IMPLEMENTATION OF LINEAR MULTIUSER DETECTORS FOR ASYNCHRONOUS CDMA SYSTEMS BY LINEAR MULTI-STAGE INTERFERENCE CANCELLATION

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ABSTRACT

The decorrelating and the linear, minimum mean-squared error (MMSE) detector for asynchronous code-division multiple-access communications ideally are infinite memory-length detectors. Finite memory approximations of these detectors require the inversion of a correlation matrix whose dimension is given by the product of the number of active users and the length of the processing window. With increasing number of active users or increasing length of the processing window, the calculation of the inverse may soon become numerically very expensive. In this paper, we prove that the decorrelating and the linear MMSE detector can both be realized by linear multi-stage interference cancellation algorithms with ideally an infinite number of stages. It will be shown that for serial multi-stage interference cancellation, depending on the signal-to-noise ratio and the number of active users, only a few stages are necessary to obtain the same BER performance as the ideal detectors. Thus, the complexity can be reduced considerably.

1. INTRODUCTION

For asynchronous code-division multiple-access (CDMA) systems a number of linear multiuser detectors have been proposed, e.g. the decorrelator [1] and the linear, minimum mean-squared error (MMSE) detector [2]. These detectors require the inversion of a correlation matrix whose dimension d equals the number of active users K in case of bit-synchronous transmission. The easiest way of obtaining the corresponding detectors for asynchronous transmission is to assume that the data can be partitioned into blocks of a certain finite length M and to view the resulting problem as the detection of an equivalent number of MK synchronous users. Thus, the dimension of this correlation matrix is proportional to the block length. To achieve the partition into finite blocks, regular symbol intervals have to be left without transmission, which degrades the the spectral efficiency and requires some form of synchronism between the users.

This can be avoided in case of continuous transmission which can be viewed as the limit $M \to \infty$. Then the above detectors require infinite memory length. Thus, they can be referred to as IIR detectors [3]. The truncation of IIR detectors was proposed in [3, 1] as a way of designing finite memory length (FIR) detectors. Whereas the so-called truncated detectors ignore the edge effect caused by symbols outside the observation window of M data symbols, the detectors can also be optimized with respect to the finite observation window [3].

However, in all of the above approaches the dimension of the correlation matrix is increased by a factor of M for asynchronous transmission compared to synchronous transmission, which significantly increases the computational complexity of matrix inversion. This is due to the fact that the above detectors are derived in the time domain. In [1] and [4], z and D-transform approaches were used, respectively, to derive the IIR detectors. In these approaches the dimension of the correlation matrix is not increased[1].

In [5], a similar Fourier-transform approach was used to show that linear multi-stage interference cancellation schemes are asymptotically equivalent to decorrelating detector, which means that they approach the IIR decorrelator as the number of stages tends to infinity. In this paper, we will generalize this approach to also include the linear MMSE detector. Moreover, we investigate the convergence speed, that is the number of stages necessary to approach the IIR detector.

In Section 2, we review the main results of [1] and [4]. In Section 3, we show that linear multiuser detectors can be realized by linear interference cancellation schemes. The computational complexity is considered in Section 4. In Section 5, we investigate the convergence speed while the conclusions are given in Section 6.

2. LINEAR MULTIUSER DETECTORS

We consider a DS-CDMA system with K users for the limiting case $M \to \infty$. Let the receiver input signal be:

$$r(t) = \sum_{k=1}^{K} \sum_{m=-\infty}^{\infty} a_k(m)\sqrt{w_k}\, s_k(t - mT - \tau_k) + n(t), \qquad (1)$$

where $n(t)$ is white Gaussian noise with power spectral density σ^2, $a_k(m) \in \pm 1$ is the binary data signal, $\sqrt{w_k}$ is the amplitude, $s_k(t)$ is the spreading signal and τ_k is the time delay of the k-th user. The data symbols are assumed to be equally probable and mutually independent. The spreading signals $s_k(t)$ are given by $s_k(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} s_k(n)\ \mathrm{rect}(t - nT_c)$, where rect$(t)$ is defined as rect$(t) = 1$ for $0 \leq$ t $\leq T_c$ and rect$(t) = 0$ otherwise. T_c is the duration of a chip and $T = NT_c$ is the code period. We suppose that the spreading signals have unit energy, e.g., spreading sequences with $|s_k(n)| = 1$. The received signal is first fed into a bank of K filters matched to the users' spreading sequences and sampled at time instances $\tau_k + mT$:

$$y_k(mT + \tau_k) = \int_{mT+\tau_k}^{mT+T+\tau_k} r(t) s_k(t - mT - \tau_k) dt. \qquad (2)$$

