ROBUST MATCHED-FIELD PROCESSING IN UNCERTAIN SHALLOW-WATER ENVIRONMENTS USING AN L_P-NORM ESTIMATOR

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ABSTRACT

An optimal approach to matched-field source localization in the presence of environmental uncertainties is the maximum *a posteriori* (MAP) estimator. The MAP estimator can be interpreted as an exponentially-weighted average over environmental realizations. In practice, only a finite number of environmental realizations can be included in this average resulting in a suboptimal processor. In this paper, we propose an L_p-norm estimator as a robust alternative to MAP in the presence of finite environmental sampling. We also show, using wavenumber gradients, that accurate localization estimates can be obtained using environmental realizations besides the precise true. Simulation results from a shallow-water environment are presented to illustrate the performance improvement.

1. INTRODUCTION

Matched-field processing (MFP) methods utilize complex multipath propagation models for source localization. However, it is well known that MFP methods can be extremely sensitive to small errors in the assumed values of the environmental parameters [1]. A statistically optimal approach to MFP source localization in the presence of environmental uncertainties is the maximum *a posteriori* probability (MAP) estimator. Using the MAP estimator, source range and depth are treated as the parameters of interest while the uncertain environmental parameters are treated as nuisance parameters. Prior probability density functions (PDF) are assumed for the uncertain environmental parameters from their uncertainty intervals based on *in situ* measurements and historical data. Location estimates are obtained by maximization over source location after integration over the uncertain environmental parameter space using the prior PDF. A MAP estimator derived in the context of robust MFP was given in [2],[3] and called the optimum uncertain field processor (OUFP). However, a major issue in using the MAP estimator is the computation required to perform the integrations. As the number of nuisance parameters grows or their uncertainty intervals become large, the MAP estimator rapidly becomes computationally intensive. A computationally efficient, approximate MAP estimator which allows the integrations to be computed off-line, prior to the processing of data, was derived in [4].

In this paper, we propose applying an L_p-norm estimator to the uncertain environment, source localization problem. It is derived from the interpretation of the MAP estimator as a weighted-averaging processor. In the limit as $p \to \infty$, the L_∞-norm estimator is in essence the ML estimator. However, in contrast to the ML estimator, the search is only conducted over the location parameter space using M sets of the environmental parameters randomly sampled from their prior PDF. We will show, using the modal horizontal wavenumbers, why accurate source location estimates can be obtained using environmental realizations other than the precise true.

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2. PRELIMINARIES

2.1. Signal model

The signals received on a vertical array of N sensors from a point source can be expressed in vector form as

$$\mathbf{y}(\omega) = s(\omega)\mathbf{a}(\omega, \mathbf{\Theta}, \mathbf{\Psi}) + \mathbf{n}(\omega), \qquad (1)$$

where the elements of $\mathbf{y}(\omega)$ are the components of the signal wavefront observed on the sensors located at depths $\mathbf{z} = [z_1^r \cdots z_N^r]^T$ at radian frequency ω. The scalar $s(\omega)$ is the complex signal amplitude at ω. The vector $\mathbf{a}(\omega, \mathbf{\Theta}, \mathbf{\Psi})$, which is called a replica vector, is the acoustic transfer function between a source at location $\mathbf{\Theta} = [r, z]$ and the array which is parameterized by the vector of environmental parameters $\mathbf{\Psi}$, i.e., sound-speed profile and bottom characteristics. The vector $\mathbf{n}(\omega)$ contains samples of complex, Gaussian noise.

