BIORTHOGONAL MODIFIED COIFLET FILTERS FOR IMAGE COMPRESSION

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ABSTRACT

The selection of filter bank in wavelet compression is crucial, affecting image quality and system design. Recently, the biorthogonal coiflet (cooklet) family of wavelet filters has been constructed [2] [4], and explicit frequency domain formulae have been developed [2] in the Bernstein polynomial basis. In this paper we use the Bernstein basis for frequency domain design and construction of biorthogonal *nearly* coiflet wavelet bases. In particular, we construct a previously unpublished *nearly* coiflet 17/11 biorthogonal wavelet filter pair. Key filter quality evaluation metrics due to Villasenor demonstrate this filter pair to be well suited for image compression. Comparison is made to the 17/11 biorthogonal coiflet (cooklet), Villasenor 10/18, Odegard 9/7, and classical CDF 9/7 wavelet bases. Simulation results with the SPIHT algorithm due to Said and Pearlman [3], and with our SR_{SFQ} [7] [5], confirm that the new 17/11 wavelet basis outperforms the others for still image compression.

1. INTRODUCTION

Much attention [6] has been given to the importance of wavelet smoothness and regularity (particularly to vanishing wavelet moments). Orthonormal filter banks, where the total number of vanishing moments of the wavelet and scaling function is maximum have been constructed by Daubechies and were named coiflets (after R. Coifman) [1]. Recently, biorthogonal filter banks where the total number of vanishing moments of the wavelet and scaling function is maximum (biorthogonal coiflets or cooklets) have been studied [2] [4]. In applications such as signal compression and denoising, it is desirable for the analysis function to have vanishing moments and regularity, and for the synthesis limit functions to possess smoothness. By dividing vanishing moments between the wavelet and scaling function, these properties may be achieved. However, as noted by Villasenor [6], regularity and smoothness are not sufficient for excellent image compression. Villasenor proposes the metrics of periodically shift-variant impulse response and step response, in addition to regularity, for predicting wavelet filter performance.

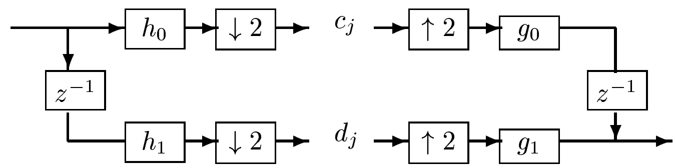

Figure 1: One-level two-channel filter bank

In this paper we demonstrate that the Bernstein polynomial basis proposed by Cooklev [2] for frequency domain construction of the biorthogonal coiflets is also an excellent design tool for the construction of *nearly* coiflet wavelet bases for which the total number of vanishing moments are *nearly* maximum. With the degree of freedom achieved by sacrificing a vanishing moment, the frequency responses of these new filters may be tailored to more closely match that of well known high performance wavelet filters. By matching the frequency response, the specific trade-off between step response and impulse response performance of the matched filters is implicitly chosen for the new filters.

Finally, although the choice of filter and the associated properties of the corresponding wavelet basis is sensitive to the compression algorithm considered, by performing simulations with two compression algorithms which differ widely in most major aspects of quantization and bit allocation (Said and Pearlman's SPIHT [3] and our SR_{SFQ} (Stack-Run Space-Frequency Quantization) [7] [5]) we demonstrate that the *nearly* coiflet 17/11 filters possess a robust form of smoothness which enables excellent image compression performance over a wide range of bit rates.

1.1. Biorthogonal Coiflet Wavelet Systems

Figure 1 shows a one level filter bank which is associated with a biorthogonal wavelet expansion. Iteration on the low-pass output will yield a multi-scale wavelet expansion.

The analysis h_0 and synthesis g_0 filters are associated with the scaling function $\phi(x)$ and the dual $\tilde{\phi}(x)$, and the wavelets $\omega(x)$ and $\tilde{\omega}(x)$ by the following equations:

$$\phi(x) = \sqrt{2} \sum_n h_0(n)\phi(2x-n) \tag{1}$$

$$\tilde{\phi}(x) = \sqrt{2}\sum_n g_0(n)\tilde{\phi}(2x-n) \quad (2)$$

$$\omega(x) = \sqrt{2}\sum_n h_1(n)\phi(2x-n) \quad (3)$$

$$\tilde{\omega}(x) = \sqrt{2}\sum_n g_1(n)\tilde{\phi}(2x-n). \quad (4)$$

Equations (1) and (2) converge to compactly supported basis functions if both the following assertions hold:

$$\sum_n h_0(n) = \sqrt{2} \quad \cap \quad \sum_n h_1(n) = \sqrt{2}.$$

For odd length linear-phase biorthogonal perfect reconstruction filter pairs, h_0 having 2N+1 coefficients and h_1 having 2N+3+4M coefficients, the following assertions hold:

$$\left.\frac{d^k H_0(\omega)}{d\omega^k}\right|_{\omega=\pi} = 0, k = 0, \ldots, 2K-1 \quad (5)$$

$$\left.\frac{d^k H_0(\omega)}{d\omega^k}\right|_{\omega=0} = \delta_k, k = 0, \ldots, 2P-1 \quad (6)$$

$$\left.\frac{d^k H_1(\omega)}{d\omega^k}\right|_{\omega=\pi} = 0, k = 0, \ldots, 2M+1. \quad (7)$$