[1] The Fourier-transform of a $d \times d$ matrix sequence yields a $d \times d$ frequency-dependent matrix. Of course the space of frequency-dependent matrices is ∞-dimensional. This corresponds to the fact that the IIR detectors can be described by such matrices.

For simplicity and without loss of generality, let us assume an ordering on the time delays τ_k such that $0 \le \tau_1 \le \tau_2 \ldots \le \tau_K < T$. Let $\boldsymbol{y}(m)$ be the output sequence of the bank of matched filters, $\boldsymbol{a}(m)$ the data sequence. Define the $K \times K$ signal crosscorrelation matrices $\boldsymbol{R}(m)$ whose entries are given by

$$R_{kj}(m) = \int_{-\infty}^{\infty} s_k^*(t-\tau_k) s_j(t-mT-\tau_j) dt. \tag{3}$$

$\boldsymbol{R}(1)$ is an upper triangular matrix with zero diagonal, $\boldsymbol{R}(m) = 0 \ \forall |m| > 1$ and $\boldsymbol{R}(m) = \boldsymbol{R}(-m)^H$. Moreover, let $\boldsymbol{W} = \mathrm{diag}([\sqrt{w_1}, ..., \sqrt{w_K}])$. With this notation and $\boldsymbol{a}(-M-1) = \boldsymbol{a}(-M+1) = 0$, the matched filter output sequence can be expressed as

$$\begin{aligned} \boldsymbol{y}(m) &= \boldsymbol{R}(-1)\boldsymbol{W}\boldsymbol{a}(m+1) + \boldsymbol{R}(0)\boldsymbol{W}\boldsymbol{a}(m) \\ &\quad + \boldsymbol{R}(1)\boldsymbol{W}\boldsymbol{a}(m-1) + \boldsymbol{n}(m), \end{aligned} \tag{4}$$

where $\boldsymbol{n}(m)$ is the matched filter output noise process with autocorrelation matrix $\mathrm{E}[\boldsymbol{n}(m)\boldsymbol{n}^H(m+j)] = \sigma^2 \boldsymbol{R}(j)$.

Using the Fourier-transform, the input-output relationship given by equation (4) can be rewritten in terms of the power spectra of the involved random processes. Since noise and data symbols are assumed to be independent, the spectrum $\boldsymbol{\Phi}_{yy}(f)$ of $\boldsymbol{y}(m)$ is given by

$$\boldsymbol{\Phi}_{yy}(f) = \boldsymbol{W}\boldsymbol{S}(f)\boldsymbol{\Phi}_{aa}(f)\boldsymbol{S}(f)\boldsymbol{W} + \boldsymbol{\Phi}_{nn}(f), \tag{5}$$

where the hermitian matrix $\boldsymbol{S}(f)$ is given by

$$\boldsymbol{S}(f) = \boldsymbol{R}^H(1)\, e^{\imath 2\pi f} + \boldsymbol{R}(0) + \boldsymbol{R}(1)\, e^{-\imath 2\pi f}, \tag{6}$$

$\boldsymbol{\Phi}_{aa}(f) = \boldsymbol{I}$ and $\boldsymbol{\Phi}_{nn}(f) = \sigma^2 \boldsymbol{S}(f)$ are respectively the power spectra of the data sequence and the noise and $\boldsymbol{I}$ is the $K \times K$ identity matrix. The cross-spectrum between the input and the output signal is

$$\boldsymbol{\Phi}_{ay}(f) = \boldsymbol{W}\boldsymbol{S}(f). \tag{7}$$

Linear IIR multiuser detectors are K-input K-output linear time-invariant filters which can be described by their transfer function $\boldsymbol{G}(f)$. Thus, the spectrum of $\boldsymbol{x}(m)$, the signal before the sign-decision, is given by

$$\boldsymbol{\Phi}_{xx}(f) = \boldsymbol{G}(f)\boldsymbol{\Phi}_{yy}(f)\boldsymbol{G}(f)^H. \tag{8}$$

