RESOLUTION ENHANCEMENT BY POLYPHASE BACK-PROJECTION FILTERING

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ABSTRACT

The method for reconstruction and restoration of super resolution images from low resolution sequences presented here is an extension of Irani and Peleg's algorithm ("Improving Resolution by Image Registration", *CVGIP: Graphical Models and Image Processing*, Vol. 53, No. 3, pp. 231-239, 1991). The input is a set of low resolution images that have been registered to a pixel translation accuracy. A high resolution image is initialized and iteratively improved by back-projecting the errors between the low resolution images and the respective images obtained by simulating the imaging system. The sub-pixel translations between the low resolution images are quantized. The imaging system's PSF and back projection function are estimated with a resolution higher than that of the super resolution image and decimated so that two banks of polyphase filters are obtained. The use of the polyphase filters allows exploitation of all the input images without any smoothing or interpolation operations.

1. INTRODUCTION

The issue of a super resolution image reconstruction from a sequence of low resolution images was first introduced by Tsai and Huang [16] in 1984. Assuming that the high resolution image is band limited, and the low resolution images do not have any noise or degradation, they addressed the interpolation issue by a frequency domain observation model, considering globally shifted versions of the same scene. The algorithms presented by Tom and Katsaggelos [14,15] have also been implemented in the frequency domain, using a ML approach with the EM method.

Ur and Gross [17] improved resolution by overcoming under-sampling with a merging process, based on the framework of the Papoulis-Brown multi-channel sampling theorem. Using controlled sub-pixel displacements, Jacquemod *et al*. [8] created an over-sampled image and deconvolved it.

Irani and Peleg [6] proposed an iterative approach similar to back-projection used in tomography, by assuming that every low resolution pixel is a "projection" of a region in the high resolution image. Later, they extended their method to improve the resolution of differently moving objects [7]. Irani and Peleg's later work was extended by Bascle *et al*. [3], who have also considered motion blur.

Stark and Oskoui [11] computed a high resolution image using projections into convex sets (POCS), taking into account blur caused by sensor geometry. Tekalp *et al*. [13] extended the POCS method by incorporating the observation noise into the problem formulation.

Using a Bayesian method for high resolution image reconstruction, Cheesman *et al.* [4] formed a likelihood function, defined as the probability of the observed data given a model of how the data was generated. Schultz and Stevenson [10] have placed the multi-frame interpolation problem into a Bayesian framework, regulating the ill-posed interpolation problem with MAP estimation.

Su and Kim [12] presented a generalized super-resolution problem where the purpose is the restoration of continuos image sequence with improved resolution. They used local sampling lattice shift estimations and a DFT based high resolution image reconstruction algorithm to perform the restoration and the resolution improvement. Eldad and Feuer [5] have shown that using constrained least squares, the problem of restoration and improving the resolution of a continuos image sequence can be reduced to several recursive equations propagating in time. Addressing the same goal, Avrin and Dinstein [2] compensated local small displacements between consecutive frames using adaptive local filters, and generated each super resolution frame from the previous super resolution frame and the back projected filtered sub-sequence of low resolution input frames.

2. RESOLUTION ENHANCEMENT USING 'BACK-PROJECTIONS'

Our algorithm is an extension of the method by Irani and Peleg [6,7], a method that is based on the assumption that every low resolution pixel is a 'projection' of its receptive field in the high-resolution image. Using Irani and Peleg's notations, let f be the original high-resolution image, let the observed degraded image sequence be $\{g_k\}$ (k representing the transform parameters), the blurring function be h, the additive noise be η_k, the 2-D geometric transformation (between f and g_k) be T_k, and the down-sampling operator be σ_k . Using this notation, the imaging model is represented by:

$$g_k(m,n)=\sigma_k\{h[T_k(f(x,y))]+\eta_k(x,y)\} \tag{1}$$

The super-resolution algorithm is iterative. It starts with an initial guess $f^{(0)}$ for the high-resolution image. The imaging process is simulated to obtain a set of low-resolution images $\{g_k^{(0)}\}$. If $f^{(0)}$ is the exact original image and the imaging simulation is perfect, then the simulated images $\{g_k^{(0)}\}$ should be identical to the observed images $\{g_k\}$. The initial guess is

improved by 'back-projecting' each pixel value in the difference images $\{g_k - g_k^{(0)}\}$onto its receptive field in $f^{(0)}$. This process is iterated to minimize a specific error function.
Denoting the blurring function PSF by h^{PSF}, the imaging process of g_k at the n_{th} iteration is represented by:

