DESIGN OF PERFECT RECONSTRUCTION FIR MULTIFILTERS

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ABSTRACT

The design of perfect reconstruction FIR multifilters is discussed in this paper. Schur algorithm is applied to factorize the polyphase matrix of multifilters into lattice blocks. The multifilters are characterized by the chain parameters in each lattice block. The complete parameterization of paraunitary multifilters and a class of biorthogonal multifilters are derived. The parameterizations are minimial and result in simple design methods using unconstrainted optimization.

1. INTRODUCTION

Multifilters are vector valued filter bank. The subband filters of the multifilters are $N \times N$ dimensional matrix filters that accept N dimensional vector valued input signal $\mathbf{x}(n) = \begin{bmatrix} x_0(n) & \cdots & x_{N-1}(n) \end{bmatrix}^T$. The bank of multifilters has the same structure as ordinary filter bank, with the extra freedom that comes with matrix coefficients. This added freedom also creates design difficulties. The major difference between multifilters and traditional filter bank is the non-commutative matrix multiplication.

Multifilters are used for multivalued signal processing, such as color image processing, transform domain vector quantization, etc. It is also used to implement traditional filter bank for parallel computation [10]. Furthermore, multfilters can be used for fast implementation of multiwavelets [6, 10]. The design of multifilters is still at an early stage of development. Johnson [11] derived the lattice structure for paraunitary 2 band 2 channel multifilters. In the author's pervious work [12], a design method for M band N channel paraunitary multifilters based on modulated structure was derived. In this paper, a complete parameterization of multifilters is derived.

The subband filters of M band N channel multifilters are matrix valued filters,

$$\mathbf{H}_i(z) = \mathbf{H}_{i,0} + \mathbf{H}_{i,1}z^{-1} + \cdots + \mathbf{H}_{i,m}z^{-m}, \quad (1)$$
$$\mathbf{F}_i(z) = \mathbf{F}_{i,0} + \mathbf{F}_{i,1}z^{-1} + \cdots + \mathbf{F}_{i,\hat{m}}z^{-\hat{m}}, \quad (2)$$

where $\mathbf{H}_{i,j}, \mathbf{F}_{i,j} \in \mathbb{R}_{N\times N}$, m and $\hat{m}$ are the degree of the matrix filters. The matrix valued coefficients do not affect the basic properties of z-transform and the noble identities can be extended naturally to matrix filter. The polyphase components of the analysis filters $\mathbf{H}_i(z)$ and synthesis filters $\mathbf{F}_i(z)$ are defined as

$$\mathbf{H}_i(z) = \sum_{k=0}^{M-1} \mathbf{E}_{i,k}(z^M)z^{-k}, \mathbf{F}_i(z) = \sum_{k=0}^{M-1} \mathbf{R}_{i,k}(z^M)z^{-(M-1-k)}.$$

The polyphase representation of multifilters in block matrix format is

$$\mathbf{E}(z) = \begin{bmatrix} \mathbf{E}_{0,0}(z) & \cdots & \mathbf{E}_{0,M-1}(z) \\ \vdots & \ddots & \vdots \\ \mathbf{E}_{M-1,0}(z) & \cdots & \mathbf{E}_{M-1,M-1}(z) \end{bmatrix}, \quad (3)$$

$$\mathbf{R}(z) = \begin{bmatrix} \mathbf{R}_{0,0}(z) & \cdots & \mathbf{R}_{M-1,0}(z) \\ \vdots & \ddots & \vdots \\ \mathbf{R}_{0,M-1}(z) & \cdots & \mathbf{R}_{M-1,M-1}(z) \end{bmatrix}, \quad (4)$$

The multifilters can be characterized by parametering the matrix polynomial of the polyphase matrix. An iterative Schur algorithm used for the reduction of the orders of polynomials [7] was extended to the matrix polynomials. It is applied to parameterize the polyphase matrix.

