

# LOSSLESS IMAGE COMPRESSION USING INTEGER COEFFICIENT FILTER BANKS AND CLASS-WISE ARITHMETIC CODING

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## ABSTRACT

A novel way of constructing integer coefficient 2-channel filter banks is proposed. A set of relationships among the filter coefficients is established in order to satisfy linear phase, perfect reconstruction, and FIR properties. The remaining degrees of freedom are used to obtain integer coefficient values by maximizing a performance evaluation function, namely subband coding gain. The number of bits required to represent the subband samples is kept low through efficient nonlinear implementation techniques. An octave-band frequency partitioning where the number of stages is determined according to the image size is employed. The subband samples are then classified into one out of a finite number of classes, and each class is coded by an arithmetic coder. The obtained compression ratios are encouraging compared to the “best” results reported so far in the literature.

## 1. INTRODUCTION

The application of nonunitary or biorthogonal filter banks (FBs) for lossy image compression has become a well established area of research. However, integer mapping from the image space to the subband domain for lossless image compression is relatively new. New applications such as telemedicine, seismic data processing, and archiving (especially of medical images) demand that efficient lossless compression schemes be developed to keep the bit rate low.

The lossless compression scheme can be decomposed into two major blocks. First, the decomposition engine transforms the image from the spatial domain to the frequency domain. Second, an appropriate coding strategy which can efficiently utilize the subband samples’ statistical characteristics is employed to reduce the number of bits needed to exactly represent the image.

If a FB with floating point coefficients is used, the subband samples will be floating point numbers. Therefore a mapping or a quantizer is needed to represent the samples by integers. The integer subband samples and the error floating point values must be coded. Another possibility is to use a FB having integer filter coefficients which can map integer pixel values to integer subband samples. Several wavelet-based integer coefficient filter banks (ICFBs) called *binary filters* can be found in [8]. Calderbank et al. [3] have shown that by using *lifting* steps, every wavelet transform, or some subclass of general FBs, can be used to map integer input signals to integer subband samples.

Several coding and prediction techniques have been proposed in the literature and we summarize some selected schemes here. The linear predictive coding, also used in the JPEG lossless standard [10], tends to be an efficient method. In [11], context modeling based prediction, quantization of Markov states, and entropy coding of the prediction errors is presented. A simple and efficient method by Said and Pearlman [7] uses a nonlinear implementation of the Haar wavelet or Walsh-Hadamard transform (WHT). A 3-stage octave-band frequency partitioning is done by WHT. An interesting issue is that they apply a predictor at each stage along with the WHT. This tends to produce better results than applying the predictor after the frequency partitioning. In [1], a coding scheme based on hierarchical enumeration techniques is used. This coding scheme has been shown to be very efficient for lossless coding of classification tables at low bit rates.

In this paper, we propose several FBs having integer filter coefficients and the perfect reconstruction property. In addition to this, such FBs need to have good energy compaction property which hopefully leads to low entropy and therefore low bit rate. In order to avoid large dynamic ranges in the subband domain, we truncate subband samples in a way such that there exists a perfect reconstruction system. This means that the filtered values in the analysis FB will be truncated,

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however, the filtered samples undergo the same type of nonlinear transform (truncation) in the synthesis FB so that the reproduced image becomes identical to the input image. To find the integer filter coefficients in the FBs, a discrete optimization algorithm was adopted where the ICFBs are optimized for subband coding gain<sup>1</sup>. Finally, we use a coding algorithm based on class-wise arithmetic coding of the subband samples as proposed in [6].

## 2. FEATURES OF THE FILTER BANK

In the case of lossy subband image compression, the best reported FBs are nonunitary with linear phase [2]. Hence, we enforce linear phase on the filter coefficients as an additional constraint for our 2-channel ICFBs, although a further motivation for this may be weak.

