

MODEL-BASED EDGE RECONSTRUCTION FOR LOW BIT-RATE WAVELET-BASED IMAGE CODING

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ABSTRACT

Low bit-rate image coding brings about obvious degradation to the compressed images, among which distortions at edges are particularly objectionable. In this paper, a model-based edge reconstruction algorithm is proposed for wavelet-based image coding at low bit-rate. Our approach applies a general model to represent varieties of edges existing in an image. Based on this model, the problem of edge reconstruction is formulated as finding original edge model parameters from the lossy image. The proposed method is able to improve the subjective visual quality and fidelity (PSNR) of images coded by wavelet-based coding using zerotree quantization.

1. INTRODUCTION

Image compression is aimed to minimize the number of bits needed to represent an image while maintaining sufficient quality. Images compressed at low bit-rate, say below 0.25bpp, suffer from the loss of details and sharpness, as well as various coding artifacts. On the other hand, with the increasing needs of image transmission and storage, the demand for higher compression is also increasing. This problem can be alleviated by effective post-processing which can improve the coding efficiency and, at the same time, maintain compatibility with the encoder. Since different methods have different artifacts, the post-processing technique should be tailor-made for a coding method.

Recently, wavelet transforms have attracted considerable attention with their application to image coding, due to their unique space-frequency characteristics. Moreover, the hierarchical wavelet image representation also allows efficient quantization and coding strategies, such as zerotree quantization [1] [2]. At low bit-rate, wavelet-based coding demonstrates some advantages over the traditional block-based methods in terms of visibility and severity of coding artifacts. However, images coded using wavelet-based methods still bear obvious artifacts around sharp edges, known as “ringings effects” and blurring effects, as a result of the considerable quantization errors of high frequency wavelet coefficients. Since edges define the most recognizable features for objects in an image, the distortions around edges are disturbing and annoying to human perception.

In this paper, we propose a novel post-processing method for recovering lossy edges by the use of a deterministic structural edge model. Based on this model, the degradation of edges due to wavelet-based coding using zerotree quantization is analyzed and the edge recovery problem is formulated as the estimation of the original edge model parameters from the compressed image. Furthermore, the parametric edge reconstruction scheme allows a flex-

ible trade-off between visual enhancement and fidelity improvement of the reconstructed images.

This paper is organized as follows. The edge model is first reviewed in Section 2. We analyze the profile of lossy edges due to wavelet-based coding in Section 3. Section 4 presents the edge reconstruction algorithm. The experimental results are shown in Section 5. Finally, we draw conclusions in Section 6.

2. EDGE ANALYSIS

2.1. Edge Model

Edges in 2-D images have a local 1-D structure feature in that there are sharp intensity changes in one direction together with little or no change in its perpendicular direction. Hence, an edge model can be represented in the 1-D form. The edge model adopted here has been described in depth by Beek in [3] where there are two assumptions of practical situation: (1) Edges of objects in the real world can be approximated by the ideal step functions; (2) The acquisition system through which the image is acquired has a point spread function of Gaussian. Therefore, an edge signal $s(x)$ with the edge center at $x = 0$ is modeled as the *Gaussian smoothed step edge* defined by

$$s(x) \equiv s(x; b, c, w) = h(x; b, c) * g(x; w), \quad (1)$$

where

$$h(x; b, c) = b + cU(x), \quad (2)$$

and

$$g(x; w) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{x^2}{2\sigma^2}\right) \quad \text{with } \sigma = w. \quad (3)$$

From (1)–(3), an edge can be represented as

$$s(x; b, c, w) = b + \frac{c}{2} \left(1 + \operatorname{erf}\left(\frac{x}{w\sqrt{2}}\right)\right), \quad (4)$$

where $\operatorname{erf}(\cdot)$ is the scaled error function, w the parameter controlling the width of the edge, c the contrast across the edge and b the intensity at the base. These parameters are depicted in Figure 1.

In a 2-D image with the x - y coordinate system, (4) becomes

$$s_{2D}(x, y; b, c, w, \theta) = b + \frac{c}{2} \left(1 + \operatorname{erf}\left(\frac{x \cos \theta + y \sin \theta}{w\sqrt{2}}\right)\right), \quad (5)$$

assuming that an edge passes through the origin at an angle θ from the y -axis. For ease of explanation, we shall explain edge detection and model parameter estimation based on (4).