2. 2 BAND N CHANNEL BLOCK FIR LOSSLESS SYSTEM

Suppose $\mathbf{E}_0(z)$ and $\mathbf{E}_1(z)$ form a lossless system

$$\mathbf{P}_D(z) = \begin{bmatrix} \mathbf{E}_0(z) \\ \mathbf{E}_1(z) \end{bmatrix} \quad (5)$$

Lossless implies the power complementary property

$$\tilde{\mathbf{P}}_D(z)\mathbf{P}_D(z) = \begin{bmatrix} \tilde{\mathbf{E}}_0(z) & \tilde{\mathbf{E}}_1(z) \end{bmatrix} \begin{bmatrix} \mathbf{E}_0(z) \\ \mathbf{E}_1(z) \end{bmatrix} \quad (6)$$
$$c^2\mathbf{I} = \tilde{\mathbf{E}}_0(z)\mathbf{E}_0(z) + \tilde{\mathbf{E}}_1(z)\mathbf{E}_1(z), \quad (7)$$

where $\mathbf{I}$ is the identity matrix and $c \neq 0$. If $\mathbf{E}_i(z)$ have the form of eq.(1), then $\mathbf{P}_D(z)$ is causal and FIR. Our aim is to factorize $\mathbf{P}_D(z)$ similar to that discussed in [5, Chapter 14.3.2].

Schur algorithm suggested the lattice structure in Figure 1a. The transfer function is given by

$$\mathbf{E}_{0_m}(z) = \Theta_{00,m}\mathbf{H}_{0_{m-1}}(z) + \Theta_{01,m}z^{-1}\mathbf{H}_{1_{m-1}}(z) \quad (8)$$
$$\mathbf{E}_{1_m}(z) = \Theta_{10,m}\mathbf{H}_{0_{m-1}}(z) + \Theta_{11,m}z^{-1}\mathbf{H}_{1_{m-1}}(z) \quad (9)$$

where the subscript k in $\mathbf{E}_{i_k}(z)$ represent the degree of the matrix filter is k.

2.1. Degree Reduction

The degree reduction requirement is satisfied by choosing

$$\begin{aligned} \Theta_{00,m} &= \Theta_{11,m} = \mathbf{I} && (10) \\ -\Theta_{01,m}^T &= \Theta_{10,m} = \mathbf{K}_m^T && (11) \\ \mathbf{K}_m &= \mathbf{H}_{1_m,0}\mathbf{H}_{0_m,0}^{-1} && (12) \end{aligned}$$

where $\mathbf{K}_m$ is the chain parameter. The structure in Figure 1a implies $\mathbf{P}_m$ can be written as

$$\mathbf{P}_m(z) = \Theta_m diag(\mathbf{I}_M, z^{-1}\mathbf{I}_M)\mathbf{P}_{m-1}(z) \quad (13)$$

where $\Theta_m = \begin{bmatrix} \Theta_{00,m} & \Theta_{01,m} \\ \Theta_{10,m} & \Theta_{11,m} \end{bmatrix}$. The reduced order FIR system $\mathbf{P}_{m-1}(z)$ is lossless bound real (LBR) if and only if $\tilde{\Theta}_m\Theta_m = \mathbf{I}$ and $\mathbf{P}_m(z)$ is LBR [5]. Thus a complete characterization of paraunitary system adds the following set of equations

$$\begin{aligned} \tilde{\Theta}_{00,m}\Theta_{00,m} + \tilde{\Theta}_{10,m}\Theta_{10,m} &= \mathbf{I} && (14) \\ \tilde{\Theta}_{11,m}\Theta_{11,m} + \tilde{\Theta}_{01,m}\Theta_{01,m} &= \mathbf{I} && (15) \\ \tilde{\Theta}_{00}^m\Theta_{01}^m + \tilde{\Theta}_{10}^m\Theta_{11}^m &= \mathbf{0} && (16) \end{aligned}$$