The IIR zero-forcing detector, which is referred to as decorrelating detector in [1], completely removes the multiple-access interference (MAI) by "inverting" the channel matrix. Therefore, the IIR decorrelator is given by a filter with transfer function

$$\boldsymbol{G}_d(f) = \boldsymbol{S}^{-1}(f) = [\mathbf{R}^H(1)\, e^{\imath 2\pi f} + \mathbf{R}(0) + \mathbf{R}(1)\, e^{-\imath 2\pi f}]^{-1}. \tag{9}$$

The assumption that the inverse exists, is well justified. It should be noted that $\boldsymbol{S}(f)$ is positive definite in this case. The signal before the sign-decision is given by

$$\boldsymbol{x}(m) = \boldsymbol{W}\boldsymbol{a}(m) + \boldsymbol{n}_e(m), \tag{10}$$

where $\boldsymbol{n}_e(m)$ is a filtered Gaussian noise vector sequence. The noise power of the k-th user's noise component is

$$N_k = \mathrm{E}[\boldsymbol{n}_e(m)\boldsymbol{n}_e^H(m)]_{kk} = \sigma^2 \int_0^1 \boldsymbol{S}^{-1}(f)_{kk}\, df \ge \sigma^2. \tag{11}$$

Therefore, the probability of error for user k is given by

$$P_k = Q\left(\sqrt{\frac{w_k}{N_k}}\right), \tag{12}$$

with the error function $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{\frac{-y^2}{2}}\ \mathrm{dy}$.

The linear IIR MMSE detector minimizes the mean square errors

$$\mathrm{MSE}_k = \mathrm{E}[(a_k(m) - x_k(m))^2], \quad k = 1, \ldots, K. \tag{13}$$

The orthogonality principle states that for the MMSE estimate

$$\mathrm{E}[(\boldsymbol{a}(m+n) - \boldsymbol{x}(m+n))\boldsymbol{y}^H(m)] = 0 \tag{14}$$

holds for all n [4]. From (14), follows $\boldsymbol{\Phi}_{xy}(f) = \boldsymbol{\Phi}_{ay}(f)$. Using (5) and (7), the transfer function of the MMSE detector is given by

$$\boldsymbol{G}_{ms}(f) = \boldsymbol{W}^{-1}(\boldsymbol{S}(f) + \sigma^2 \boldsymbol{W}^{-2})^{-1}. \tag{15}$$

In case of a binary data signal the detector is followed by a sign-decision device. Thus, the multiplication with the diagonal matrix $\boldsymbol{W}^{-1}$ in (15) has no effect on the detected data symbols and can be omitted. From (14), it follows that the MMSE estimate is orthogonal to the error signal. This can be used to find the following expression for the MMSE of user k

$$\mathrm{MMSE}_k = 1 - \mathrm{E}[a_k(m)x_k(m)]. \tag{16}$$

Since the cross-spectrum $\boldsymbol{\Phi}_{ax} = \boldsymbol{W}\boldsymbol{G}_{ms}(f)\boldsymbol{S}(f)$, the MMSE of user k can be written as

$$\mathrm{MMSE}_k = \sigma^2 \int_0^1 [(\boldsymbol{W}^2\boldsymbol{S}(f) + \sigma^2 \boldsymbol{I})^{-1}]_{kk} df. \tag{17}$$

The signal-to-interference ratio (SIR) is defined as the ratio of the desired signal power to the sum of the powers due to noise and MAI at the output of the filter [6]. It can be shown that the MMSE solution also maximizes the SIR and that this maximum value is given by

$$\mathrm{MSIR}_k = \frac{\mathrm{E}[x_k(m)^2]}{\mathrm{E}[(a_k(m) - x_k(m))^2]} = \mathrm{MMSE}_k^{-1} - 1. \tag{18}$$