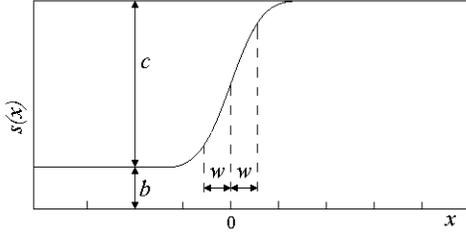


Figure 1: 1-D edge model with edge center at $x = 0$.

2.2. Edge Detection

Canny edge detection [4] is applied by convolving the signal $s(x)$ with the derivative of a Gaussian function $g_d(x; \sigma_d)$ with σ_d controlling the correctness of edge detection and accuracy of edge localization. Without considering the noise, the detection output is

$$d(x; c, w, \sigma_d) = \frac{c}{\sqrt{2\pi(w^2 + \sigma_d^2)}} e^{-\frac{x^2}{2(w^2 + \sigma_d^2)}} = c \cdot g(x; \sigma_1), \quad (6)$$

where $\sigma_1 = \sqrt{w^2 + \sigma_d^2}$. An edge point is identified by checking out the local maximum of the magnitude in the response of (6).

Edges of 2-D images are often not isolated but belong to some curves which generally are the boundaries of the image structure. Usually, long edge curves are more important for human perception compared with short ones. Therefore, edges are reconstructed along the edge curves of significant length (see Section 4).

2.3. Model Parameter Estimation

A multi-point estimation method, which is based on several sampled values near the peak of the response of edge detection, was proposed in [3] for estimating the model parameters of a detected edge. Given the response of edge detection expressed in (6) whose local maximum is recognized as an edge, the detected edge may not be at the true position because of the discretization of the signal. We indicate “edge point” as the edge on the sampled grid of the discrete signal, and “edge center” as the true position of the edge in the continuous version. Edge points can be identified during edge detection, as shown in (6). For a signal $s_0(x)$ with unit sampling interval, if the edge point is at $x = 0$ and the true edge center is at $x = x_0$ ($|x_0| < 0.5$), i.e.,

$$s_0(x) = s(x - x_0; b, c, w), \quad (7)$$

then the response of edge detection in (6) becomes

$$d(x; c, w, \sigma_1) = s_0(x) * g'_d(x; \sigma_d) = c \cdot g(x - x_0; \sigma_1). \quad (8)$$

By sampling (8) at $x = -a, 0, a$, we obtain three measurements:

$$d_1 \equiv d(0; c, w, \sigma_d) = \frac{c}{\sqrt{2\pi(w^2 + \sigma_d^2)}} e^{-\frac{x_0^2}{2(w^2 + \sigma_d^2)}}, \quad (9)$$

$$d_2 \equiv d(a; c, w, \sigma_d) = \frac{c}{\sqrt{2\pi(w^2 + \sigma_d^2)}} e^{-\frac{(a-x_0)^2}{2(w^2 + \sigma_d^2)}}, \quad (10)$$

$$d_3 \equiv d(-a; c, w, \sigma_d) = \frac{c}{\sqrt{2\pi(w^2 + \sigma_d^2)}} e^{-\frac{(-a-x_0)^2}{2(w^2 + \sigma_d^2)}}. \quad (11)$$

From (4), (9)–(11), where $a = 1$ is a practical choice in the sampled 1-D signal, parameters c, w, b and the subpixel position of the edge center, x_0 , can be computed by

$$w^2 = \frac{a^2}{\ln\left(\frac{d_1 d_1}{d_2 d_3}\right)} - \sigma_d^2, \quad (12)$$

$$c \approx d_1 \sqrt{\frac{2\pi a^2}{\ln\left(\frac{d_1 d_1}{d_2 d_3}\right)}} \left(\frac{d_2}{d_3}\right)^{\frac{1}{4a}}, \quad (13)$$

$$x_0 = \frac{a \ln(d_2/d_3)}{2 \ln\left(\frac{d_1 d_1}{d_2 d_3}\right)}, \quad (14)$$

$$b = s_0(x_0) - \frac{c}{2}. \quad (15)$$

In a discrete signal, $s(x_0)$ in (15) can be obtained by the linear interpolation between two nearest sampled points. It is shown in [3] that with a suitable choice of filter scale σ_d for edge detection, parameter estimation can be performed with sufficient accuracy.