Using Cholesky factorizations, the lattice coefficients are modified as follows, so that FIR LBR constraint is satisfied;

$$\begin{aligned} \Theta_{00,m} &= \left[\mathbf{I} + \mathbf{K}_m^T\mathbf{K}_m\right]^{\frac{-T}{2}} && (17) \\ \Theta_{01,m} &= -\mathbf{K}_m^T\left[\mathbf{I} + \mathbf{K}_m\mathbf{K}_m^T\right]^{\frac{-T}{2}} && (18) \\ \Theta_{10,m} &= \mathbf{K}_m^T\left[\mathbf{I} + \mathbf{K}_m^T\mathbf{K}_m\right]^{\frac{-T}{2}} && (19) \\ \Theta_{11,m} &= \left[\mathbf{I} + \mathbf{K}_m\mathbf{K}_m^T\right]^{\frac{-T}{2}} && (20) \end{aligned}$$

2.2. Complete Factorization of $\mathbf{P}_D(z)$

Since the signal flow directions are consistant with the block description, causality is satisfied.

Theorem 1 *$\mathbf{P}_D(z)$ is a 2 band N channel block FIR lossless system with degree D, if and only if it can be factorized as*

$$\mathbf{P}_D(z) = \mathbf{C}\Theta_D\Lambda(z)\Theta_{D-1}\cdots\Theta_1\Lambda(z)\begin{bmatrix} \Theta_{00,0} \\ \Theta_{11,0} \end{bmatrix} \quad (21)$$

where $\mathbf{C} \in \mathbb{R}_{N\times N}$ is diagonal and invertible, $\Theta_i \in \mathbb{R}_{M\times M}$ is orthogonal and the matrix coefficients are given by eq.(17-20), $\Theta_{00,0}$ and $\Theta_{11,0}$ have the form of eq.(17) and eq.(20) respectively, and $\Lambda(z) = diag(\mathbf{I}_N, z^{-1}\mathbf{I}_N)$.

2.3. Minimality

Since $\mathbf{P}_D(z)$ can be implemented as eq.(21) which has $D\times N$ delays, we have

$$det\mathbf{P}_D(z) \le D \times N \quad (22)$$

From eq.(21), we also have $deg\ det\mathbf{P}_D(z) = D\times N$. But we know that $deg\mathbf{P}_D(z) \ge deg\ det\mathbf{P}_D(z)$ [5], so that

$$deg\mathbf{P}_D(z) \ge D \times N \quad (23)$$

From the above two inequalities we conclude $deg\mathbf{P}_D(z) = D\times N$. This proves that the structure is minimial.

2.4. Multifilters Implementation

Based on Theorem 1, a complete parameterization of the polyphase matrix of 2 band N channel FIR paraunitary multifilters is given by Figure 1b

$$\mathbf{E}(z) = \mathbf{C}\Theta_D\Lambda(z)\Theta_{D-1}\cdots\Theta_1\Lambda(z)\Theta_0 \quad (24)$$

where $\Theta_0 = \begin{bmatrix} \Theta_{00,0} & \Theta_{01,0} \\ \Theta_{10,0} & \Theta_{11,0} \end{bmatrix}$, is orthogonal.

3. M BAND N CHANNEL BLOCK FIR LOSSLESS SYSTEMS

Suppose $\mathbf{E}_0(z), \ldots, \mathbf{E}_{M-1}(z)$ form a lossless System

$$\mathbf{P}_D(z) = \begin{bmatrix} \mathbf{E}_0(z) \\ \vdots \\ \mathbf{E}_{M-1}(z) \end{bmatrix} \quad (25)$$

If $\mathbf{E}_i(z)$ are given by eq.(1), $\mathbf{P}_N(z)$ is causal and FIR. Our aim is to find a structure for $\mathbf{P}_N(z)$, similar to [5, Chapter 14.4]. Apply the Schur algorithm, the structure in Figure 2a is considered. The transfer function of the lattice block is given by

$$\mathbf{P}_m(z) = \Theta_m diag(\mathbf{I}_{N(M-1)}, z^{-1}\mathbf{I}_N)\mathbf{P}_{m-1}(z) \quad (26)$$

