FILTERED ERROR ADAPTIVE IIR ALGORITHMS AND THEIR APPLICATION TO ACTIVE NOISE CONTROL

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ABSTRACT

This paper is concerned with systems in which the output error of an adaptive IIR filter is subsequently filtered, e.g., an active noise control system where a transfer function models the path between the injection point of the cancellation noise and the point where the residual error is measured. We propose a family of algorithms suited to this type of scenarios, deriving conditions for their deterministic convergence. The analysis of the convergence is particularized to the Filtered-U Recursive LMS algorithm, a popular scheme whose global convergence has never been proved formally. Finally, some results based on real measurements are also presented.

1. INTRODUCTION

In some adaptive systems applications, such as active noise control (ANC), the error signal cannot be obtained directly but only a filtered version of it, as shown in Figure 1. An adaptive identification algorithm tries to obtain a good estimate of the unknown system θ_0 which makes it possible to reduce the error, without direct access to $e(n) = y(n) - \hat{y}(n)$, since a transfer function $S(z)$ filters it. The first important result in this direction is due to Morgan [9], who extended the LMS algorithm to handle a linear filter $S(z)$ in the path of the estimated output. He proposed the filtering of the regressor signal by an estimate of $S(z)$, and provided conditions for the convergence of the LMS algorithm in terms of the maximum difference in the estimation of the phase of $S(z)$. This algorithm, called FxLMS, was independently derived by Widrow *et al.* [12] and Burgess [2], and has been studied extensively since then [4], [11], [1]. On the other hand, adaptive IIR algorithms have gained attention due to the potential saving in the number of computations, especially for applications with long impulse responses. IIR filters seem highly desirable for active noise controllers used for systems with acoustic feedback. Their use was first considered for the ANC problem by Eriksson *et al.* [3], in the so-called Filtered-U Recursive LMS (FuRLMS) algorithm. This is essentially a gradient algorithm,

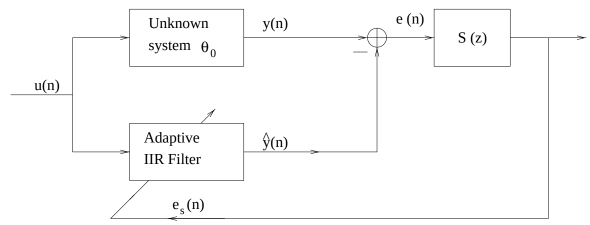

Figure 1: Adaptive IIR filtering with filtered error.

with both the input and the estimated output filtered by an estimate of the cancellation path transfer function. The convergence of this algorithm cannot be guaranteed; it has the same drawbacks as the gradient-based adaptive IIR algorithm, with a multimodal surface where different local minima may exist. In this paper we perform a stability analysis of a family of adaptive IIR algorithms in a deterministic environment, for applications such as ANC. Sufficient conditions for the convergence of the algorithms are obtained, being the FuRLMS a particular case of our study.

2. CONVERGENCE ANALYSIS

We will start our analysis with a family of adaptive IIR algorithms known as CRA (Composite Regressor Algorithm) [6], which includes the output error and equation error formulations, and the a posteriori output error algorithm as particular cases. Before pursuing any modification of the original family of algorithms, we will concentrate on the convergence of such a family in the presence of a filter $S(z)$ in the error path. The unknown system is supposed to have a rational transfer function of order N, of the form $H(z) = B(z)/(1-A(z))$. The estimated parameter vector $\theta(n)$ and the regressor vector $\phi(n)$ are

$$\begin{aligned}\theta(n) &= [a_1(n), \cdots, a_N(n), b_0(n), \cdots, b_N(n)]^T \\ \phi(n) &= [x(n-1), \cdots, x(n-N), u(n), \cdots, u(n-N)]^T\end{aligned}$$