2.2. The maximum *a posteriori* probability estimator

The MAP estimator derived in [2] assumes uniform PDF $p(\mathbf{\Psi})$ for the uncertain environmental parameters. If we further assume the noise is white and the replica vectors are normalized to have unit norm, the MAP estimator can be written as

$$\hat{\mathbf{\Theta}} = \arg\max_{\mathbf{\Theta}} \int_{\mathbf{\Psi}} \exp\left\{ \frac{\sigma_a^2 \left| \mathbf{a}^H(\mathbf{\Theta}, \mathbf{\Psi})\mathbf{y} \right|^2}{2(\sigma_a^2+1)} \right\} \, p(\mathbf{\Psi}) \, d\mathbf{\Psi}, \tag{2}$$

where σ_a^2 is the signal amplitude variance. Dependence on ω has been dropped for convenience, since we will be assuming a monochromatic signal. The white noise assumption was made for clarity in the subsequent development. It is a straightforward modification to (2) if non-white noise is assumed.

For numerical implementation of the MAP estimator, the integral is approximated by a sum over M realizations of the environment, i.e., M integration steps,

$$\hat{\mathbf{\Theta}} = \arg\max_{\mathbf{\Theta}} \sum_{i=1}^{M} \exp\left\{ \frac{\sigma_a^2 \left| \mathbf{a}^H(\mathbf{\Theta}, \mathbf{\Psi}_i)\mathbf{y} \right|^2}{2(\sigma_a^2+1)} \right\}. \tag{3}$$

The M environmental realizations are taken from samples of the probability distributions of the environmental parameters. In [3], a Monte Carlo approach to computing (3) was proposed. Notice that the argument of the exponential in (3) is simply the conventional Bartlett processor scaled by $\frac{\sigma_a^2}{2(\sigma_a^2+1)}$. Therefore, (3) can be interpreted as an exponentially-weighted average of Bartlett surfaces over M combinations of the environmental parameters. If we assume $\mathbf{y}$ is also normalized to unit norm, then $\left| \mathbf{a}^H(\mathbf{\Theta}, \mathbf{\Psi}_i)\mathbf{y} \right|^2$ would equal one if $\mathbf{\Theta}$ and $\mathbf{\Psi}_i$ were perfectly matched to the data. Any other values of $\mathbf{\Theta}$ and $\mathbf{\Psi}_i$ would produce a value between zero and one. Thus, those values of $\left| \mathbf{a}^H(\mathbf{\Theta}, \mathbf{\Psi}_i)\mathbf{y} \right|^2$ nearer to one, which correspond to *better* replica-data matches, are given more relative weight in the averaging process of (3).

We observe that the exact implementation of the MAP estimator in (2) assumes infinitesimally-spaced samples of the environmental parameters. Thus, the exact true environment and combinations of the environmental parameters very close to the true are all included in the averaging process. This results in a clustering of reinforcing peaks near the true value of $\mathbf{\Theta}$. Therefore, the MAP estimator as implemented in (3) with finite sampling of the environmental parameters can be suboptimal . The minimum sampling density of the environmental parameter space using (3), to obtain an accurate approximation to (2), would be dependent on the sensitivity of the model to each of the environmental parameters. Also, since model sensitivity increases with frequency, sampling density would also need to be increased with frequency. However, finer sampling of the environmental parameter space rapidly leads to a computationally-intensive processor which can become computationally prohibitive. In [3] for example, 51^7 integration steps were required at each $\mathbf{\Theta}$ for the OUFP to compute a localization estimate if all combinations of the environmental parameters were utilized. Even using a Monte Carlo approach can require thousands of integration steps which results in a computationally-prohibitive estimator. A more computationally efficient approach is required for real-world systems application.