Because the MAI is not completely removed by the MMSE detector, there generally is no simple relationship between the SIR and BER like (12) in case of the decorrelating detector. However, in [7] it was found that the BER can be well-approximated by assuming that the output MAI-plus-noise is Gaussian. Using the Gaussian approximation the BER is given by

$$\tilde{P}_k = Q\left(\sqrt{\mathrm{MSIR}_k}\right). \tag{19}$$

3. LINEAR INTERFERENCE CANCELLATION

In this section, we will show that linear interference cancellation schemes are asymptotically equivalent to the linear multiuser detectors described in the previous section. A linear interference cancellation scheme is one where no hard-decisions, e.g., sign-decisions, are made in any of the stages, but there is only one decision device after the interference canceller. In [8], it was shown that multi-stage serial interference cancellation is asymptotically equivalent to the decorrelator for synchronous CDMA. In [5], it was found that both multi-stage, parallel and serial interference cancellation are asymptotically equivalent to the IIR decorrelator for asynchronous CDMA. For linear parallel interference cancellation only, this was also found independently in [9]. Here, we generalize the approach of [5] to also include the MMSE detector.

The decorrelator and the MMSE detector both require the inversion of the Fourier-transform $\boldsymbol{C}(f)$ of a correlation matrix which is respectively given by

$$\boldsymbol{C}_d(f) = \boldsymbol{S}(f),\ \boldsymbol{C}_{ms}(f) = \boldsymbol{S}(f) + \sigma^2 \boldsymbol{W}^{-2}. \tag{20}$$

Note that the decorrelator and the MMSE detector differ only in the diagonal elements of $\boldsymbol{C}(f)$.

The basic idea behind our derivation of the different linear interference cancellation schemes is to apply an iterative algorithm for matrix inversion to solve the following system of linear equations in the frequency domain:

$$\boldsymbol{C}(f)\boldsymbol{X}(f) = \boldsymbol{Y}(f). \tag{21}$$

However, an algorithm in the frequency domain would not solve the implementation problem because it ideally would require the Fourier-transform of a sequence of infinite length. Therefore, a time-domain version of the algorithm is obtained by applying the inverse Fourier-transform to each of the step equations of the frequency-domain algorithm. The resulting time-domain step equations generally require the knowledge of some future signal elements. Thus, to actually implement this algorithm an appropriate amount of delay has to be introduced between the stages.

The simplest iterative scheme is the Jacobi iteration. Let us consider the following splitting of the correlation matrix $\boldsymbol{C}(f)$:

$$\boldsymbol{C}(f) = \boldsymbol{C}_L(f) + \boldsymbol{D} + \boldsymbol{C}_U(f), \tag{22}$$

where $\boldsymbol{C}_L(f)$ is lower triangular with zero diagonal, $\boldsymbol{D}$ is a diagonal matrix, and $\boldsymbol{C}_U(f)$ is upper triangular with zero diagonal. Note that $\boldsymbol{D} = \boldsymbol{I}$ for the decorrelator because the code sequences are normalized to have unit energy, and $\boldsymbol{D} = \boldsymbol{I} + \sigma^2 \boldsymbol{W}^{-2}$ for the MMSE detector. Moreover, $\boldsymbol{C}_L(f) = \boldsymbol{C}_U(f)^H$ since $\boldsymbol{C}(f)$ is hermitian. In the frequency domain the transition from $\boldsymbol{X}^i(f)$ to $\boldsymbol{X}^{i+1}(f)$ is given by

$$\boldsymbol{X}^{i+1}(f) = \boldsymbol{D}^{-1}\left[\boldsymbol{Y}(f) - (\boldsymbol{C}_L(f) + \boldsymbol{C}_U(f))\boldsymbol{X}^i(f)\right]. \tag{23}$$

If $\|\boldsymbol{C}_L(f) + \boldsymbol{C}_U(f)\| < 1$ for any induced matrix norm, the Jacobi iteration converges to $\boldsymbol{X}^\infty(f) = \boldsymbol{C}(f)^{-1}\boldsymbol{Y}(f)$ for any $\boldsymbol{X}^0(f)$ [10]. Thus, it is asymptotically equivalent to the decorrelator or the MMSE detector. The time domain step equation is given by the inverse Fourier-transform of (23) as

$$\begin{aligned} \boldsymbol{x}^{i+1}(m) \;=\; \boldsymbol{D}^{-1}\big[&\boldsymbol{y}(m) - \boldsymbol{R}(-1)\boldsymbol{x}^i(m+1) \\ &-(\boldsymbol{R}(0) - \boldsymbol{I})\boldsymbol{x}^i(m) - \boldsymbol{R}(1)\boldsymbol{x}^i(m-1)\big]. \end{aligned} \tag{24}$$