$$g_k^{(n)}=[T_k(f^{(n)})*h^{PSF}]\downarrow S \quad (2)$$

where $\downarrow S$ denotes a down-sampling operator by factor S, and $*$ is the convolution operator. The iterative update scheme of the high resolution image is expressed by:

$$f^{(n+1)} = f^{(n)} + \frac{1}{K}\sum_{k=1}^{K} T_k^{-1}\left\{\left[\left(g_k - g_k^{(n)}\right)\uparrow S\right]*h^{BP}\right\} \quad (3)$$

where K is the number of low resolution images, $\uparrow S$ is an up-sampling operator by factor S, and h^{BP} is a 'back-projection' kernel. The choice of h^{BP} influences the convergence of the algorithm and the characteristics of the final solution. In order to assure convergence and decrease noise amplification, Irani and Peleg recommend using $h^{BP}=(h^{PSF})^2$.

3. POLYPHASE FILTERS

Polyphase filters are used in certain realizations of multirate filtering operations in order to save computational efforts [9]. In the case where low pass filtering is followed by a decimation with factor M, the operation can be performed with less computations if the polyphase components of $h[n]$ are used to filter delayed and decimated versions of the discrete-time signal. These polyphase components are defined by-

$$p_m[i'] = h[i'M+m]. \quad (4)$$

As one can see, the frequency response of the mth filter is a frequency-shifted version of the baseband prototype. With the use of these polyphase components (in reverse order), expansion followed by filtering can also be performed with computational savings.

4. POLYPHASE BACK-PROJECTING SUPER RESOLUTION IMAGE RESTORATION

The goal of the work described here is to form a restored high resolution image from a given set of blurred low resolution images. Each one of the blurred low-resolution images $g_{i,j}(k,l)$, $k,l=0,1,2,...,N-1$, and $i,j\in[0,Q-1]$ is a translated and decimated observation of a low pass-filtered version of the original high-resolution image $f(m,n)$, $m,n=0,1,2,...,SN-1$. The integer S is the resolution enhancement factor, e.g. the ratio between the number of rows (columns) of the image f and the number of rows (columns) of the decimated images $g_{i,j}$.
Assume that the images $g_{i,j}$ are registered with respect to image $g_{0,0}$ (a version of g that is decimated, but not translated) up to a pixel accuracy, and that the sub-pixel translations between the images, $\Delta X_{i,j}$ and $\Delta Y_{i,j}$, are known. The translations $\Delta X_{i,j}$ and $\Delta Y_{i,j}$ are uniformly quantized with a quantization step of $1/Q$. Therefore, there are only Q^2 possible distinct translations.

We require that R, the ratio between Q (number of quantization levels) and S (resolution enhancement factor) is an integer. Without loss of generality, let $g_{i,j}$ be the image with translation parameters $\Delta X_{i,j}=i/Q$ and $\Delta Y_{i,j}=j/Q$.
Let $h^{PSF}(p,q)$ be the known real symmetric FIR point spread function of the blurring low-pass filter. Assume that $h^{PSF}(p,q)$ is known at a resolution level higher by a factor Q than the resolution of $g_{i,j}$. If $h^{PSF}(p,q)$ is unknown, it can be estimated from the degraded images using conventional well known techniques [1].
The value of $Mask_{i,j}$ is defined to be 1 if an image with translation parameters $\Delta X_{i,j}=i/Q$ and $\Delta Y_{i,j}=j/Q$ belongs to the given set of images, and 0 if not. The algorithm presented here initializes an estimate of the original image f, and improves it in an iterative way.

The proposed algorithm:
The idea behind our algorithm is the follows: we implement any 2-D geometric translation used during the imaging system simulation and the 'back-projection' operation by polyphase filtering, without the need to perform smoothing interpolation operations. Since, in general, the number of quantization levels (Q) is larger than the resolution enhancement factor (S), the value of the 2-D geometric transform (performed at the high-resolution level) contains an integer part and a fraction part. The integer part of the transform is applied by adjusting the indexes of the receptive field, and the fraction part of the translation parameters is compensated by the use of the polyphase filters. Since the number of quantization levels, Q, can be as large as needed, using our algorithm one can use all of the observed low-resolution images at hand, without any restrictions regarding the translation parameters. At the present, we assume that the geometric transform contains translation only. In the case that the geometric transform is more complex, registration should be performed in order to compensate for all the distortions except for pure translations.
The first step is to initialize the super-resolution image $f^{(0)}(x,y)$. The estimation of $f^{(0)}$ is done as follows:

Initial estimation of the restored image:
For $i,j=0,R,...,S-R$

$$f^0\left(kS+\frac{i}{R}, lS+\frac{j}{R}\right) = g_{i,j}(k,l) \quad (5)$$

Note that if for a specific translation (i,j) a degraded image does not exist ($Mask_{i,j}=0$), $g_{i,j}$ should be estimated by averaging its closest present images.
The initial values of the pixels of $f^{(0)}$ are the respective pixels of the lower resolution images $g_{i,j}$. Now, let $h^{PSF}(p,q)$ mark the FIR point spread function (at a resolution higher than $g_{i,j}$ by a factor of Q), and $h^{BP}(p,q)$ mark the 'back-projection' kernel (at the same resolution as $h^{PSF}(p,q)$). The polyphase decomposition's of h^{PSF} and h^{BP} are marked by $\{h_{i,j}^{PP\text{-}PSF}\}$ and $\{h_{i,j}^{PP\text{-}BP}\}$, respectively. Allowing I to be the iteration index, marking the modulus operator of a by b as $((a))_b$ and the integer part of a real number by $\lfloor\bullet\rfloor$, the proposed super-resolution image restoration algorithm is as follows:

Initializing:

I=1, C= a convergence constant factor.

$$h^{PP-PSF}_{i,j}(x,y) = h^{PSF}(xR+i, yR+j) \qquad i,j=0,1,...,R-1. \tag{6}$$

$$h^{PP-BP}_{i,j}(x,y) = h^{BP}(xR+i, yR+j) \qquad i,j=0,1,...,R-1. \tag{7}$$

Iterative process:

For $i,j=0,1,...,Q-1$

If $Mask_{i,j}=1$

Simulate the imaging process (as shown in Figure 1):

$$g^{I}_{i,j}(k,l) = \frac{\left(f^{I-1}\left(kS+\left\lfloor \frac{i}{R}\right\rfloor, lS+\left\lfloor \frac{j}{R}\right\rfloor\right) * h^{PP-PSF}_{((i))_R,((j))_R}(x,y)\right)\downarrow S}{\sum_{s,t} h^{PP-PSF}_{((i))_R,((j))_R}(s,t)} \tag{8}$$

Calculate the difference between the simulated and given data:

$$Dg^{I}_{i,j}(k,l) = g_{i,j}(k,l) - g^{I}_{i,j}(k,l) \tag{9}$$

Update the restored image (as shown in Figure 2):

$$f^{I}\left(x+\left\lfloor \frac{i}{R}\right\rfloor, y+\left\lfloor \frac{j}{R}\right\rfloor\right) = \tag{10}$$

$$f^{I}\left(x+\left\lfloor \frac{i}{R}\right\rfloor, y+\left\lfloor \frac{j}{R}\right\rfloor\right) + C\,\frac{\left(Dg^{I}_{i,j}(k,l)\uparrow S\right) * h^{PP-BP}_{R-1-((i))_R,R-1-((j))_R}(x,y)}{\sum_{s,t} Mask_{s,t}\cdot \sum_{s,t} h^{PP-BP}_{R-1-((i))_R,R-1-((j))_R}(s,t)}$$

Increase I and repeat iterating until minimizing-

$$e^{I} = \sqrt{\sum_{i,j}\left(Dg^{I}_{i,j}\right)^2}$$

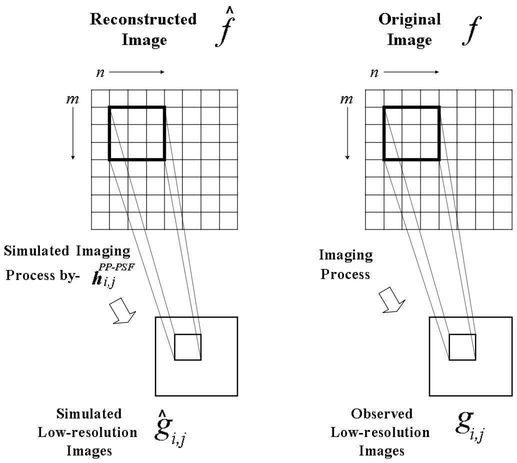


Figure 1. Simulation of the imaging process.

The efficient implementation of the filtering followed by decimation in eq. 8 and the interpolation followed by filtering in eq. 10 is done by polyphase decomposition [9].

5. EXPERIMENTAL RESULTS

The experimental result presented in Figure 3 was obtained using synthetically filtered, translated and decimated images using an averaging h^{PSF} blurring function and generating one degraded image for each one of the quantized translation parameters. The back-projection kernel used here was $h^{BP}=(h^{PSF})^2$.