3.1. Degree Reduction

The paraunitary matrix $\mathbf{P}_m(z)$ can be written as

$$\mathbf{P}_m(z) = \mathbf{P}_{m,0} + \cdots + \mathbf{P}_{m,m}z^{-m} \quad (27)$$

where $\mathbf{P}_{m,i} \in \mathbb{R}_{MN\times N}$ and the degree m condition is equivalent to $\mathbf{P}_{m,m} \neq \mathbf{0}$. The paraunitary condition imples that [5, Chapter 14.2]

$$\tilde{\mathbf{P}}_{m,m}\mathbf{P}_{m,0} = \mathbf{0} \quad (28)$$

Consider $\mathbf{P}_{m-1}(z)$

$$\mathbf{P}_{m-1}(z) \equiv \underbrace{(\mathbf{I} - \mathbf{v}_m\tilde{\mathbf{v}}_m + z\mathbf{v}_m\tilde{\mathbf{v}}_m)}_{\tilde{\mathbf{V}}_m(z)}\mathbf{P}_m(z) \quad (29)$$

Choose $\tilde{\mathbf{v}}_m = \mathbf{P}_{m,m}/\|\mathbf{P}_{m,m}\|$, thus the noncausal term in eq.(29) becomes

$$z\mathbf{v}_m\tilde{\mathbf{v}}_m\mathbf{P}_{m,0} = \mathbf{0} \quad (30)$$

Furthermore the coefficients of z^{-m} in $\mathbf{P}_{m-1}(z)$ is given by

$$\begin{aligned} (\mathbf{I} - \mathbf{v}_m\tilde{\mathbf{v}}_m)\,\mathbf{P}_{m,m} &= \|\mathbf{P}_{m,m}\|\,(\mathbf{I} - \mathbf{v}_m\tilde{\mathbf{v}}_m)\,\mathbf{v}_m \\ &= \|\mathbf{P}_{m,m}\|\,(\mathbf{v}_m - \mathbf{v}_m) = \mathbf{0} \quad (31) \end{aligned}$$

where we assume $\tilde{\mathbf{v}}_m\mathbf{v}_m = 1$. So $\mathbf{P}_{m-1}(z)$ is causal and FIR with degree $< m$. Moreover

$$\mathbf{P}_m(z) = \mathbf{V}_m(z)\mathbf{P}_{m-1}(z) \quad (32)$$

Since $deg(\mathbf{V}_m(z)) = 1$, hence $\mathbf{P}_{m-1}(z)$ cannot have degree smaller than $m-1$. Thus the degree of $\mathbf{P}_{m-1}(z)$ is precisely $m-1$. A similar prove as Section 2.3 will show the following parameterization of $\mathbf{V}(z)$ is minimal.

3.2. Size Reduction

Follow the construction of $\mathbf{V}(z)$ in [4],

$$\mathbf{V}(z) = diag(\mathbf{I}_N(M-1), z^{-1}\mathbf{I}_N)\mathbf{U} \tag{33}$$

where $\mathbf{U} \in \mathbb{R}_{NM \times NM}$ and $\mathbf{U}^T\mathbf{U} = \mathbf{I}$. Since $\mathbf{U}$ is paraunitary, it can be written as

$$\mathbf{U} = \Theta \begin{bmatrix} \mathbf{S} & \mathbf{0} \\ \mathbf{0} & \mu \end{bmatrix} \tag{34}$$

where $\mathbf{S} \in \mathbb{R}_{N(M-1) \times N(M-1)}$ and $\mathbf{S}^T\mathbf{S} = \mathbf{I}$, $\mu = \pm\mathbf{I}$, and $\Theta = \Theta_{N(M-2)}\Theta_{N(M-3)} \cdots \Theta_0$ where Θ_m has the form in Figure 2 and $\mathbf{K}_m$ has the same form as that in the factorization of 2 band N channel paraunitary multifilters. Thus, $\Theta_m^T\Theta_m = \mathbf{I}$, and $\Theta^T\Theta = \mathbf{I}$.

The paraunitary matrix $\mathbf{U} \in \mathbb{R}_{NM \times NM}$ has been reduced in size to $\mathbf{S} \in \mathbb{R}_{N(M-1) \times N(M-1)}$. The complete factorization of $\mathbf{U}$ is obtained by repeated size reduction. Figure 2b shows the details of the building block.