3. EDGE ANALYSIS IN IMAGES WITH QUANTIZATION ERROR

We now analyze the edge distortions due to the quantization error of wavelet coefficients based on the consideration of the frequency properties of the edge signal $s(x)$ in (1). For simplicity, the problem is discussed in the 1-D continuous form and the results can be adapted to the 2-D discrete case. Denote the Fourier transform of $s(x)$ by

$$S(\omega_x) = \frac{c}{j\omega_x} e^{-\frac{w^2 \omega_x^2}{2}} + 2\pi(b + \frac{c}{2})\delta(\omega_x),$$

where ω_x is the spatial frequency and $\delta(\cdot)$ the Dirac delta function. We show the amplitude responses $|S(\omega_x)|$ for edges with different model parameters in Figure 2.

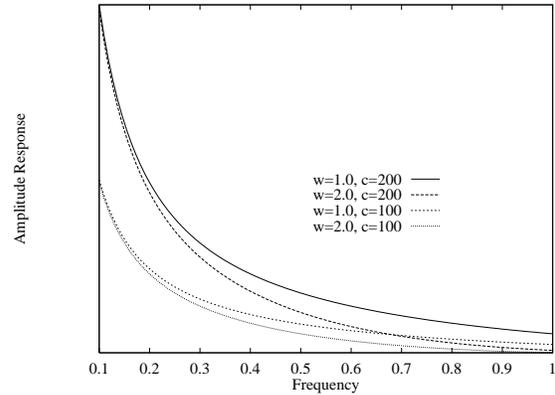


Figure 2: Amplitude response of the 1-D edge model.

From Figure 2, it is observed that the amplitude response of the edge model decays with a rate related to w and c of the edge. Similar results are obtained for discrete wavelet transform (DWT). In wavelet-based coding, DWT decomposes an image over several scales where most energy is compacted to the low-pass band with a small number of coefficients of large magnitude, and high frequency components are dispersed among the high frequency

subbands with a large number of coefficients of small magnitude. Furthermore, zerotree quantization of wavelet coefficients results in keeping the higher bits of larger coefficients and discarding smaller ones. Therefore, such a quantization scheme causes a considerable truncation of high frequency energy, and introduces relatively little effect on low frequency components. Thus, the wavelet coefficients in high frequency subbands contributed to edges are normally of small magnitude and discarded during quantization. Based on these facts, we give the following formulation of edge distortion, in which edges in a compressed image is represented in the 1-D form.

Ideally speaking, it is assumed that during wavelet-based coding using zerotree quantization, the edge signal $s_0(x)$ of (7) is locally filtered with a low-pass zero phase filter $f(x)$ of unity integral and accompanied with quantization noise $q_n(x)$ in the output. We therefore propose to model the coded signal by

$$s_1(x) = s(x - x_0; w, b, c) * f(x) + q_n(x). \quad (16)$$

Low pass filter $f(x)$ widens w by a factor λ ($\lambda > 1$) which is related to the amount of high frequency components truncated and the original model parameters. If we consider $f(x)$ as a FIR filter which has a symmetrical (about origin) pulse response of unity integral, it can be proved that the filtering of $s_0(x)$ by $f(x)$ leaves the position of the edge center x_0 unmoved, as well as the value of the edge center $s(x_0)$ unchanged, and causes no effect on c and b , so (16) can be written in the form of

$$s_1(x) = s(x - x_0; w_1, b, c) + q_n(x), \quad (17)$$

where $w_1 = \lambda \cdot w$. Then, edge detection and parameter estimation of the signal $s_1(x)$ are developed in the following sections.

3.1. Edge Detection in the Lossy Signal

The response of edge detection using Canny detector for the compressed lossy signal $s_1(x)$ is expressed as

$$d_t(x) = s_1(x) * g'_d(x) = c \cdot g(x - x_0; w_2) + q_n(x) * g'_d(x), \quad (18)$$

where $w_2 = \sqrt{w_1^2 + \sigma_d^2}$. Thus, as long as the noise $q_n(x)$ can be averaged out, the local maximum in (18) is the same as that in (8).