In order to guarantee PR through the encoding-decoding system, we constrain the decimated analysis and synthesis polyphase matrices,  $\mathbf{P}(z)$  and  $\mathbf{Q}(z)$ , as [9]

$$\mathbf{Q}(z) = z^{-k} \mathbf{P}^{-1}(z), \quad (1)$$

where  $k$  is an integer. Then, the reconstructed signal becomes equal to a delayed version of the input signal. If using FIR analysis filters, FIR synthesis filters are obtained by setting appropriate terms to zero in the determinant of  $\mathbf{P}(z)$ . For the 2-channel case, the filter relationships are:  $G_{LP}(z) = H_{HP}(-z)$  and  $G_{HP}(z) = -H_{LP}(-z)$  where  $\mathbf{H}(z)$  and  $\mathbf{G}(z)$  denote analysis and synthesis filters, respectively, and LP and HP denote “lowpass” and “highpass”, respectively.

### 2.1. Gain Optimized Integer Coefficient Filter Banks

The constraints imposed on the 2-channel FB in the previous section (PR and linear phase) leaves a number of free parameters that can be used to enhance the performance. Here the last parameters are fixed by optimizing coding gain in the same way as for lossy image compression systems [4]. A compact formula to evaluate the generalized coding gain for nonunitary, nonuniform FBs can be found in [5]. Since optimization is conducted in a statistical manner, we include the underlying statistics of an image by approximating it to an AR(1) process with nearest sample correlation  $\rho = 0.95$ . Hence, the maximum 1-dimensional theoretical coding gain is 10.11 dB [4].

<sup>1</sup>Subband coding gain may be a very confusing term in lossless coding because it is usually linked to quantization noise. However, the coding gain is directly based on the spectral flatness measure which is totally determined by the signal’s power spectral density

Secondly, the filter coefficients are restricted to have only integer values. Therefore the optimization becomes a constrained maximization problem. In the optimization routine we apply a strategy where the problem is recursively divided into two subproblems by setting upper and lower bounds on the set-constrained variables. The routine implements a ‘branch-and-bound’ method on top of CONSTR and thus the Optimization Toolbox in MATLAB is required. We list the following selected cases of the gain optimized ICFBs below. The list shows the analysis and synthesis lowpass filter coefficients and their respective number of zeros at  $z = -1$ . Lowpass: Analysis ( $h$ .) / synthesis ( $g$ ) filters:

- 4/4I ICFB  
 $h_4 = [-1, 4, 4, -1]/6$  and  $g_4 = [1, 4, 4, 1]/5$ .  
 1/1 zeros at  $z = -1$ . Gain = 6.16dB.
- 4/4II ICFB  
 $h_4 = [-1, 3, 3, -1]/4$  and  $g_4 = [1, 3, 3, 1]/4$ .  
 1/3 zeros at  $z = -1$ . Gain = 6.03 dB.
- 5/3I ICFB  
 $h_5 = [-1, 2, 6, 2, -1]/8$  and  $g_3 = [1, 2, 1]/2$ .  
 2/2 zeros at  $z = -1$ . Gain = 6.28 dB.
- 5/3II ICFB  
 $h_5 = [-1, 2, 4, 2, -1]/6$  and  $g_3 = [1, 2, 1]/2$ .  
 0/2 zeros at  $z = -1$ . Gain = 6.26 dB.
- 6/2 ICFB  
 $h_6 = [1, -1, 32, 32, -1, 1]/32$  and  $g_2 = [1, 1]/2$ .  
 1/1 zeros at  $z = -1$ . Gain = 5.06 dB.
- 2/6 ICFB  
 $h_2 = [1, 1]/2$  and  $g_6 = [-1, 1, 8, 8, 1, -1]/8$ .  
 1/3 zeros at  $z = -1$ . Gain = 5.65 dB.

The selected wavelet-based ICFBs from [8] which have been used in this paper are specified below.