Cooklev has demonstrated that if we represent the low-pass filter in Bernstein basis form:

$$H_0(x) = \sum_{n=0}^{N} d(n)\begin{pmatrix} N \\ i \end{pmatrix} x^i(1-x)^{N-i}, forx = sin^2(\omega/2),$$

then from equations (5-7) the d(n) are necessarily

$$d(n) = \begin{cases} 1 & : \quad 0 \le n < P \\ arbitrary & : \quad P \le n \le N-K \\ 0 & : \quad N-K < n \le N \end{cases}.$$

To form the biorthogonal coiflets the total number of vanishing moments of all limit functions is maximum, so K+P=max=N+1 and

$$\left.\frac{d^k H_1(\omega)}{d\omega^k}\right|_{\omega=0} = 0, k = 0, 2, \ldots, 2M. \quad (8)$$

For even length filters a Bernstein basis representation of the low-pass filter can also be found (see [2]).

2. *NEARLY* COIFLET BIORTHOGONAL WAVELETS

Regularity is important for filter performance, but other factors also play a large role. Beginning with the biorthogonal coiflet wavelet bases, we choose to sacrifice a zero of H_0 at $\omega = \pi$. This will reduce the regularity of the low-pass analysis filter. However, the number of vanishing moments of the analysis filters remains the same for this class of *nearly* coiflet biorthogonal wavelet filters. We may now tailor of the frequency response characteristic of the filter; in particular, by increasing the magnitude of the highest non-zero Bernstein basis the width of the low-pass filter's pass-band may be increased.

Table 1: Filter Bank Evaluation for Image Compression

	Step Overshoot [1]		Impulse Resp. [2]	
Filter	Average	Max	Average	Min
CDF 9/7	**.002**	**.004**	.142	.092
Odegard 9/7	.006	.010	**.164**	**.107**
Villasenor 18/10	.022	.034	.104	.088
Cooklet 17/11	**.003**	**.005**	.108	.088
Nearly Cooklet 17/11	.007	.013	.125	.092

For example, the Villasenor 18/10 filter may be shown to be a member of the *nearly* coiflet family (even length filters) for which the coefficients of the low-pass filter's Bernstein basis are d(n) = [1,1.02310534,1.78409122,0,0] (N=4, P=1, K=2, M=2). The Bernstein basis coefficients for the CDF 9/7 pair are d(n) = [1,1.30689881,0,0] (N=3, P=1, K=2, M=0). For the 17/11 biorthogonal coiflet (reversed cooklet) they are d(n) = [1,1,1,0,0] (N=4, P=3, K=2, M=1).

Choosing the coefficients d(n)=[1,1,1.3,0,0,0] (N=4, P=2, K=2, M=1) we obtain a new *nearly* coiflet 17/11 filter pair.

The Villasenor 18/10 filters increase the biorthogonal coiflet's Bernstein coefficients increasing the pass-band of the filter. The CDF 9/7 and 17/11 filters also exchange the role of the analysis and synthesis filters [2].

Figure 2 shows the frequency responses for the low-pass analysis filters. Note that the pass-band of the *nearly* coiflet 17/11 pair is more closely matched to the pass-bands of the CDF 9/7 and Villasenor 18/10 than is the pass-band of the original cooklet. The 3 dB cut-off of the low-pass analysis filter is a useful point to match as it well represents the width of the filter pass-band. This matching will result in a near (empirical) optimal trade-off between the step and the impulse response performance of the new filters.

Expanding Villasenor's filter metrics to include both minimum and average impulse response (as measure of the detail preserving ability of the low pass filter), as well as both maximum and average step response overshoot (as a measure of the filter ringing) we obtain Table 1.

Table 1 demonstrates that short filters do not necessarily produce fewer ringing artifacts than longer filters. In particular, the first side-lobe overshoot of the Odegard 9/7 filters is greater than that of the cooklet 17/11 filters and comparable to that of the biorthogonal *nearly* coiflet filters. For the *nearly* coiflet filters the overshoot and impulse response both increase as the last Bernstein coefficient is increased

[1] Fractional overshoot of the second side-lobe of the step response.
[2] Magnitude of the shift-variant impulse response.

