}$. Let $\mathcal{Q}_\mu$ denote the class of $1 \times M$ FIR orthonormal polyphase vectors of degree less than or equal to μ. The **compaction problem** is the following:

$$\max_{\mathbf{e}_0(z)\in\mathcal{Q}_\mu} \int_{-\pi}^{\pi} \mathbf{e}_0^\dagger(e^{j\omega})\mathbf{S}_{\mathbf{xx}}(e^{j\omega})\mathbf{e}_0(e^{j\omega})\frac{d\omega}{2\pi} \tag{5}$$

Considering Fig. 3, the objectives can be written as

$$\min_{\mathbf{v}_n,\mathbf{U}} \prod_{i=0}^{M-1} [\mathbf{U}\mathbf{R}_{\mathbf{ww}}(0)\mathbf{U}^\dagger]_{ii} \quad \text{(coding)} \tag{6}$$

$$\max_{\mathbf{v}_n,\mathbf{u}_0} \mathbf{u}_0^\dagger\mathbf{R}_{\mathbf{ww}}(0)\mathbf{u}_0 \quad \text{(compaction)} \tag{7}$$

where $\mathbf{R}_{\mathbf{ww}}(0)$ is the autocorrelation matrix of $\mathbf{w}(n)$. In the coding problem, $\mathbf{U}$ has to be the KLT for $\mathbf{w}(n)$ and in the compaction problem $\mathbf{u}_0$ has to be the unit-norm eigenvector of $\mathbf{R}_{\mathbf{ww}}(0)$ corresponding to the maximum eigenvalue. Let λ_i's be the eigenvalues of $\mathbf{R}_{\mathbf{ww}}(0)$. Hence one can rewrite the problems as:

$$\min_{\mathbf{v}_n} \prod_{i=0}^{M-1} \lambda_i \quad \text{(coding)}, \quad \max_{\mathbf{v}_n}\max_i \lambda_i \quad \text{(compaction)} \tag{8}$$

Hence both problems are parametrized by μ unit-norm vectors of length M. The total number of free parameters is therefore $\mu(M-1)$. If $\mu = 0$, there is nothing to optimize. In this case $\mathbf{E}(z) = \mathbf{E}_0 = \mathbf{U}$ is the KLT for the input vector $\mathbf{x}(n)$ in the coding problem and $\mathbf{u}_0^\dagger$ is the first row of the KLT in the compaction problem. The matrix $\mathbf{U}$ diagonalizes $\mathbf{R}_{\mathbf{xx}}(0)$, the $M \times M$ autocorrelation matrix of the input. The solution for the case where the filter orders are unconstrained has recently been established. We will refer to this case as $\mu = \infty$, although the degree is formally undefined because the filters are not causal. The optimum solution $\mathbf{E}(e^{j\omega})$ that maximizes the coding gain, diagonalizes $\mathbf{S}_{\mathbf{xx}}(e^{j\omega})$ at each frequency. This in particular implies the diagonalization of the autocorrelation matrix $\mathbf{R}_{\mathbf{xx}}(0)$ (which was both necessary and sufficient condition for the transform coding case). Diagonalization of the psd matrix at each frequency however, is not sufficient for $\mathbf{E}(e^{j\omega})$ to maximize the coding gain [10]. There should be an additional ordering of the eigenvalues of the psd matrix at each frequency (spectral majorization) [10]. If $x(n)$ is WSS, then these eigenvalues are $S_{xx}(e^{j(\omega+i2\pi/M)})$, $i = 0, \ldots, M-1$. For the two-channel case and for a restricted class of input psd, we show in [3] that, if μ is the degree of the optimum FIR filter bank, then $\mathbf{S}_{\mathbf{xx}}(e^{j\omega})$ should be decorrelated and majorized only at $\lceil \mu/2 \rceil$ discrete frequencies. In [3] we show how to find those frequencies.

4. FIR PRINCIPAL COMPONENT FILTER BANKS

It turns out that both the KLT ($\mu = 0$) and the optimum ideal $\mathbf{E}(e^{j\omega})$ ($\mu = \infty$) that minimize ϕ (coding) also maximize $\max_i(\sigma_{x_i}^2)$ (compaction). More is true as they achieve a fascinating **majorization** property described as follows: Let us order the subband variances such that $\sigma^2_{x_0} \geq \sigma^2_{x_1} \geq \ldots \geq \sigma^2_{x_{M-1}}$. Among all orthogonal transform coders, the KLT has the property that the partial sum $\sum_{i=0}^{P} \sigma^2_{x_i}$ is maximized for each $P = 0, \ldots, M-1$ The same property holds for orthornormal subband coders with no order constraints ($\mu = \infty$). That is, for each P, $\sum_{i=0}^{P} \sigma^2_{x_i}$ is the largest for the optimal one. Whenever this happens, we call the filter bank a **principal component filter bank (PCFB)**. In particular, when $P = 0$, this says that $\sigma^2_{x_0}$ should be maximized by the choice of $H_0(e^{j\omega})$. That is, $H_0(e^{j\omega})$ should be an **optimum compaction filter** [3, 10].