In the original algorithm the regressor element $x(n)$ is a convex combination of the output $y(n)$ and the estimated output $\hat{y}(n)$. Now, $y(n)$ cannot be accessed, so the definition of $x(n)$ must be changed to

$$x(n) = [1-\gamma(n)]\hat{y}(n) + \gamma(n)[e_s(n) + \hat{y}(n)] \quad (1)$$

$\gamma(n)$ is a composition parameter which determines the composition of the output in the regressor; e.g., for $\gamma(n) = 0$, $x(n) = \hat{y}(n)$. The update equations governing the algorithm are

$$\hat{y}(n) = \theta^T(n)\phi(n) \quad (2)$$

$$e(n) = y(n) - \hat{y}(n) \quad (3)$$

$$\theta(n+1) = \theta(n) + \frac{\mu}{1+\mu\phi^T(n)\phi(n)} e_s(n)\phi(n) \quad (4)$$

where $e_s(n) = S(z)\{e(n)\}$ (see Figure 1). For notational simplicity, a mixed notation is used for denoting the filtering operation. The case $\gamma(n) = \frac{\mu\phi^T(n)\phi(n)}{1+\mu\phi^T(n)\phi(n)}$, when $S(z) = 1$, is the a posteriori output error algorithm, since $x(n) = \theta^T(n+1)\phi(n)$ in this case [6].

Next we present a deterministic analysis of the convergence in the absence of disturbance noise. For this purpose, let $V(n) = ||\tilde{\theta}(n)||^2$. We rewrite the update equation (4) in a form more useful for analysis purposes:

$$\theta(n+1) = \theta(n) + \mu\bar{e}_d(n)\phi(n) \quad (5)$$

$$\tilde{\theta}(n+1) = \tilde{\theta}(n) - \mu\bar{e}_d(n)\phi(n) \quad (6)$$

where $\bar{e}_d(n) = \frac{\mu}{1+\mu\phi^T(n)\phi(n)} e_s(n)$ represents a distorted version of the a posteriori error, and $\tilde{\theta}(n) = \theta_0 - \theta(n)$, with θ_0 the true parameter vector. After taking the squared norm in both sides of (6), we can write $V(n+1) - V(n)$ as

$$-2\mu\phi^T(n)\tilde{\theta}(n)\bar{e}_d(n) + \mu^2\phi^T(n)\phi(n)\bar{e}_d^2(n)$$

It can be easily shown that

$$\phi^T(n)\tilde{\theta}(n) = (1-A(z))\{e(n)\} + A(z)\{\gamma(n)e_s(n)\} \quad (7)$$

And from the update equations (4) and (5), $\phi^T(n)\tilde{\theta}(n)$ can be written as a function of the distorted a posteriori error, $\bar{e}_d(n)$. Thus, we find that

$$V(n+1) - V(n) = -2\mu\bar{e}_d(n)H(z,n)\{\bar{e}_d(n)\} \quad (8)$$

with

$$H(z,n) = f(z,n) + \mu\left(f(z,n) - \frac{1}{2}\right)\phi^T(n)\phi(n) \quad (9)$$

and

$$f(z,n) = \frac{1-A(z)}{S(z)} + A(z)\gamma(n)$$

Following [5], the passivity of the operator $H(z,n)$ is sufficient to make the error $\bar{e}_d(n)$, and consequently $e_s(n)$, to decrease to zero for a bounded input $u(n)$. The proof of the convergence of $\theta(n)$ would require some additional technical assumptions on the input. This and other details related to this type of proofs can be pursued as in [5] when dealing with output error methods.

The previous algorithm can be modified to include a compensating filter $C(z)$ that helps to improve the convergence, as done in the hyperstability-based algorithms HARF and SHARF [8]. The mission of this filter is the smoothing of the error, in such a way that a good estimate of the denominator of the unknown transfer function $1 - A(z)$ may ensure convergence. In the case of the CRA family, a modification in the definition of $x(n)$ in (1) must be done to accommodate this smoothing:

$$x(n) = [1-\gamma(n)]\hat{y}(n) + \gamma(n)[e_s(n) + \hat{y}(n)] + \gamma(n)(C(z)-1)\{e_s(n) + \hat{y}(n) - x(n)\} \quad (10)$$

In the update equation (5) $\bar{e}_d(n)$ is written now as

$$\frac{1}{1+\mu\phi^T(n)\phi(n)}\left(e_s(n) + (C(z)-1)\{e'(n)\}\right) \quad (11)$$

where $e'(n) = e_s(n) + \hat{y}(n) - x(n)$. $f(z,n)$ in (9) can be computed now to give

$$\frac{1-A(z)}{C(z)S(z)}(1-\gamma(n)) + \frac{1-A(z)}{S(z)}\gamma(n) + A(z)\gamma(n)$$