It can easily be seen that for $D_{kk} = 1$ Eqn. (24) describes the linear version of the interference cancellation scheme proposed, e.g., in [11]. Since the signals of all users are treated in parallel in each stage, Eqn. (24) describes a linear multi-stage parallel interference canceller. In (24), the knowledge of future estimates $\boldsymbol{x}^i(m+1)$ of the previous stage is required. Thus, it necessary to introduce a delay of one bit duration between the single stages. It should be noted that the Jacobi iteration does not converge for all possible $\boldsymbol{C}(f)$. Moreover, the convergence very slow. Therefore, it will not be considered in the sequel.

The convergence of the iteration scheme can be sped up by a scheme that is serial in-between the users because the parallel treatment of the users in the Jacobi iteration does not use the most recently available information. In the i-th stage of the interference cancellation for user k, the estimates of the signals of the users $1,\dots,k-1$ of that stage can be used instead of the estimates of the previous stage. This is the idea of the Gauß-Seidel (GS) iteration. Thus, instead of (23) we now have

$$\boldsymbol{X}^{i+1}(f) = \boldsymbol{D}^{-1}\big[\boldsymbol{Y}(f) - \boldsymbol{C}_U(f)\boldsymbol{X}^i(f) - \boldsymbol{C}_L(f)\boldsymbol{X}^{i+1}(f)\big], \tag{25}$$

with time-domain step equation for user k given by

$$\begin{aligned} x_k^{i+1}(m) \;=\; \frac{1}{D_{kk}}\Bigg[&y_k(m) - \sum_{l=k+1}^{K} R_{kl}(0)x_l^i(m) + R_{kl}(1)x_l^i(m-1) \\ &-\sum_{l=1}^{k-1} R_{kl}(-1)x_l^{i+1}(m+1) + R_{kl}(0)x_l^{i+1}(m)\Bigg]. \end{aligned} \tag{26}$$

Since $\boldsymbol{S}(f)$ is positive definite and hermitian, the GS iteration converges to $\boldsymbol{X}^\infty(f) = \boldsymbol{S}(f)^{-1}\boldsymbol{Y}(f)$ [10]. The GS iteration defines a linear multi-stage serial interference cancellation scheme. Serial cancellers have been proposed, e.g., in [12].

In the time-domain step equation (26) for user k, the knowledge of future estimates $x_l^{i+1}(m+1)$ of the users $1,\dots,k-1$ of the actual stage is required. Therefore, a delay of one bit duration has to be introduced between the calculation of the estimates for the different users. Thus, the delay between the single stages is equal to K times the bit duration. It is possible to reduce the delay between the single stages to be equal to one bit duration if one imposes an ordering of the users on the time delays τ_k such that $T > \tau_1 \geq \ldots \geq \tau_K \geq 0$. Since $\boldsymbol{R}(-1)$ is upper triangular in this case, no future estimates of the actual stage are needed. However, the fastest convergence can usually be achieved by an ordering of the users according to their signal strength. To avoid the large delay between the stages we modify (26) by replacing $x_l^{i+1}(m+1)$ with $x_l^i(m+1)$, that is we use future estimates of the previous stage instead of the actual stage. The resulting time-domain step equation can be implemented with a delay equal to one bit duration between the stages. We have found this modified version of the GS iteration to perform equally well as the original one.

4. COMPUTATIONAL COMPLEXITY

The FIR detectors require the inversion of an $MK \times MK$ correlation matrix. It is reasonable to assume that, in a time-varying environment, this matrix has to be updated every M symbols . Not taking into account the block structure and using the Cholesky factorization the complexity would be $(MK)^2/6 + 3MK/2 + 1/3$ multiplications/symbol. However, using the block LU factorization [10, Sect. 4.5.1] results in $M(K^3/3 + 5K^2)$ multiplications to calculate the inverse and detect MK data symbols. Thus the complexity is $K^2/3 + 5K$ multiplications/symbol which is independent of M. The iterative methods require $2K^2$ multiplications for the calculation of the step equation. Thus, the complexity is $2SK$ multiplications/symbol. Therefore, the iterative methods are less complex if