3. THE L_P-NORM ESTIMATOR

As discussed previously, the MAP estimator computes robust localization estimates by forming an exponentially weighted average over environmental realizations. The MAP estimator also inherently assumes a continuum of environmental parameter samples. Therefore, with a finite sampling of the environmental parameter space (e.g., $M \leq 100$), a smaller number of better replica-data matches are possible. Thus, there is a smaller occurrence of the clustering of peaks in the vicinity of the true value of $\mathbf{\Theta}$. This increases the possibility of the true peak being averaged out and obscured by sidelobes reinforced over the averaging process. We can compensate for this by giving $\left| \mathbf{a}^H(\mathbf{\Theta}, \mathbf{\Psi}_i)\mathbf{y} \right|^2$ values corresponding to better replica-data matches more significance; greater relative weight in the averaging process. This is accomplished by replacing the exponential weighting in (3) by a p-power weighting,

$$\hat{\mathbf{\Theta}} = \arg\max_{\mathbf{\Theta}} \sum_{i=1}^{M} \left(\left| \mathbf{a}^H(\mathbf{\Theta}, \mathbf{\Psi}_i)\mathbf{y} \right|^2 \right)^p, \tag{4}$$

where p would assume a large value, e.g. $p \geq 10$. This is essentially an L_p-norm over environmental realizations. These types of norms accentuate large values and attenuate small values. In this way, peaks resulting from better replica-data matches are less likely to be averaged out.

In the limit as $p \to \infty$, (4) becomes

$$\hat{\mathbf{\Theta}} = \arg \max_{\mathbf{\Theta}, \mathbf{\Psi}_1 \leq \mathbf{\Psi}_i \leq \mathbf{\Psi}_M} \left| \mathbf{a}^H(\mathbf{\Theta}, \mathbf{\Psi}_i)\mathbf{y} \right|^2. \tag{5}$$

The averaging is eliminated. The estimator of (5) simply finds the best match over source location $\mathbf{\Theta}$ using the M realizations of the environmental parameters. Equation (5) is the L_∞-norm estimator. It resembles the ML estimator, however it differs in that only the location parameter space is searched over. Environmental parameter estimates are not obtained. Since it is unlikely that the precise true combination of the environmental parameters are included in the M realizations, the L_∞-norm estimator inherently assumes that other combinations of the environmental parameters can also produce accurate localization estimates. We will show the validity of this assumption next.

4. ENVIRONMENTAL SIMILARITY

In this section we will show why, in the context of localization, other environmental realizations can appear similar to the precise true and can produce accurate localization estimates. We begin by decomposing the replica vectors using the modefunctions ϕ_m of the environment [5]. We can express $\mathbf{a}(\boldsymbol{\Theta}, \boldsymbol{\Psi})$ as

$$\mathbf{a}(\boldsymbol{\Theta}, \boldsymbol{\Psi}) = \exp\left\{j\frac{\pi}{4}\right\} \boldsymbol{\Omega}(\mathbf{z}, \boldsymbol{\Psi})\mathbf{G}(\boldsymbol{\Theta}, \boldsymbol{\Psi})\boldsymbol{\mu}(\boldsymbol{\Theta}, \boldsymbol{\Psi}). \quad (6)$$

where the columns of $\boldsymbol{\Omega}(\mathbf{z}, \boldsymbol{\Psi})$ are the modefunctions sampled at the receiver depths, the diagonal matrix

$$\mathbf{G}(\boldsymbol{\Theta}, \boldsymbol{\Psi}) = diag[\exp\{-\gamma_1 r\}, \ldots, \exp\{-\gamma_Q r\}] \quad (7)$$

contains the modal attenuation coefficients γ_i, and

$$\boldsymbol{\mu}(\boldsymbol{\Theta}, \boldsymbol{\Psi}) = \begin{bmatrix} \left(\frac{2\pi}{k_1 r}\right)^{\frac{1}{2}} \phi_1(z) \exp\{jk_1 r\} \\ \vdots \\ \left(\frac{2\pi}{k_Q r}\right)^{\frac{1}{2}} \phi_Q(z) \exp\{jk_Q r\} \end{bmatrix}. \quad (8)$$

Since the modal phases are composed of the products of the wavenumbers k_i and source range r, the effects of environmental mismatch are most significant in the modal phases. The modal attenuation terms given by the $\exp\{-\gamma_i r\}$ in $\mathbf{G}(\boldsymbol{\Theta}, \boldsymbol{\Psi})$ also contain a multiplication by r. However, errors in the modal attenuation are much less significant in comparison, since the γ_i are typically orders of magnitude smaller than the k_i.