3.3. Complete Factorization of $\mathbf{P}_D(z)$

The M band N channel FIR lossless system can be factorized by repeated degree reductions. The resulting system is causal because the signal flow directions are consistant with the block description.

Theorem 2 *Let $\mathbf{P}_D(z)$ be a M band N channel block FIR lossless system with degree D. Then it can be factorized as*

$$\mathbf{P}_D(z) = \mathbf{V}_D(z)\mathbf{V}_{D-1}(z)\cdots\mathbf{V}_1(z)\mathbf{U}_0 \begin{bmatrix} \mathbf{I} & \mathbf{0} & \cdots & \mathbf{0} \end{bmatrix}^T, \tag{35}$$

where $\mathbf{V}_i(z)$ has the form given by eq.(33) and $\mathbf{U}_0 \in \mathbb{R}_{NM \times N}$ satisfies $\mathbf{U}_0^T\mathbf{U}_0 = \mathbf{I}$ thus can be implemented by the size reduction technique in Section 3.2.

3.4. Multifilters Implementation

Base on Theorem 2, a complete parameterization of the polyphase matrix of M band N channel paraunitary FIR multifilters is given by Figure 2c

$$\mathbf{E}(z) = \mathbf{V}_D(z)\mathbf{V}_{D-1}(z)\cdots\mathbf{V}_1(z)\mathbf{U}_0 \tag{36}$$

4. 2 BAND N CHANNEL BLOCK FIR BIORTHOGONAL SYSTEM

Suppose $\mathbf{E}_0(z)$, $\mathbf{E}_1(z)$, $\mathbf{G}_0(z)$ and $\mathbf{G}_1(z)$ form a biorthogonal system

$$\mathbf{P}_D(z) = \begin{bmatrix} \mathbf{E}_0(z) \\ \mathbf{E}_1(z) \end{bmatrix}, \tag{37}$$

$$\mathbf{R}_D(z) = \begin{bmatrix} \mathbf{G}_0(z) & \mathbf{G}_1(z) \end{bmatrix}, \tag{38}$$

$$\mathbf{R}_D(z)\mathbf{P}_D(z) = \begin{bmatrix} \mathbf{G}_0(z) & \mathbf{G}_1(z) \end{bmatrix} \begin{bmatrix} \mathbf{E}_0(z) \\ \mathbf{E}_1(z) \end{bmatrix}, \tag{39}$$

$$c^2\mathbf{I} = \mathbf{G}_0(z)\mathbf{E}_0(z) + \mathbf{G}_1(z)\mathbf{E}_1(z). \tag{40}$$

Our aim is to find a structure for $\mathbf{P}_D(z)$ and $\mathbf{R}_D(z)$ similar to [5, Chapter 7.2]. Using Schur algorithm, consider the structure in Figure 3a. Let $\mathbf{E}_{0_{m-1}}(z)$ and $\mathbf{E}_{1_{m-1}}(z)$ be

$$\mathbf{E}_{0_{m-1}}(z) = \mathbf{E}_{0_{m-1},0} + \mathbf{E}_{0_{m-1},1}z^{-1} + \cdots + \mathbf{E}_{0_{m-1},m-1}z^{-m+1} \tag{41}$$

$$\mathbf{E}_{1_{m-1}}(z) = \mathbf{E}_{1_{m-1},0} + \mathbf{E}_{1_{m-1},1}z^{-1} + \cdots + \mathbf{E}_{1_{m-1},m-1}z^{-m+1} \tag{42}$$

where $\mathbf{E}_{i_{m-1},j} \in \mathbb{R}_{2N \times N}$ and the degree $m-1$ condition is equivalent to $\mathbf{E}_{i_{m-1},m-1} \neq 0$. The transfer functions are

$$\mathbf{E}_{0_m}(z) = \mathbf{E}_{0_{m-1}}(z) + \mathbf{K}_m\mathbf{E}_{1_{m-1}}z^{-1} \tag{43}$$

$$\mathbf{E}_{1_m}(z) = \mathbf{K}_m\mathbf{E}_{0_{m-1}}(z) + \mathbf{E}_{1_{m-1}}z^{-1} \tag{44}$$

Thus the degree of $\mathbf{E}_{0_m}(z)$ and $\mathbf{E}_{1_m}(z)$ equal to m. If $\mathbf{E}_{1_{m-1}}(z) = z^{-m+1}\mathbf{E}_{0_{m-1}}(z^{-1})$ then $\mathbf{E}_{1_m}(z) = z^{-m}\mathbf{E}_{0_m}(z^{-1})$.