For $\gamma(n) = \frac{\mu\phi^T(n)\phi(n)}{1+\mu\phi^T(n)\phi(n)}$ this condition is an extension of the convergence results of HARF to the filtered error case. The well-known sufficient condition for the convergence of HARF in the sufficient order case is the passivity of

$$H(z,n) = \frac{1-A(z)}{C(z)} + \frac{\mu}{2}\phi^T(n)\phi(n) \quad (12)$$

equivalent to the verification of the strictly positive real (SPR) condition by $\frac{1-A(z)}{C(z)}$. With a transfer function $S(z)$ in the error path, this result is generalized to the passivity of

$$\frac{1-A(z)}{S(z)C(z)} + \mu\left(\frac{1-A(z)}{S(z)} + A(z) - \frac{1}{2}\right)\phi^T(n)\phi(n)$$

Thus, now it is not sufficient that $\frac{1-A(z)}{S(z)C(z)}$ be SPR to ensure convergence for any μ. The update parameter μ must be small enough, with an exact bound very difficult to obtain.

2.1. Convergence of the FuRLMS algorithm

In some cases it may be more appropriate to filter the regressor, analogously to the FuRLMS, instead of using a compensating filter. Let us think, for example, in a non-minimum phase transfer function $S(z)$, which would require an unstable filter $C(z)$ with the previous approach.

A filtering of the regressor by $S(z)$ avoids such a problem. The HARF update equation for a filtered regressor can be written as

$$\theta(n+1) = \theta(n) + \frac{\mu}{1+\mu\phi_f^T(n)\phi(n)} e_s(n)\phi_f(n) \qquad (13)$$

with $\phi_f(n)$ equal to

$$[x_f(n-1), \cdots, x_f(n-N), u_f(n), \cdots, u_f(n-N)]^T$$

where the input $u(n)$ and the output $x(n)$ have been filtered by $1/C(z)$ to yield $u_f(n)$ and $x_f(n)$. The analysis of the convergence follows the pattern exposed above. The passivity must be imposed now on $H(z,n)$ of the form

$$\frac{1-A(z)}{C(z)S(z)} + \mu\left(\frac{1-A(z)}{C(z)S(z)} - \frac{1-A(z)}{C(z)}\right)\phi_f^T(n)\phi(n) + \frac{\mu}{2}\phi_f^T(n)\phi_f(n)$$

Notice that for $S(z) = 1$, the passivity of $H(z,n)$ reduces to $\frac{1-A(z)}{C(z)}$ being SPR. The non-commutativity of the time-varying operators has been neglected when approximating $\phi_f^T(n)\theta(n+1)$ by

$$\frac{1}{C(z)}\{\phi^T(n)\theta(n+1)\}$$

This approximation is very convenient for the analysis, and its accuracy is good for slow adaptation [7]. In the slow adaptation case, corresponding to μ being chosen small, the equations can be replaced by simplified approximations which reduce the algorithmic complexity. The simplified version turns out to be the FuRLMS algorithm:

$$\theta(n+1) = \theta(n) + \mu\phi_f(n)e_s(n) \qquad (14)$$

with $\phi_f(n)$ equal to

$$[\hat{y}_f(n-1), \cdots, \hat{y}_f(n-N), u_f(n), \cdots, u_f(n-N)]^T$$

The time-varying operator $H(z,n)$ is now

$$H(z,n) = \frac{1-A(z)}{C(z)S(z)} - \frac{\mu}{2}\phi_f^T(n)\phi_f(n) \qquad (15)$$

The passivity of $\frac{1-A(z)}{C(z)S(z)}$ is not sufficient for the convergence of the algorithm, although we can expect convergence for a sufficiently small μ dependent on the case. However, an exact bound on μ is difficult to infer. The same condition for the convergence is obtained with SHARF (Simplified Hyperstable Adaptive Recursive Filter) [8], with $C(z)$ as a compensating filter in the error path, although the bounds on μ are expected to be different in both cases.