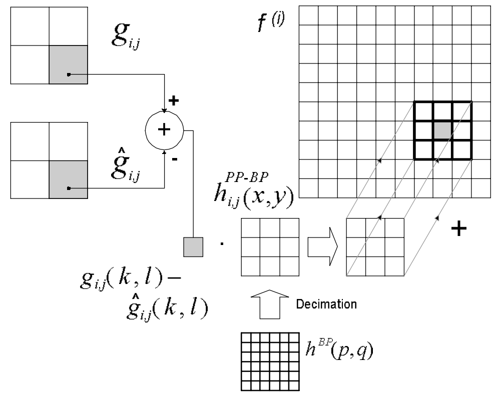
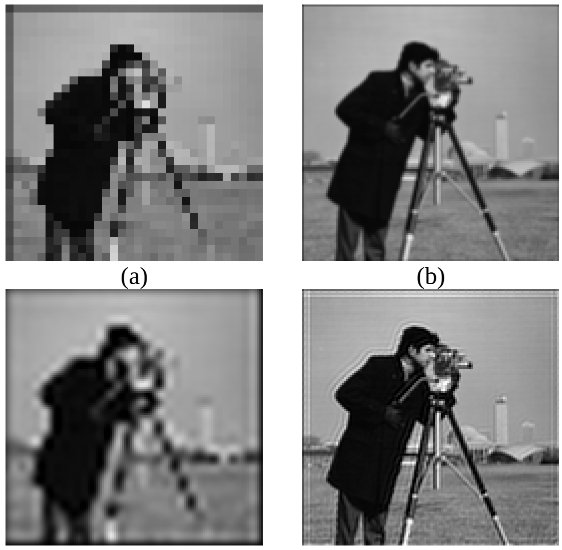


Figure 2. Improvement of the initial guess by polyphase 'back-projecting'.



(a) (b)

(c) (d)

Figure 3. 'Cameraman' image result (S=4, Q=8): (a) low-resolution image (zoomed); (b) initial estimation; (c) interpolation result; (d) super-resolution result.

The experimental data used to generate the result presented in Figure 4 was digitized by low-resolution scanning of an image translated by hand. The translation parameters were estimated using the sub-pixel registration scheme described in [6]. The imaging system's PSF was estimated with a derivative of an edge response. Here we have also used $h^{BP}=(h^{PSF})^2$.

6. SUMMARY AND CONCLUSIONS

An extension of the super resolution image restoration algorithm proposed by Irani and Peleg [6,7] has been presented. In the proposed algorithm, the translation parameters between the low resolution images are quantized, and the images with the various translation quantization levels are 'back projected' and simulated using polyphase filters.

Using the phase shifts of the polyphase filters, we can use every low resolution image available in order to restore the super resolution image, without the need of any smoothing.
We have shown results obtained by applying our algorithm to low resolution images that are synthetically degraded versions of a known image, and low resolution images that are low resolution scanned versions of a translated image.

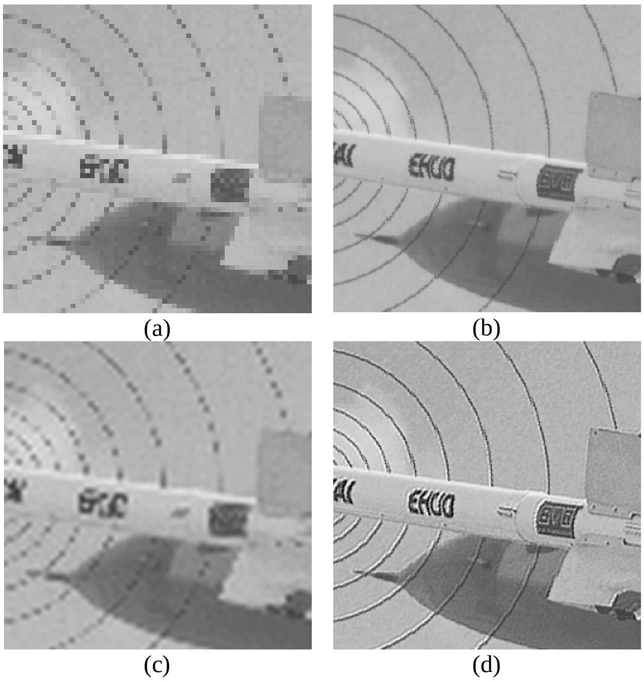


Figure 4. 'Ehud' image result (S=4, Q=16):
(a) low-resolution image (zoomed); (b) initial estimation;
(c) interpolation result; (d) super-resolution result.

7. ACKNOWLEDGMENT

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