4.1. Complete Factorization of $\mathbf{P}_D(z)$ and $\mathbf{R}_D(z)$

Since the signal flow directions are consistant with the block description, causality is satisfied.

Theorem 3 *Let $\mathbf{P}_D(z) = \begin{bmatrix} \mathbf{E}_0(z) \\ \mathbf{E}_1(z) \end{bmatrix}$ be a 2 band N channel block FIR system with degree D such that $\mathbf{E}_1(z) = z^{-D}\mathbf{E}_0(z)$. Then it can be factorized as*

$$\mathbf{P}_D(z) = \mathbf{T}_D\Lambda(z)\mathbf{T}_{D-1}\cdots\mathbf{T}_1\Lambda\mathbf{P}_0 \tag{45}$$

where $\mathbf{T}_i = \begin{bmatrix} \mathbf{I} & \mathbf{K}_i \\ \mathbf{K}_i & \mathbf{I} \end{bmatrix}$, and $\mathbf{P}_0 \in \mathbb{R}_{2N \times N}$.

$\mathbf{R}_D(z)$ is chosen to be the inverse matrix of $\mathbf{P}_D$.

$$\mathbf{R}_D(z) = \mathbf{P}_0^{-1}\,\Gamma(z)\mathbf{S}_1\cdots\mathbf{S}_{D-1}\,\Gamma(z)\mathbf{S}_D \tag{46}$$

where $\mathbf{S}_i = \begin{bmatrix} \mathbf{I} & -\mathbf{K}_i \\ -\mathbf{K}_i & \mathbf{I} \end{bmatrix}$, and $\Gamma(z) = diag(z^{-1}\mathbf{I}_N, \mathbf{I}_N)$.

4.2. Implementation

The analysis filters shown in Figure 3b is given by

$$\mathbf{H}_0(z) = \mathbf{S}\mathbf{E}_{0_D}(z^2) + z^{-1}\mathbf{S}\mathbf{E}_{1_D}(z^2) \tag{47}$$

$$\mathbf{H}_1(z) = \mathbf{S}\mathbf{E}_{0_D}(z^2) - z^{-1}\mathbf{S}\mathbf{E}_{1_D}(z^2) \tag{48}$$

where $\mathbf{S}$ is the permutation matrix with ± 1 as element. Each row of the $\mathbf{H}_0(z)$ and $\mathbf{H}_1(z)$ is symmetry and antisymmetry around z^{-D}. Thus the multifilters is linearphase.

5. M BAND N CHANNEL BLOCK FIR BIORTHOGONAL SYSTEM

Suppose $\mathbf{E}_0(z), \ldots, \mathbf{E}_{M-1}(z)$ and $\mathbf{G}_0, \cdots, \mathbf{G}_{M-1}$ form an invertible system

$$\mathbf{P}_D(z) = \begin{bmatrix} \mathbf{E}_0(z) \\ \vdots \\ \mathbf{E}_{M-1}(z) \end{bmatrix}, \tag{49}$$

$$\mathbf{R}_D(z) = \begin{bmatrix} \mathbf{G}_0(z) & \cdots & \mathbf{G}_{M-1}(z) \end{bmatrix}, \tag{50}$$

$$\mathbf{R}_D(z)\mathbf{P}_D(z) = c^2\mathbf{I}. \tag{51}$$

5.1. Size Reduction

Construct $\mathbf{V}(z)$ as

$$\mathbf{V}(z) = diag(\mathbf{I}_N(M-1, z^{-1}\mathbf{I}_N)\mathbf{U} \tag{52}$$