- 9/7 ICFB  
 $h_9 = [1, 0, -8, 16, 46, 16, -8, 0, 1]/64$  and  
 $g_7 = [-1, 0, 9, 16, 9, 0, -1]/16$ .  
 2/4 zeros at  $z = -1$ . Calculated gain = 6.18 dB.
- 13/7 ICFB  
 $h_{13} = [-1, 0, 18, -16, -63, 144, 348, 144, -63, -16, 18, 0, -1]/512$  and  $g_7 = [-1, 0, 9, 16, 9, 0, -1]/16$ .  
 4/4 zeros at  $z = -1$ . Calculated gain = 6.24 dB.
- 13/11 ICFB  
 $h_{13} = [-3, 0, 22, 0, -125, 256, 724, 256, -125, 0, 22, 0, -3]/1024$  and  
 $g_{11} = [3, 0, -25, 0, 150, 256, 150, 0, -25, 0, 3]/256$ .  
 2/6 zeros at  $z = -1$ . Calculated gain = 6.10 dB.

- 17/11 ICFB  
 $h_{17} = [3, 0, -52, 0, 348, -256, -972, 2304, 5442, 2304, -972, -256, 348, 0, -52, 0, 3]/8192$  and  
 $g_{11} = [3, 0, -25, 0, 150, 256, 150, 0, -25, 0, 3]/256$ .  
 4/6 zeros at  $z = -1$ , Calculated gain = 6.17 dB.

## 2.2. Implementation of Integer Coefficient FBs

In the case of lossless compression, the above mentioned ICFBs need to be implemented in an efficient way. Otherwise, the dynamic range of the subband samples becomes very large. This will lead to higher entropy, and thus higher bit rate. Hence, it is very important to have a proper scaling at the analysis ICFB which leads to an efficient implementation having an exact inverse at the synthesis ICFB. The implementation techniques for 9/7 and 2/2 ICFBs are given as examples. Assume  $x$  and  $y$  are input pixel values and subband samples, respectively.

The nonlinear analysis 9/7 ICFB algorithms are given by

$$y_{2i+1} = x_{2i+1} - \lfloor 9(x_{2i+2} + x_{2i})/16 \rfloor + \lfloor (x_{2i+4} + x_{2i-2})/16 \rfloor, \quad (2)$$

$$y_{2i} = x_{2i} + \lfloor (y_{2i+1} + y_{2i-1})/4 \rfloor. \quad (3)$$

The synthesis 9/7 ICFB algorithms are

$$x_{2i} = y_{2i} - \lfloor (y_{2i+1} + y_{2i-1})/4 \rfloor, \quad (4)$$

$$x_{2i+1} = y_{2i+1} + \lfloor 9(x_{2i+2} + x_{2i})/16 \rfloor - \lfloor (x_{2i+4} + x_{2i-2})/16 \rfloor. \quad (5)$$

The implementation of 2/2 ICFB used in [7] is given below.

Analysis 2/2 ICFB:

$$y_{2i} = \lfloor (x_{2i} + x_{2i+1})/2 \rfloor, \quad (6)$$

$$y_{2i+1} = x_{2i} - x_{2i+1}. \quad (7)$$

Synthesis 2/2 ICFB:

$$x_{2i} = y_{2i} + \lfloor (y_{2i+1} + 1)/2 \rfloor, \quad (8)$$

$$x_{2i+1} = x_{2i} - y_{2i+1}. \quad (9)$$

## 3. CLASS-WISE ARITHMETIC CODING SCHEME

The coding scheme is based on class-wise arithmetic coding of the subband samples, and is a modified extension of the work in [6].

After octave-band frequency partitioning, the subband samples at each decomposition level are classified into 4 classes based on the block energy<sup>2</sup> to minimize

<sup>2</sup>The block energy is defined as the mean square value of the samples in a block

the overall entropy. For the three highest frequency subbands the block size is  $8 \times 8$ , for the subbands at the next level  $4 \times 4$  blocks are used, and at all other levels, including the lowpass-lowpass frequency subband, the block sizes equal  $2 \times 2$ .

