TRIPULSE: AN ACCURATE ORIENTATION/ATTITUDE ESTIMATION SYSTEM FOR SATELLITE BORNE PHASED ARRAYS

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ABSTRACT

Tripulse is a novel orientation/attitude estimation system that is designed to accurately estimate the orientation of a satellite borne phased array relative to one or more earth stations. This system has an accuracy potential that is significantly better than conventional Earth-Moon-Sun attitude sensors. Tripulse has conceptual similarities to amplitude comparison monopulse systems used in tracking radars. Detailed Tripulse statistical performance analyses for noise, beamforming quantization errors, and hardware failures are presented.

I. INTRODUCTION

This work describes a novel orientation estimation system called *Tripulse* that is designed to accurately estimate the orientation and attitude of a satellite borne phased array relative to one or more earth stations.

An active phased array antenna communication satellite system can require a higher degree of pointing accuracy than that which can be obtained using Earth-Moon and Sun sensors. Accurate attitude determination is an important factor in issues of delivering maximum signal power (equivalent isotropic radiated power – EIRP) to localized areas, and also to allow for maximum frequency reuse with minimal interference. Earth-Moon-Sun sensor systems typically provide an attitude accuracy of ~ 0.1 deg . This work demonstrates that a Tripulse system can maintain a specification accuracy of ~ 0.01 deg. over normal 15 year lifetimes of a communications satellite.

II. BACKGROUND

The Tripulse system is conceptually similar to amplitude comparison monopulse systems used in tracking radars, see for example, Dunn *et al* [1]. Our array model consists of individual active elements that can be assigned a distinct amplitude and phase. In general, the array geometry is quite flexible. In this work we consider a square array antenna with N^2 elements. The origin of the coordinate system is taken at the center of the array. The x, y axes lie in the plane of the array passing through the center of symmetry and are directed along the rows and columns of the array. The z axis is perpendicular to plane of the array. The array elements are indexed by numbers (m, n). For a right-handed coordinate system place the $(1,1)$ element in the upper left hand corner of the array. The array is divided into four clockwise arranged quadrants, $Q_1, \ldots, Q_4$, starting with the upper lefthand quadrant.

The ground receiving antenna has spherical polar coordinates, R, θ, ϕ. Here θ and ϕ are respectively the polar and azimuthal angles. The elevation angle is equal to $\pi/2 - \theta$.

The projections of the vector $\vec{R}$ onto the cartesian axes are, $T_x \stackrel{\text{def}}{=} \sin\theta \cos\phi$, $T_y \stackrel{\text{def}}{=} \sin\theta \sin\phi$, and $T_z \stackrel{\text{def}}{=} \cos\theta$. The originally estimated angular coordinates of the target, $\theta_0\phi_0$, are represented by initial cartesian projections, T_{x0}, T_{y0}, T_{z0}.

The coherent signals from the m, n array elements that are received at the ground station are of the form,

$$s(m,n) \propto \frac{1}{R} e^{jkR} e^{jkd(c(m)T_x + c(n)T_y)}. \qquad (1)$$

The wave-number $k \stackrel{\text{def}}{=} 2\pi/\lambda$; d's is the array element spacing; and $c(m) = m - (N+1)/2$; for $m = 1, 2, \ldots, N$,

Let $\{S_{Q_i}\}$ represent the sum of all the individual coherent transmitted elemental signals in each of the quadrants that are steered to the initial estimate of the receiver orientation. The sum and delta beams in the Tripulse/monopulse methods are defined by

$$\begin{aligned} S_\Sigma &= S_{Q_1} + S_{Q_2} + S_{Q_3} + S_{Q_4}, \\ S_{\Delta x} &= S_{Q_1} - S_{Q_2} - S_{Q_3} + S_{Q_4}, \\ S_{\Delta y} &= S_{Q_1} + S_{Q_2} - S_{Q_3} - S_{Q_4}. \end{aligned} \qquad (2)$$