In [3], it is indicated that, with the consideration of various errors of the practical situation, the setting of $\sigma_d = 1.0$ is a reasonable value for edge detection in original images. In our experiments, it was found that $\sigma_d \in (1.3, 1.6)$ is suitable for most images coded using wavelet-based coding at low bit-rate.

3.2. Model Parameter Estimation in the Lossy Signal

Firstly, the initial estimated values of parameters c and w , denoted by \bar{c} and \bar{w} , of the lossy signal $s_1(x)$ are obtained by (12) and (13) from the response of (18). Then, the parameter estimation of the original signal $s_0(x)$, where the computed values of c , w and b are denoted by \hat{c} , \hat{w} and \hat{b} respectively, is given as follows:

$$\hat{c} \approx \bar{c}, \quad (19)$$

$$\hat{w} \approx \frac{\bar{w}}{\lambda}, \quad (20)$$

$$\hat{b} \approx s_2(0) - \frac{\bar{c}}{2}, \quad (21)$$

where $s_2(x)$ is the Gaussian smoothed version of $s_1(x)$; λ is the widening coefficient and its empirical setting can be determined by the experimental analysis. Thus, the original edge model parameters are able to be estimated from the compressed signal.

4. EDGE RECONSTRUCTION ALGORITHM

We extend the results from the 1-D case to the 2-D image. In order to obtain effective edge recovery, we introduce a *confident function*, a *reconstruction model* and a *projection operation*, which play important roles in our algorithm.

Let Φ be the compressed image and W its quantized wavelet coefficient array in which most coefficients are zero. A pixel at (x, y) in Φ is denoted as $p(x, y)$, and the distance between $p(x_1, y_1)$ and $p(x_2, y_2)$ is defined by

$$D(p(x_1, y_1), p(x_2, y_2)) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}.$$

Let I_e^j be the set which contains edge points of the j th edge curve detected in Φ , and $L(I_e^j)$ be the length of curve I_e^j . Suppose there are n curves in Φ . We define two sets, I_E and I_R , as follows,

$$\begin{aligned} I_E &= \{p(x, y) | p(x, y) \in I_e^j, L(I_e^j) > L_0 \text{ with } 1 \leq j \leq n\} \\ I_R &= \{p(x, y) | D(p(x, y), p(x_e, y_e)) \leq D_0, p(x_e, y_e) \in I_E \\ &\quad \text{and } p(x_e, y_e) = \arg \min_{p(x_e, y_e)} D(p(x, y), p(x_e, y_e))\}, \end{aligned}$$

where L_0 and D_0 are two thresholds for length filtering and modulus thresholding respectively. I_E contains edge points of the edge curves of significant length. I_R is the edge region to be recovered.

4.1. Model-Based Edge Reconstruction

For a certain pixel $p(x, y) \in I_R$, its intensity value is denoted by $\Phi(x, y)$ and its model-based approximation $\Theta(x, y)$ is given by

$$\Theta(x, y) = \hat{b}_e + \frac{\hat{c}_e}{2} (1 + \text{erf}(\frac{l}{\hat{w}_e})), \quad (22)$$

where $l = D(p(x, y), p(x_e, y_e))$ and $p(x_e, y_e) \in I_E$ is the nearest edge point to $p(x, y)$; \hat{c}_e , \hat{b}_e and \hat{w}_e are the estimated model parameters of the original image associated with pixel $p(x_e, y_e)$.

The use of $\Theta(x, y)$ is mainly for deblurring the lossy edges. In practice, the approximation $\Theta(x, y)$ may not be close to the original intensity of $p(x, y)$. We observe from experiments that the validity of the model-based approximation decreases with the increase of l . However, $\Theta(x, y)$ provides a good intensity tendency for each $p(x, y) \in I_R$. This is attributed to the regularized edge structure defined by the edge model. In order to measure the reliability of the approximation for edge reconstruction, with some prior knowledge, we construct a confident function Γ as

$$\Gamma(l) = e^{-l^\alpha} \text{ with } \alpha \geq 0, \quad (23)$$

where α is an empirical factor.