To investigate how the modal phases impact localization, we substitute (6) into (5) and expand out the magnitude-squared inner product which gives

$$\mathbf{y}^H \boldsymbol{\Omega}(\mathbf{z}, \boldsymbol{\Psi})\mathbf{G}(\boldsymbol{\Theta}, \boldsymbol{\Psi})\boldsymbol{\mu}(\boldsymbol{\Theta}, \boldsymbol{\Psi})\boldsymbol{\mu}^H(\boldsymbol{\Theta}, \boldsymbol{\Psi})\mathbf{G}(\boldsymbol{\Theta}, \boldsymbol{\Psi})\boldsymbol{\Omega}^T(\mathbf{z}, \boldsymbol{\Psi})\mathbf{y}. \quad (9)$$

The outer product $\boldsymbol{\mu}(\boldsymbol{\Theta}, \boldsymbol{\Psi})\boldsymbol{\mu}^H(\boldsymbol{\Theta}, \boldsymbol{\Psi})$ is the only term in (9) which contains the modal phases. This outer product produces a Hermitian matrix with element i, l equal to

$$\frac{\phi_i(z)\phi_l(z)}{(k_i k_l)^{\frac{1}{2}}} \exp\{j\Delta K_{i,l} r\}, \quad (10)$$

where $\Delta K_{i,l} = k_i - k_l$. We observe that the modal phase of the L_∞-norm estimator is only dependent on the relative differences between the wavenumbers and not their precise values. Hence, environmental realizations whose relative wavenumber differences are close to those at the true environment can produce similar ambiguity surfaces.

A method for quantifying the wavenumber differences of the environmental realizations is to use the wavenumber gradient. The wavenumber gradient (WG) is the gradient of the curve fit to the wavenumbers. Environmental realizations which have WG's similar to that at the true will have similar relative wavenumber-differences. As an example, consider a set of environmental realizations whose wavenumbers lie on different lines with varying slopes. It is easy to see that realizations which have identical slopes, or gradients, will have identical relative wavenumber differences. Therefore, a measure of similarity between environmental realizations is the error between the WG at the true environment and the WG at other realizations. The WG error is given by

$$e_{wg} = \sum_{i=Q_{\min}}^{Q_{\max}} |g_t(i) - g(i)|^2, \quad (11)$$

where $g_t(i)$ and $g(i)$ are the samples of the numerically computed WG at the true environment and an alternate realization, respectively. Notice that e_{wg} is only computed over the *effective* modes, $Q_{\min}$ to $Q_{\max}$, those modes with significant amplitude as sampled by the receiving array. We can compute (11) in simulation studies to substantiate our assertion that environmental realizations, other than the precise true, yielding small WG errors produce similar ambiguity surfaces and accurate localization estimates.

5. SIMULATION RESULTS

In this section, we will compare the performance of the MAP estimator with that of the L_∞-norm estimator for a simulated shallow-water environment. Both estimators used 100 randomly selected environmental realizations from the PDF of the environmental parameters to process the data. Localization performance was determined based on 100 Monte Carlo trials for each signal-to-noise ratio from -5 dB to 40 dB. A correct localization was defined as a estimate within a region of ± 300 m in range and ± 4 m in depth of the true source position. The additive noise in each trial is independent, zero-mean Gaussian. In each of the 100 trials a unique, randomly selected environmental realization and source position was chosen. The receiving array consisted of 20 elements spaced at 5 m with the shallowest element at a depth of 5 m. A single observation of the array output was used in each trial.