Defining $X \stackrel{\text{def}}{=} kd(T_x - T_{x0})$, and $Y \stackrel{\text{def}}{=} kd(T_y - T_{y0})$, the analytical expressions for these signals are:

$$\begin{aligned} S_\Sigma &= K \frac{\sin(XN/2)}{\sin(X/2)} \frac{\sin(YN/2)}{\sin(Y/2)}, \\ S_{\Delta_x} &= j2K \frac{\sin^2(XN/4)}{\sin(X/2)} \frac{\sin(YN/2)}{\sin(Y/2)}, \\ S_{\Delta_y} &= j2K \frac{\sin(XN/2)}{\sin(X/2)} \frac{\sin^2(YN/4)}{\sin(Y/2)}. \end{aligned} \qquad (3)$$

The notation has been simplified by grouping the constants multiplying all the signals into a single generic complex coefficient K. From these results we see that the sum and delta signals differ in phase by $\pi/2$. The imaginary parts of the ratio of the delta to the sum beams are usually referred to as the *Monopulse ratios*,

$$\mathcal{R}_x \stackrel{\mathrm{def}}{=} \Im m\left[\frac{S_{\Delta x}}{S_\Sigma}\right] = 2\frac{\sin^2(XN/4)}{\sin(XN/2)}, \tag{4}$$

$$\mathcal{R}_y \stackrel{\mathrm{def}}{=} \Im m\left[\frac{S_{\Delta y}}{S_\Sigma}\right] = 2\frac{\sin^2(YN/4)}{\sin(YN/2)}.$$

The process that is used to estimate the orientation errors is as follows:

1. Measure the ratios, $\hat{\mathcal{R}}_x, \hat{\mathcal{R}}_y$.
2. With the measured values of $\hat{\mathcal{R}}_x, \hat{\mathcal{R}}_y$ solve the transcendental equations Eqs. (4) for $\hat{X}N/4, \hat{Y}N/4$ for the errors,

$$\hat{\epsilon}_x \stackrel{\mathrm{def}}{=} \hat{T}_x - T_{x0}, \quad \hat{\epsilon}_y \stackrel{\mathrm{def}}{=} \hat{T}_y - T_{y0},$$

The estimated angles can be computed using,

$$\hat{\theta} = \sin^{-1}\sqrt{\hat{T}_x^2 + \hat{T}_y^2}; \quad \hat{\phi} = \tan^{-1}\left(\frac{\hat{T}_y}{\hat{T}_x}\right).$$

The sum beam maximum coincides will the initial orientation estimate of the ground station receiver. The initial estimates of the ground station's orientation angles can be obtained using conventional attitude measurement systems [2]. The Tripulse algorithm finds the position of the sum beam maximum relative to the true orientation of the ground station receiver. This is significant as it is well known from spectral estimation theory that the maximum likelihood estimate (MLE) for a *single complex sinusoid* in complex Gaussian white noise corresponds to the maximum of the periodogram [3]. It is also well know that 1-D direction of arrival estimation (DOA) for point sources in the far field of a line phased array is *completely equivalent* to temporal spectral estimation. The form of the periodogram for temporal spectral estimation and the power spectrum for the sum beam of a linear array are *identical*. Accordingly, the maximum of sum beam power spectrum for a uniform linear array will correspond to the MLE for the DOA estimate of a single point source .

From an obvious extension of this discussion to 2-D systems, it is evident that the Tripulse methods generate the MLE for the ground station spatial frequencies.

III. TRIPULSE SYSTEM

1. The Tripulse system must transmit three different time multiplexed coherent pulses corresponding to the sum and two delta beams. For each pulse the satellite borne transmitting antenna is electronically steered to the initial orientation angle estimate of the ground station. The initial orientation angle estimates are obtained either using separate Earth-Moon-Sun sensors, or the estimates from a previous Tripulse estimation sequence.