where $\mathbf{U} \in \mathbb{R}_{NM\times NM}$ is invertible. Let $\mathcal{G}$ be the group of invertible matrices satisfy

$$\forall \mathbf{T} \in \mathcal{G} \qquad \mathbf{T} = \Upsilon \begin{bmatrix} \mu & \mathbf{0} \\ \mathbf{0} & \mathbf{S} \end{bmatrix} \tag{53}$$

where $\mathbf{T} \in \mathbb{R}_{Nk\times Nk}$, $\mathbf{S} \in \mathcal{G}$ and $\mathbf{S} \in \mathbb{R}_{N(k-1)\times N(k-1)}$, $\mu = \pm\mathbf{I}$ and $\Upsilon = \Upsilon_{N(k-2)}\Upsilon_{N(k-3)}\cdots\Upsilon_0$ where $\Upsilon_m = \begin{bmatrix} \mathbf{I} & \mathbf{K}_m \\ \mathbf{K}_m & \mathbf{I} \end{bmatrix}$ as shown in Figure 4. Following the results in Section 3.2, any matrix $\mathbf{U} \in \mathcal{G}$ can be reduced in size by eq.(53) to $\mathbf{S} \in \mathcal{G}$. A complete parameterization of $\mathbf{U}$ can be obtained by repeated size reduction. Figure 4 shows the details of the building block for this class of biorthogonal multifilters. The inverse of $\mathbf{U}$ is given by

$$\begin{aligned} \mathbf{U}^{-1} &= \begin{bmatrix} \mu & \mathbf{0} \\ \mathbf{0} & \mathbf{S}^{-1} \end{bmatrix} \Upsilon^{-1}, & (54) \\ \Upsilon^{-1} &= \Upsilon_0^{-1}\cdots\Upsilon_{N(M-3)}^{-1}\Upsilon_{N(M-2)}^{-1}, \\ \Upsilon_m^{-1} &= \begin{bmatrix} \mathbf{I} & -\mathbf{K}_m \\ -\mathbf{K}_m & \mathbf{I} \end{bmatrix}. & (55) \end{aligned}$$

5.2. Synthesis of $\mathbf{P}_D(z)$

Similar to the orthogonal M band N channel FIR lossless system, an M band N channel FIR biorthogonal system with degree D can be synthesized by Theorem 4.

Theorem 4 *Any degree D M band N channel block FIR biorthogonal system $\mathbf{P}_D(z)$ can be synthesized by*

$$\mathbf{P}_D(z) = \mathbf{V}_D(z)\mathbf{V}_{D-1}(z)\cdots\mathbf{V}_1(z)\mathbf{P}_0 \begin{bmatrix} I & \mathbf{0} & \cdots & \mathbf{0} \end{bmatrix}^T \tag{56}$$

where $\mathbf{V}_i(z)$ has the form as eq.(52) and $\mathbf{P}_0 \in \mathcal{G}$.

5.3. Implementation

Base on Theorem 4, a complete parameterization of the polyphase matrix of M band N channel biorthogonal FIR multifilters is given by Figure 2c.

$$\mathbf{E}(z) = \mathbf{V}_D(z)\mathbf{V}_{D-1}(z)\cdots\mathbf{V}_1(z)\mathbf{P}_0 \tag{57}$$

6. DISCUSSIONS AND CONCLUSIONS

A complete parameterization of paraunitary multifilters by block lattice structure is derived from an iterative Schur algorithm. The design of multifilters can be formulated as unconstrained optimization problems of the appropriate lattice (chain) parameters and objective functions. Similarly, a class of biorthogonal multifilters are parametrized by block lattice structure derived from an iterative Schur algorithm. The lattice structure for 2 band N channel multifilters completely parametrize the class of multifilters that have linear phase subband filters. A class of M band N channel multifilters are parameterized by cascade of the lattice block in 2 band N channel case. The presented design methods have been implemented in Matlab and Matlab optimization toolbox. The convergence rate of the algorithm is fast, and high performance multifilters are obtained. However, design example is not presented due to limited space. Furture research should be directed to the structure of linear phase paraunitary multifilters and nonlinear phase biorthogonal multifilters.

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