The environment was a two-layer, range-independent shallow water waveguide with a linear sound-velocity profile. This environment was developed at the Naval Postgraduate School for localization algorithm testing at the Naval Undersea Warfare Center. It contained five uncertain environmental parameters whose ranges of uncertainty are given in Table 1. The range-depth search region was 900 to 5000 m in range and 10 to 90 m in depth. A narrowband source at 700 Hz was used. Figure 1 presents the simulation results for this environment. The performance of the L_∞-norm estimator is significantly better than that of the MAP estimator. This is due to the fact that with only 100 environmental

PARAMETER	RANGE
surface sound-speed	1530±5 m/s
bottom sound-speed	1490±5 m/s
bottom depth	100±5 m
subbottom sound-speed	1604±25 m/s
bottom attenuation	0.192±0.125 dB/λ

Table 1: Ranges of environmental uncertainty for Naval Postgraduate School environment.

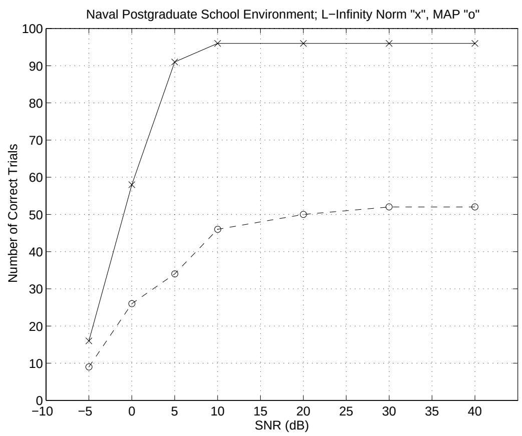

Figure 1: Performance of estimators using Naval Postgraduate School environment.

realizations the MAP estimator does not benefit from the clustering of reinforcing peaks near the true source location as discussed in Section 2. However, the L_∞-norm estimator only needs a single environment producing a small WG error to achieve an accurate localization estimate. Clearly in this case, 100 environmental realizations were a sufficient amount in which to obtain the small WG error needed.

We can also use this simulation example to illustrate the relationship between WG error and correct localization estimates. For each of the 96 correct trials at a SNR of 40 dB, we computed the WG errors between the true environment in that trial and each of the 100 environmental realizations used by the L_∞-norm estimator to process the data. We then ranked the 100 environmental realizations according to their WG error from smallest to largest. This allows us to histogram the ranking of the environmental realization which produced the localization estimate in each of the correct trials. Figure 2 shows the resulting histogram. We see that in over 50 of the 96 correct trials the environmental realization which produced the localization estimate had the smallest WG error. Over all, in more than 90% of the trials the WG error of the environmental realization which produced the localization estimate was within the 10 smallest.

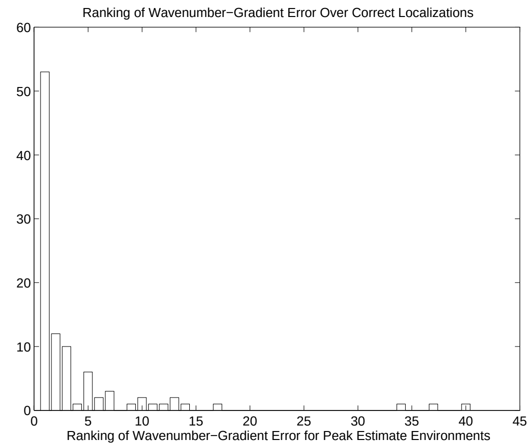

Figure 2: Histogram of rank of WG errors in correct trials.

6. CONCLUSIONS

In this paper, it was shown that the MAP estimator applied to matched-field source localization can be suboptimal when a finite sampling of the environmental parameters is used. Using a L_∞-norm estimator in the presence of finite environmental sampling can provide significant localization performance improvement over the MAP estimator. It was also shown that for the localization problem, environmental realizations besides the precise true can provide accurate localization estimates. Simulation results from a shallow water environment were presented to support these findings.

7. REFERENCES

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