After classification, all samples belonging to each class at each level of decomposition is independently encoded with an adaptive arithmetic coder using uniform initial alphabet probabilities.

The block classification map is independently encoded at each decomposition level using first-order adaptive arithmetic encoding and transmitted as side information. In addition the maximum symbol magnitude of each class is transmitted to the decoder.

## 4. PRACTICAL CODING RESULTS

To evaluate the performance of our ICFBs we select five ISO images given in Table 1. All of them have 256 gray levels (i.e., 8 bits per pixel). The selected test images possess rather different frequency characteristics in the subband domain, and include three natural images, an aerial image, and a digitized finger print.

We apply the ICFBs in a dyadic type of frequency partitioning where the total number of stages depends on the image size. The frequency partitioning is terminated if the number of decomposition levels exceeds six or if the horizontal or vertical size of the lowpass-lowpass frequency band becomes less than 8.

The exact bit rates using different ICFBs for the selected images are given in Table 1. As the images have different information content and space and frequency characteristics, a single ICFB is not necessarily optimal for all cases. The simulation results show that none of the tested FBs has the highest performance for all images<sup>3</sup>.

From the simulation results it can be concluded that the 2/2 ICFB is clearly inferior to the other FBs. Also note that the 5/3I ICFB, which falls under the class called *maximum regular* wavelet filter banks, and the 5/3II ICFB, which does *not* possess the *maximum regularity* property, perform quite close to each other. Furthermore, for the particular five images tested, there is also a tendency that the ICFBs with the longest filters on the average perform best. In particular, the 17/11 ICFB has the lowest total bit rate among the 8 ICFBs considered.

<sup>3</sup>For the current implementation, the 2/6 ICFB does not perform well. This is due to the fact that it is not feasible to scale down the lowpass samples in magnitude as derived in [8]. Thus, the amplitudes of the lowpass subband samples increase at each stage of decomposition. Consequently, the simulation results for this FB are not listed in the table.

Table 1: Exact bit rates of the chosen ISO images.

Image	Size	Color plane	ICFB						
			2/2	5/3I	5/3II	9/7	13/7	13/11	17/11
“Lenna”	512×512	Y	4.624	4.268	4.288	<b>4.260</b>	4.264	4.274	4.268
“Barbara”	512×512	Y	5.342	4.878	4.882	4.803	4.785	4.789	<b>4.763</b>
“Goldhill”	512×512	Y	5.083	<b>4.790</b>	4.809	4.803	4.805	4.826	4.821
“Aerial1”	1024×1024	G	6.271	5.561	5.588	5.417	5.425	<b>5.394</b>	5.395
“Finger”	512×512	Mono	5.789	5.500	5.520	5.465	5.466	5.472	<b>5.465</b>

## 5. CONCLUSIONS

We have presented a new class of ICFBs where the filter coefficients are optimized for subband coding gain. MATLAB software was used to find the gain optimized filter coefficients in the discrete domain. By adopting the same implementation techniques for ICFBs given in [8], we showed that odd and even length ICFBs can be implemented. The assumed coding strategy, which is also used in lossy compressions, performs fairly well. It is interesting to note that several FBs are in first place for different images. This indicates that optimality really depends on the image statistics. It may also be debated whether coding gain optimization is a good tool to minimize entropy, especially since the nonlinear signal operation cannot be included in this type of optimization.

The simple 5/3 ICFBs seem to be good candidates for practical implementation as they perform well and have low implementation complexity.

The performance of the coding scheme can be improved by finding better initial alphabet probabilities for the arithmetic coder, as well as by exploiting the statistical dependencies in the block classification map through conditional arithmetic coding of the classification map entries [6]. Furthermore, a strategy of using a predictor along with the ICFB, as mentioned in [7], can be used to increase the performance.

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