In the image Φ , the intensities of the pixels in I_R contains quantization noise which is exhibited as the ringing effect around edges. A 2-D Gaussian filter $g_s(x, y; \sigma_s)$ is adopted with $\sigma_s = 1.0$ for reducing this noise, since a small spread parameter σ_s adapts to a rapidly-varying signal better than a large one [5]. The filtered result of each pixel $p(x, y) \in I_R$ is denoted by

$$\bar{\Phi}(x, y) = \Phi * g_s(x, y). \quad (24)$$

The deviation of $\bar{\Phi}(x, y)$ introduced by Gaussian filtering may decrease with the increase of l , so we weights $\bar{\Phi}(x, y)$ by $1 - \Gamma(l)$.

Both $\Theta(x, y)$ of (22) and $\bar{\Phi}(x, y)$ of (24) are incorporated into the edge reconstruction of $p(x, y)$, since they are respectively aimed at two artifacts existing around the lossy edges due to low



Figure 3: Result of Flower (512×512 , 8bpp). Left: Coded image (32.02dB, 0.10bpp). Right: Reconstructed image (32.34 dB, +0.32dB).

bit-rate wavelet image coding. We introduce a reconstruction model with $\Gamma(l)$ adjusting the balance between them as follows:

$$\hat{\Phi}(x, y) = \Gamma(l)\Theta(x, y) + (1 - \Gamma(l))\bar{\Phi}(x, y). \quad (25)$$

By the tuning of α in (23) and (25), the influence of the approximation $\Theta(x, y)$ on each reconstructed pixel $p(x, y)$ can be adjusted. For an image, with an appropriate setting of α , the largest PSNR gain can be attained. On the other hand, if we do not care about the image fidelity and choose a smaller α , we can implement edge enhancement with sharp edge structure. Thus, α provides us a good trade-off on the quality of the reconstructed image.

4.2. Projection Operation in the Wavelet Domain

After edge reconstruction of (25), we require a projection operation defined as follows to ensure the validity of the post-processing.

$$W'(i, j) = \begin{cases} W(i, j) & \text{if } W(i, j) \neq 0 \\ \pm T_0 & \text{if } W(i, j) = 0 \text{ and } |\hat{W}(i, j)| > T_0 \\ \hat{W}(i, j) & \text{otherwise} \end{cases}$$

where \hat{W} is the wavelet coefficient array of the image $\hat{\Phi}$ obtained from (25); (i, j) is the coordinate of wavelet coefficients; T_0 is the quantization threshold of W ; W' is the reconstructed wavelet coefficient array. The inverse DWT of W' gives the last result.

5. EXPERIMENTAL RESULTS

The proposed approach has been tested using twenty images coded by wavelet-based codec SPIHT [2]. We use $\sigma_d = 1.5$, $\lambda = 1.3$, $D_0 = \sqrt{8}$ and $\alpha = 0.6$ in our experiments. The results show that our method can improve the PSNR of all these images except three which have few notable edge structures. On the other hand, the visual quality of all these images is improved. Table 1 shows the improvements in PSNR of standard images Lena, Peppers and

Table 1: PSNR(dB) of reconstructed images (512×512 , 8bpp).

| bit-rate (bpp) | Lena | | Peppers | | Flower | |
|-------------------|-------|-------|---------|-------|--------|-------|
| | PSNR | Gain | PSNR | Gain | PSNR | Gain |
| 0.20 | 33.26 | +0.11 | 32.83 | +0.10 | 36.25 | +0.12 |
| 0.15 | 32.02 | +0.13 | 31.68 | +0.13 | 34.78 | +0.32 |
| 0.10 | 30.45 | +0.23 | 30.10 | +0.26 | 32.34 | +0.32 |
| 0.08 | 29.60 | +0.25 | 29.12 | +0.23 | 31.16 | +0.34 |

Flower. Figure 3 demonstrates the improvement in visual quality of the image Flower coded by SPIHT at 0.10bpp.

6. CONCLUSIONS

A new post-processing technique for images coded by wavelet-based coding at low bit-rate has been presented. It reconstructs the distorted edges in order to improve the image quality in terms of both fidelity and visual perception. Experimental results show that it performs well for most images with notable edge structures. The proposed approach is promising in stretching the performance of wavelet-based image coding at low bit-rate.

7. REFERENCES

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