We show in Sec. 5 that an optimum filter bank of a finite nonzero degree ($0 < \mu < \infty$) does not satisfy such a majorization property except in the special two-channel case. We show this by exhibiting examples where a filter bank has the maximum coding gain but none of its filters is an optimum compaction filter. That is, the majorization property is violated for $P = 0$. This suggests the following:
Fact. In general, there does not exist an FIR $M-$channel PCFB for finite nonzero degree μ.
Proof. Assume on the contrary that there always exists a PCFB $\mathbf{E}^p(z)$ of degree $0 < \mu < \infty$. Then for all $\mathbf{E}(z) \in \mathcal{O}_\mu$, $\sum_{i=0}^{P} \sigma^2_{x_i}$ is maximized by $\mathbf{E}^p(z)$, for each $P = 0, \ldots, M-1$. This implies two things: $\sigma^2_{x_0}$ is maximized by $\mathbf{E}^p(z)$ (optimum compaction gain), and $\prod_{i=0}^{M-1} \sigma^2_{x_i}$ is minimized by $\mathbf{E}^p(z)$ (optimum coding gain). The first one is by definition ($P = 0$), while the second one is due to a well-known result in linear algebra (see [2, page 199]) that says: given two nonnegative sets of M numbers, say $a_0, \ldots, a_{M-1}$ and $b_0, \ldots, b_{M-1}$ if $\sum_{i=0}^{P} a_i \geq \sum_{i=0}^{P} b_i$, $P = 0, 1, \ldots, M-1$, with equality for $P = M-1$, then $\prod_{i=0}^{M-1} a_i \leq \prod_{i=0}^{M-1} b_i$. The set $a_0, \ldots, a_{M-1}$ is said to majorize the set $b_0, \ldots, b_{M-1}$. Hence the set of variances $\sigma^2_{x_0}, \ldots, \sigma^2_{x_{M-1}}$ of $\mathbf{E}^p(z)$ majorizes every other set of variances of $\mathbf{E}(z) \in \mathcal{O}_\mu$ and therefore $\prod_{i=0}^{M-1} \sigma^2_{x_i}$ is minimized by $\mathbf{E}^p(z)$. So, if a PCFB exists, it solves both optimization problems. Since we have examples in the rest of the paper that show that there is no single filter bank that achieves both the maximum compaction and coding gains, we conclude that a PCFB of a given degree does not always exist. ■

4.1. A Simple Example

Let the input process be AR(1) with the correlation coefficient of $\rho = 0.9$. Let the number of channels be $M = 3$ and $\mu = K = 1$ Assume that the filter orders are less than or equal to $N = 4$. Note that these are the smallest numbers for which we can expect to have a counter example. This is because the coding and compaction problems are the same if either $M = 2$ or $N < M$ [3]. Now, since the maximum filter order is 4, we can write $\mathbf{v}_1 = [\cos(\alpha)\ sin(\alpha)\ 0]^T$. Hence the two problems can be formulated by one single parameter α. Hence we can plot the coding and compaction gains versus α as in Fig. 5 where we kept the range of α from 0 to $\pi/2$. This is because the plots are symmetric with respect to both 0 and $\pi/2$. From the plot we see that the the two problems have different answers. The value of α that maximizes the coding gain is $\alpha_{coding} = 0.1507\pi$ where as $\alpha_{comp} = 0.1695\pi$ maximizes the compaction gain. For these choices of α, the coding gains are $G_{coding} = 3.2176$, and $G_{coding} = 3.2052$ respectively and the compaction gains are $G_{comp} = 2.7672$ and $G_{comp} = 2.7682$. Hence among the class of orthonormal filter banks with $\mu = 1$ and the maximum filter order $N = 4$, there does not exist a PCFB. If it existed, then it would have achieved both the maximum compaction and coding gains. From the plot, we see that there is no value of α for which both gains are maximized.