$$S < K/6 + 5/2. \tag{27}$$

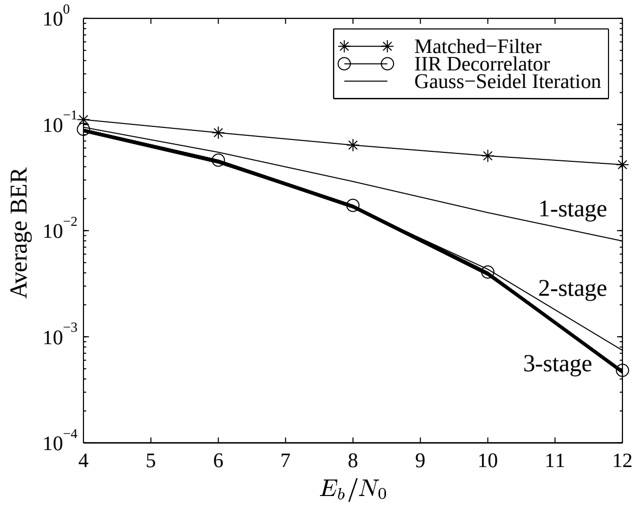

Figure 1: Average BER of the Gauß-Seidel Iteration, $K = 10$.

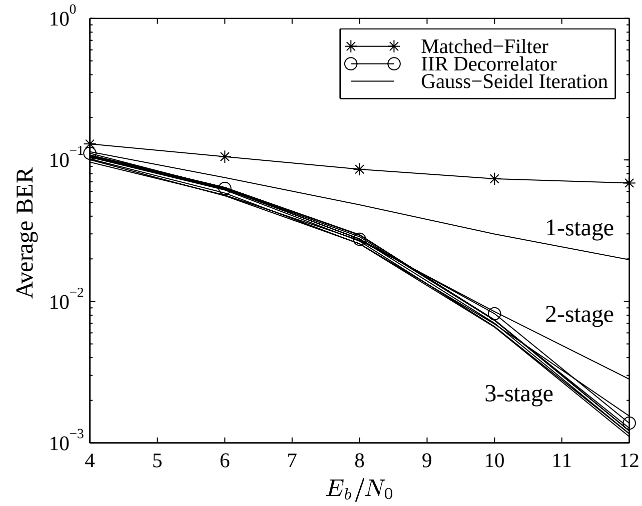

Figure 2: Average BER of the Gauß-Seidel Iteration, $K = 15$.

5. NUMERICAL RESULTS

In this section, based on numerical examples, we investigate how many stages S are required for the GS method to achieve IIR detector performance. The results are based on an asynchronous system with perfect power control using the decorrelating detector. The spreading sequences are randomly selected from a Gold code of length $N = 31$. All bit-error-rates are averaged over the users and over 1000 randomly chosen delay constellations.

In Figs. 1 and 2 the bit-error-rates are depicted for 10 and 15 users, respectively. The BER of the matched filter detector and the IIR detector are also included. It can be seen that S depends on the SNR or the desired BER. If the SNR is less than 10 dB, two stages are sufficient two achieve IIR detector performance, whereas 3 stages are needed if 10 dB $\leq$ SNR $\leq$ 12 dB. For $K = 10$ and $K = 15$, Eqn. (27) respectively gives $S < 4.17$ and $S < 5$. Thus, in both cases, the GS method is more efficient than the block LU decomposition.

6. CONCLUSIONS

We have shown that the linear decorrelating detector and the linear MMSE detector can be realized by linear multi-stage interference cancellation algorithms with ideally an infinite number of stages. It was found that a parallel linear interference cancellation scheme is equivalent to a Jacobi iteration, whereas a Gauß-Seidel iteration corresponds to a serial linear canceller.

Numerical examples demonstrate that the number of stages required to achieve IIR detector performance depends on the number of users, the signal-to-noise ratio, and the choice iterative algorithm. For a small number of users and moderate signal-to-noise ratios (SNR ≤ 12 dB), only a few, typically 3, stages of the Gauß-Seidel iteration are needed.

Thus, linear interference cancellation schemes offer a way of implementing linear multiuser detectors very efficiently, with small detection delay and low memory consumption.

7. REFERENCES

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