Table 2: New 17/11 *Nearly* Coiflet Filters

h_0	0.8402696692, 0.4090630083, -0.1073757602, -0.0621741791, 0.0533641923, 0.0073357876, -0.0135767155, -0.0006712263, 0.0010068394
g_0	0.7568252267, 0.4226067872, -0.0331456304, -0.0814830079, 0.0082864076, 0.0124296114

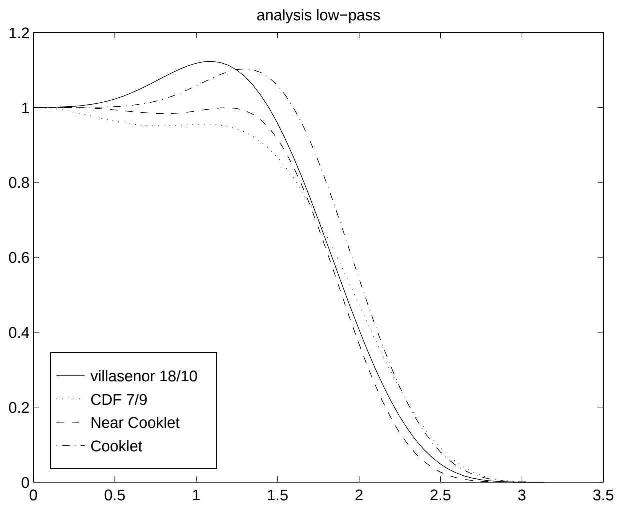

Figure 2: Analysis Low-Pass Filters H_0

in magnitude. This results in an improved impulse response (the filters are able to retain more fine detail energy in lower scale image subbands) at the expense of an increased overshoot artifact. Finding the ideal trade-off between the two desirable filter characteristics of low overshoot and high impulse response yields a filter pair which may perform well for image compression.

3. SIMULATION RESULTS

The standard for assessment of image compression performance is PSNR. To compare filter performance we use two wavelet compressors [3] [7] at a wide variety of bit rates for the popular test images Lena, Goldhill, and Barbara. The quantization and compression methods for Said and Pearlman's SPIHT (Set Partitioning in Hierarchical Trees) coder, and for our SR_{SFQ} coder are completely different. The characteristics of the test images are also quite distinct.

Table 3 shows in detail the superiority of the new *nearly* coiflet 17/11 filters for image compression with SPIHT (figures in bold indicate the highest performing filters). Experiments with our SR_{SFQ} algorithm also demonstrated the consistent advantage of the new filters.

Figures 4-5 show the reconstructed compressed barbara image for the SPIHT coding method at a compression ratio of 32:1 for the CDF 9/7 and new 17/11 filters. Clearly the new 17/11 filters retain more of the checkered pattern

Table 3: Image Compression Performance

		Lena	Barbara	Goldhill
Filter	Rate (b/p)	PSNR (dB)		
CDF 9/7	.125	31.09	24.59	27.52
	.25	34.12	27.36	29.56
	.50	37.20	31.18	32.22
	1.0	40.38	36.15	35.85
Villasenor 18/10 filter	.125	**31.22**	24.58	27.45
	.25	34.21	27.52	29.50
	.50	37.27	31.44	32.18
	1.0	40.39	**36.60**	35.87
Odegard 9/7 filter	.125	31.07	**24.65**	27.44
	.25	34.09	27.38	29.49
	.50	37.17	31.55	32.11
	1.0	40.36	36.29	35.78
Cooklet 17/11 filter	.125	31.06	24.37	27.37
	.25	34.08	27.14	29.40
	.50	37.18	31.17	32.06
	1.0	40.27	36.31	35.76
New 17/11 filter	.125	**31.22**	24.61	**27.54**
	.25	**34.23**	**27.56**	**29.60**
	.50	**37.30**	**31.56**	**32.26**
	1.0	**40.45**	36.57	**35.91**

on the side of the tablecloth, a behaviour which we expect from longer filters. The difference between the 17/11 and 18/10 filters is more difficult to demonstrate in hardcopy. However, in high quality reproduction it is easily seen that the new filters exhibit less ringing than the Villasenor 18/10 filters. In fact, close inspection of the figures shows in this case the new 17/11 filters exhibit less visually disturbing ringing than the CDF 9/7 filters. In effect, for still image coding [1], the new filters allow the best of both worlds – high frequencies are retained without significant ringing artifacts.

4. SUMMARY AND CONCLUSIONS

As our experiments indicate, biorthogonal coiflet filter banks are very competitive for image compression applications. In particular, they exhibit very little edge ringing. The *nearly* coiflet filters which we construct in this work, and in particular our new 17/11 filter pair, are clearly capable of achieving state-of-the art performance levels for lossy wavelet compression of gray scale images.

Our design methodology is instructive for the design of other *nearly* coiflet filter banks. We retain an almost maximum number of vanishing moments for the filters. These vanishing moments are known to be important for filter smoothness and regularity which are well documented to be impor-

[1] For video coding even length filters are preferred [6].

tant for image compression. By sacrificing one vanishing moment and more closely matching the frequency response of the well known CDF 9/7 and Villasenor 18/10 filters, superior image compression performance can be achieved with short filters that exhibit little ringing and good energy compaction of small details (impulses and thin lines).

We expect that this family of *nearly* coiflet filters will be very strong competitors for incorporation in future wavelet image and video compression standards.

ACKNOWLEDGMENTS

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Figure 3: Original Barbara Image

Figure 4: Compressed 32:1 [SPIHT & CDF 9/7]

Figure 5: Compressed 32:1 [SPIHT & *nearly* coiflet 17/11]