5. EFFICIENCY OF THE SUBOPTIMUM DESIGN

In the introduction, we have outlined an algorithm proposed by Moulin et al. [5]. They constrain the first filter of the filter bank to be an optimum compaction filter. In the previous section we have shown an example where this constraint resulted in loss of coding gain. Another issue with the algorithm of [5] is the fact that the optimum compaction filter $H_0(z)$ is not uniquely determined from its magnitude square (or the product filter) $|H_0(e^{j\omega})|^2$. Since the latter can be spectrally factorized in many ways, we see that one spectral factor may give better coding gain than the others although they all have the same coding gain. This indeed turns out to be the case as we show in Example 2. In that example, we show also that even if one uses the compaction filter that has the best phase response (best spectral factor of $|H_0(e^{j\omega})|^2$), one can still increase the coding gain by brute force optimization of the filter bank. We want to remark that the coding gain loss due to constraining the first filter to be optimum compaction filter is not significant for most of the practical signals we have considered. Below are some examples that confirm this observation.
Example 1. Let us consider the counter example of the previous section. Let the input be MA(1) instead of AR(1) with arbitrary correlation. Then one can verify by explicitly plotting the coding and compaction gains versus the parameter α that both achieve the maximum at the same value of α. This means that the best coding gain is achieved by designing a compaction filter first. This determines $\mathbf{v}_1$ and the first row of $\mathbf{U}$. The best filter bank that maximizes the coding gain is then obtained by using the KLT for the output of $\mathbf{V}_1(z)$. In the previous section, the difference in the coding gains was very small, for this example it is identically zero.
Example 2. Let the input be MA(1) with $r(1) = 0.3$. Let $M = 3$ and $\mu = 5$ so that the maximum filter order, $N \leq 17$. The best compaction gain is $G_{comp} = 1.4920$ achieved by the best compaction filter magnitude response. There are 8 possible phase responses for $H_0(e^{j\omega})$ that yield the same magnitude response $|H_0(e^{j\omega})|^2$ (assuming real coefficients). Among them there is one filter that achieves the maximum coding gain of $G_{coding} = 1.0944$. The minimum phase filter has the coding gain of $G_{coding} = 1.0653$ which is worse. By brute-force optimization, one can find a filter bank that has the coding gain of $G_{coding} = 1.0951$. This has a compaction gain of $G_{comp} = 1.4910$, slightly worse than the optimum. Hence this is an example where the phase response of the compaction filter $H_0(e^{j\omega})$ does affect the coding gain and even with a best phase, the coding gain is not the maximum achievable. On the other hand the numerical differences are

not significant at all.

Example 3. Let the input be an AR(12) process. Let $M = 8$ and the degree $\mu = 5$ so that the maximum filter order $N \leq 47$. This is the example where we obtained the most discrepancy between the two solutions. The coding gain for the suboptimum method of Moulin et al. is $G_{coding} = 5.3948$. By brute-force optimization of vectors $\mathbf{v}_n$, we find that we can achieve a coding gain of $G_{coding} = 5.9642$. The previous solution has the maximum compaction gain $G_{comp} = 5.9190$ while the latter solution has $G_{comp} = 5.0228$.

6. CONCLUSION

In this paper we have seen that the optimum $M-$channel FIR orthonormal filter bank of a given nonzero degree μ does not posses the majorization property that is encountered both in the transform coding ($\mu = 0$) and ideal subband coding ($\mu = \infty$). We have shown that there does not always exist a principal component filter bank of a given degree. If one designs first a compaction filter and then completes it by a KLT, we have seen that the phase response of the compaction filter has a role. Although these observations are theoretically important, on the practical side, we have seen that designing any optimum compaction filter and then completing it by a KLT as in [5] results in very little loss of coding gain.

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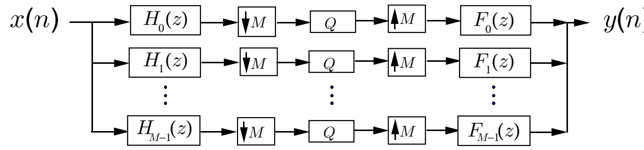

Figure 1. $M-$**channel filter bank.**

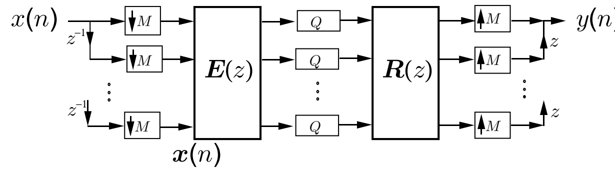

Figure 2. Polyphase representation.

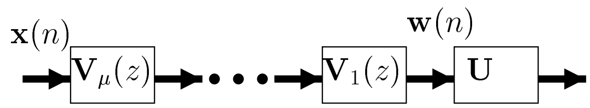

Figure 3. Householder factorization of $\mathbf{E}(z)$

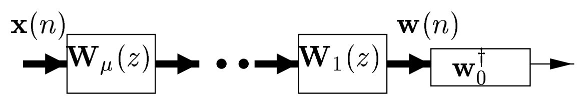

Figure 4. Householder factorization of $\mathbf{e}_0^{\dagger}(z)$

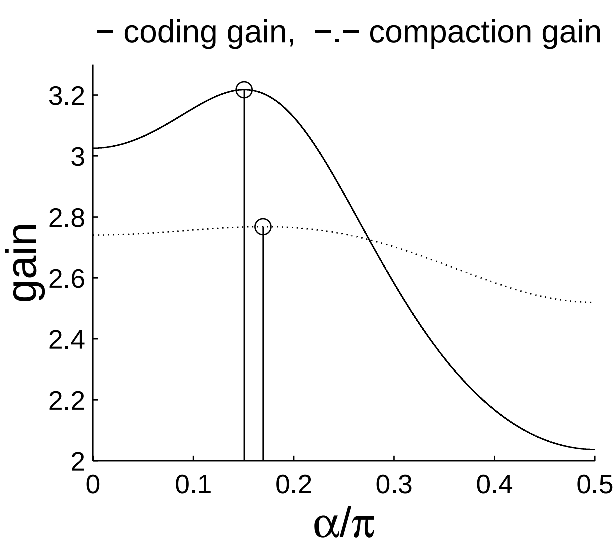

Figure 5. Coding and compaction gains versus the parameter α**. The two plots have maxima at different values of** α**.**