2. The receiving antenna (either a phased array or a dish) is steered to maximize the received signals.

3. Coherent detection of the Tripulse signals are performed at the receiver with one or more reference signals that are phased-locked to the input Tripulse tone at the transmitter. The coherent detection system architecture must be specifically designed to compensate for Doppler phase shifts due to satellite motion and independent phase noise effects due to non-synchronized clocks on the satellite and the ground receiving station.

IV. TRIPULSE PERFORMANCE ANALYSIS

4.1 Statistical noise analysis

The estimates of the orientation errors, $\hat{\epsilon}_x$, $\hat{\epsilon}_y$, are obtained by solving the transcendental equations generated by the functional form of the Δ/Σ ratios given by Eq. (4). For small errors in the initial orientation estimates, these functions can be approximated by their leading order expansion terms,

$$\mathcal{R}_x \cong XN/4; \quad \mathcal{R}_y \cong YN/4. \tag{5}$$

For a typical Ku band system, $N^2 = 256$, $d = 3\lambda$. The initial Earth-Moon-Sun attitude estimates $T_x - T_{0x}$ and $T_y - T_{y0}$ are $\sim 0.1\pi/180$. The higher order terms in the expansions illustrated in Eq. (5) are only $\sim 2\%$ of the first order linear terms. Using the linear term, we calculate the statistics of the projection errors in terms of the corresponding noise statistics of the Δ_x/Σ ratios,

$$E\{\epsilon_x\} \cong \frac{4}{kdN} E\Big\{\mathcal{R}_x\Big\},$$

$$\mathbf{var}\{\epsilon_x\} \cong \left(\frac{4}{kdN}\right)^2 \mathbf{var}\{\mathcal{R}_x\}. \tag{6}$$

The demodulated signals have been low-passed filtered and have a base-band bandwidth W. The signals are subsequently sampled at the frequency $W = 1/T$, where T is the sampling period. In the moderate SNR regime the statistics the noise can be approximated by the statistics of complex AWGN with a power spectral density, $\mathcal{N}_0$ [4], The variance of the noise samples has an implicit bandwidth dependence, $\sigma^2 \equiv \mathcal{N}_0 W$. The expansion of the variance in a power series of the ratio of the sampled noise variance to the square magnitude of the sum signals gives,

$$\mathbf{var}\left(\hat{\mathcal{R}}_x\right) = \left(1 + |S_{\Delta x}/S_\Sigma|^2\right)\frac{\sigma^2}{2|S_\Sigma|^2} + \mathcal{O}\left(\frac{\sigma^4}{|S_\Sigma|^4}\right). \tag{7}$$

For practicable systems, $|S_\Sigma|^2 >> \sigma^2$, and $|S_{\Delta x}/S_\Sigma|^2 << 1$, therefore

$$\mathbf{var}\{\epsilon_x\} \cong \left(\frac{\lambda}{\pi d_x}\right)^2 \frac{2}{N^6}\frac{\mathcal{N}_0 W}{|K|^2}. \tag{8}$$

Here $|K|^2/(\mathcal{N}_0 W)$ is the link budget value of the SNR at the receiver of the signal transmitted from *a single element* of the phased array. There is *no* array gain factor in this single element SNR.

The Tripulse system operates near broadside corresponding to small polar angles. As $\mathbf{var}(\epsilon_x) = \mathbf{var}(\epsilon_y)$, the polar azimuthal angles are related by,

$$\mathbf{var}(\theta) \sim 2\mathbf{var}(\epsilon_x); \;\; \mathbf{var}(\phi) \cong \frac{\mathbf{var}(\epsilon_x)}{\sin^2\theta}. \tag{9}$$