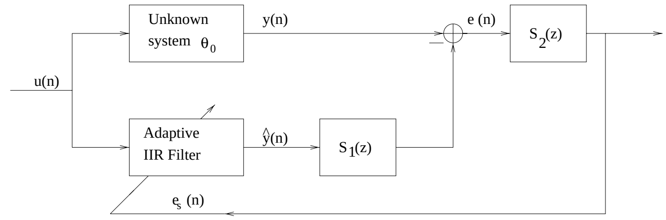

Figure 2: ANC configuration

3. PRACTICAL EXAMPLE

In Figure 2 a basic active adaptive noise canceller configuration is shown. $S_1(z)$ describes the loudspeaker system, including the D/A converter, reconstruction filter, power amplifier, loudspeaker, and acoustic transfer function from loudspeaker to summing junction. $S_2(z)$ includes the acoustic path from the summing junction to the error microphone, error microphone, preamplifier, antialiasing filter, and A/D converter. In practice, as shown by measurements, $S_1(z)$ often contains nonminimum-phase zeros, making the inverse noncausal. This drawback is decreased by the inherent delay of the acoustic system, which in many cases is long enough. Thus, we can redraw the cancellation system in Figure 1, where the error filter is now $S(z) = S_1(z)S_2(z)$, and $S_1(z)$ is included in the system under identification. The transfer function $S(z)$ affects the behavior of the adaptive algorithm, as was shown above for a variety of adaptive IIR algorithms. The transfer functions involved in the example were chosen as the identified filters in a real setup. Measurements were taken on a PVC duct with 8 m length and a circular cross section of 8 cm radius. The reference input $u(n)$ is a pseudorandom sequence, representing the sampled undesired noise. The system transfer function $H(z) = z^{-d}B(z)/A(z)$ identified has $d = 24$, with $B(z)$ and $A(z)$ sixth and tenth order polynomials in z^{-1}, respectively. Its pole-zero plot is shown in Figure 3. The filter $S(z)$ used has an impulse response shown in Figure 4. The algorithm chosen was HARF with filtered regressor. Its increased complexity with respect to FuRLMS allows a higher value of the stepsize parameter. A 64 coefficient filter in the regressor was proved to be sufficient to compensate for the error path filter. With $\mu = 0.1$, a reduction of the noise level of 33 dB was achieved after 50,000 iterations, as shown in Figure 5.

4. FINAL CONSIDERATIONS

The convergence analysis exposed in this paper has been done under the important premise of sufficient order identification. The robustness of the family of algorithms under study in the undermodelled case has not been proved yet, which makes us to be very cautious when using them in real applications under general conditions, for which the

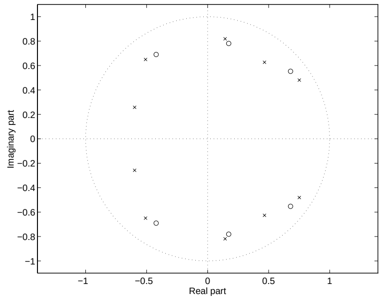

Figure 3: Pole-zero diagram of the transfer function $H(z)$.

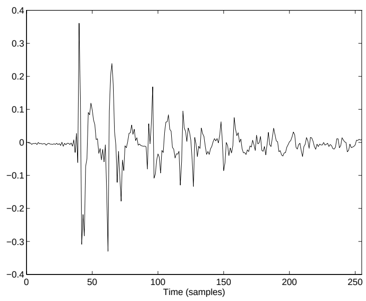

Figure 4: Impulse response of the error path transfer function $S(z)$.

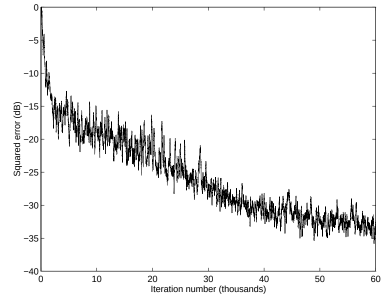

Figure 5: Squared residual error.

systems involved are not given exactly by rational transfer functions. However, we cannot forget the final goal of every ANC system: the minimization of the residual error $e_s(n)$. Even though convergence to the true values of the plant is not achieved, significant reductions in the noise level can be obtained, as proved through extensive simulations.

In the deterministic case there is no advantage in using output-error based methods instead of a simple least-squares approach. However, the passivity notions detailed above can be extended to the analysis of stochastic algorithms (i.e., with noise). In addition, the analysis of the bias in the presence of noise, done in [10] for a subset of the exposed family of algorithms in the case $S(z) = C(z) = 1$, can be also undertaken. These results will be the object of a future work.

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