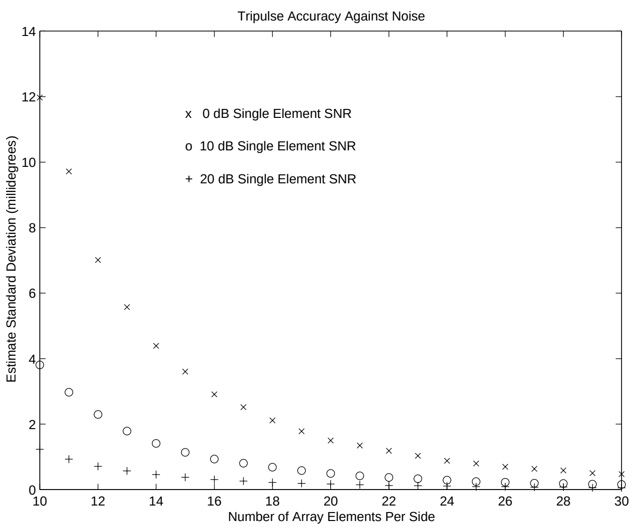

Figure 1: Tripulse robustness to noise

In Fig. 1. we show the *Monte-Carlo* simulation results for the standard deviation of the polar angle estimates for different single element SNR levels as a function of the number of elements in the phased array. The initial angular estimate was taken at a fixed error of 0.1 degrees. The simulation results are in accord with the theoretical results of Eq. (8,9).

4.2 Tripulse measurement time estimates

The required measurement times for large order arrays can be very short because measurement SNR's are enhanced significantly by the large antenna gain in the sum beam. From Eq. (8), with $T_{\text{int}} \equiv 1/W$, the integration time necessary to satisfy a specification $\delta_{\text{spec}} \equiv \mathbf{std}(\epsilon_x) \equiv \mathbf{std}(\epsilon_y)$ on the orientation-projection error as a function of the single element link budget value, $|K|^2/\mathcal{N}_0$, is

$$T_{\text{int}} \geq \frac{1}{\delta_{\text{spec}}^2}\frac{2}{N^6}\left(\frac{\lambda}{\pi d_x}\right)^2 \frac{\mathcal{N}_0}{|K|^2}. \tag{10}$$

For a Ku band GEO satellite, assume: a single element downlink budget value, $|K|^2/\mathcal{N}_0 \sim 37$dBHz; a square array with $N^2 = 256$; $d = 3\lambda$; and an orientation-projection specification $\delta_{\text{spec}} = 3.7 \times 10^{-5}$, corresponding to a *3 sigma* (estimate within 3 standard deviations of correct value) angular specification of .01 degrees. Substituting these values into Eq. (10), we calculate a required measurement time of $\sim 160\mu$sec.

In some cases the interference spectral density will be dominated by intermods rather than receiver noise. The intermodulation spectrum can also be modeled as being uniform (white) over the receiver bandwidth. For these cases the analysis proceeds the same as above, where the noise spectral density $\mathcal{N}_0$ is replaced by the effective intermodulation spectral density, $\mathcal{I}_0$. Here again, 37 dBHz is a representative number for the downlink budget, and the required integration time will still be $\sim 160\mu$sec.

4.3 Sensitivity to quantization errors

Figure 2. illustrates the sensitivity of the Tripulse estimate to random phase quantization errors in the beamformer phase states. Simulations were performed using phase errors that were uniformly distributed over a width equal to the quantization level associated with the highest bit state. For example, a five bit quantizer will have a uniformly distributed phase error of $\pm 1/2(2\pi/2^5)$. As we see, nominal Tripulse 3 sigma orientation error specifications of ≤ 0.01 deg . are met for array orders exceeding ~ 320 elements for a five bit quantizer.

4.4 Sensitivity to faulty elements

We now estimate the effects of element failure on the accuracy of the orientation estimation. For a given percentage of "dead" elements, we performed a Tripulse orientation estimate for each random arrangement of the dead elements throughout the array. The estimation error statistics are based upon 20K trials (random arrangements) for each indicated number of failed elements. The results illustrated in Fig. 3 indicate that in the absence of receiver noise that 3 sigma specifications of ~ 0.01 deg . can be maintained for up to $\sim 15\%$ element failures. Satellites will be designed so that less than 10% of the elements will have failed by the end of the life cycle.

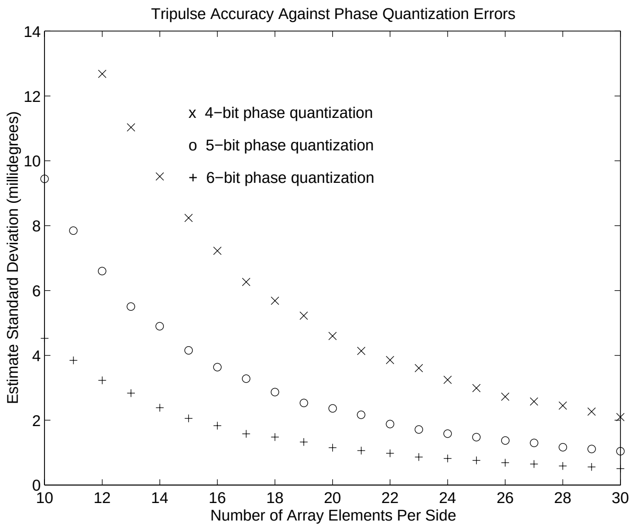

Figure 2: Tripulse robustness to phase quantization errors

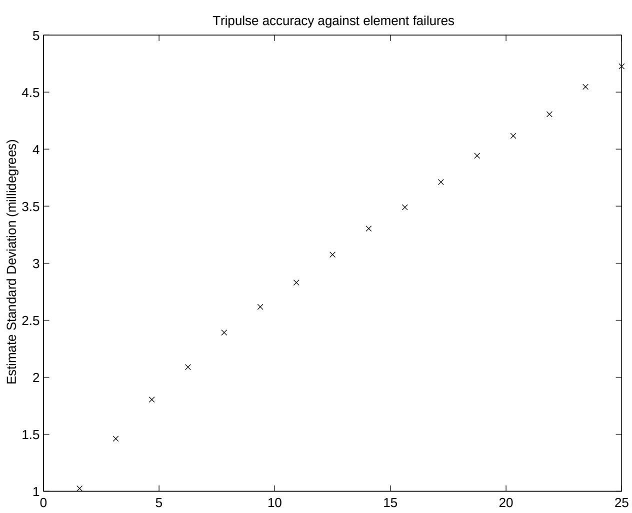

Figure 3: Tripulse robustness to failed elements

V. ATTITUDE ESTIMATION

The goal of three-axis attitude estimation is to find the physical rotation the array coordinate system has undergone relative to some fixed reference coordinate system, assuming that the position of the array is known. The direction to a remote receiver in the reference coordinate system is known *a priori*. The Tripulse technique measures the direction to a remote receiver relative to the array coordinate system. The attitude is determined by finding the transformation that registers these direction vectors.

In order to solve for the attitude of the phased array, direction measurements to at least two remote receivers are necessary, and more can be used to increase accuracy and robustness.

Let $l = 1, \ldots, L$ index the remote receivers. The direction to remote receiver l is represented by the unit vector $\mathbf{t}'_l$ in the reference coordinate system and by $\mathbf{t}_l = (T_{l,x}, T_{l,y}, T_{l,z})$ in the array coordinate system. The Tripulse procedure estimates $T_{l,x}$ and $T_{l,y}$ and we calculate $T_{l,z} = \sqrt{1 - T_{l,x}^2 - T_{l,y}^2}$. The direction vectors are related by the rotation matrix $\mathbf{R}$,

$$\mathbf{t}_l = \mathbf{R}\,\mathbf{t}'_l. \tag{11}$$

Using this relation and the least-squares procedure of [5], Tripulse measurements are used to solve for the unknown attitude of the array.

VI. REFERENCES

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