INSTANTANEOUS PARAMETER ESTIMATION BASED ON CONTINUOUS WAVELET TRANSFORM AND SOME IMPROVEMENTS

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ABSTRACT

In this paper,. an novel method based on the phase information of continuous wavelet transform to estimate the instantaneous parameter of AM-FM signal is introduced, and some strategies, including the determination of initial value in iteration and post-processing to the estimated results, to improve the performance of the algorithm are proposed. Compared to several other instantaneous parameter estimators, such as CDF, Teager-Kaiser energy operator, and some TFR-based estimators, the proposed method has the advantages of noise resistance and accuracy by exploiting the time-scale localization of the wavelet transform. Simulation results testify that the proposed strategies improve the performance of the CWT based iteration algorithm greatly, and the method has excellent performance including robustness and accuracy in noisy condition.

1. INTRODUCTION

AM-FM(amplitude and/or frequency modulated) signals of the form $s(t) = a(t)\cos(\phi(t))$ such as chirp signals, are widely used in various areas (e.g., seismic, sonar, radar, communications systems, etc.). Also, signals with such time-vary instantaneous frequency $\omega_i(t) = \phi'(t)$ or amplitude a(t) are often required for speech modeling. Some bio-signals such as bat echoes are proved to be of the similar form. Since the instantaneous parameters like the frequency modulation law and amplitude modulation law can provide very helpful information for analysis of nonstationary signals and estimation of important parameters, such as Doppler frequency associated with radia velocity in radar and sonar situations[6], methods for estimating instantaneous parameters especially in noisy situations accurately and efficiently are a topic of increased recent concern.

2. BACKGROUND

A variety of methods have been developed for the estimation of instantaneous parameters. Some of them can estimate the instantaneous parameters (amplitude and frequency) jointly, such as Teagar -Kaiser Energy Algorithm(TKA)[2], AS(analytic signal) method[4], and some predictive and modeling methods[1][3], etc. Others are instantaneous frequency(IF) estimators, such as CFD(central finite difference of the phase of analytic signal)[9], and frequency tracking methods based on following the peaks of the ridge on TFR representations like STFT, WVD[5]. Consider the frequency estimation performance of these methods, TKA can yield distortion in some special wideband signals[4], and when the signal is contaminated by significant noise, the TKA will become invalid unless multiband filtering is applied and a special procedure for determining which filter contains the instantaneous frequency[2]. AS also needs filtering in noisy conditions[4]. CDF becomes unreliable in noisy signals. The TFR will become blurred due to presence of noise[5]. Thus how to get noise-resistant, accurate, efficient estimation of instantaneous frequency is still an open problem.

We here introduce an efficient method using an iterative algorithm based on continuous wavelet transform for instantaneous parameter estimation. Owing to the time-frequency locally filtering property of wavelet transform, the method has strong ability to resist significant noise. Since the iteration is a procedure of automatically optimizing, the method can yield accurate estimation, and it is also highly efficient and has very low computation complexity. A distinguished character of the method compared to other TFR methods is that it is deduced from the phase information other than energy of transform coefficients. We also propose some skills for improving the performance of the algorithm.

3. CONTINUOUS WAVELET TRANSFORM, RIDGE AND INSTANTANEOUS FREQUENCY

In time-scale domain, the scalogram of a FM signal can yield a ridge which corresponds to the frequency modulation law in a nonlinear way.[10][11] The generation of the ridge can be explained as follows.

Given a FM signal in its analytic form:

$$s(t) = A_s(t)e^{j\phi_s(t)} \tag{1}$$

and suppose that it is time-asymptotic, that is

$$\left|\phi_s'(t)\right| >> \left|A_s'(t)/A_s(t)\right| \tag{2}$$

then its wavelet transform with respect to an asymptotic wavelet function

$$g(t) = A_g(t)e^{j\phi_g(t)} \tag{3}$$

reads

$$\begin{aligned} WT(b,a) &= \frac{1}{2a}\int_{-\infty}^{\infty} s(t)g^*(\frac{t-b}{a})dt \\ &= \frac{1}{2a}\int_{-\infty}^{\infty} A_s(t)A_g(t)e^{j\left[\phi_s(t)-\phi_g(\frac{t-b}{a})\right]}dt \\ &= \frac{1}{2a}\int_{-\infty}^{\infty} A_s(t)A_g(t)e^{j\Phi_{b,a}(t)}dt \end{aligned} \tag{4}$$

By expanding $\Phi_{b,a}(t)$ into a Taylor series around $t = t_s$ such that $\Phi'_{b,a}(t_s) = 0, \Phi''_{b,a}(t_s) \neq 0,$

which is called stationary point, we get

$$\begin{aligned} \Phi_{b,a}(t) &= \Phi_{b,a}(t_s) + \Phi'_{b,a}(t_s)(t-t_s) + \frac{1}{2}\Phi''_{b,a}(t_s)(t-t_s)^2 + \ldots \\ &\approx \Phi_{b,a}(t_s) + \frac{1}{2}\Phi''_{b,a}(t_s)(t-t_s)^2 \end{aligned} \tag{5}$$

If the amplitude of analyzing wavelet is gaussian, for example, the morlet wavelet $g(t) = e^{-t^2/2}e^{j\omega_0 t}$, then the wavelet transform is approximated by a Gaussian integration as

$$WT(b,a) \approx \frac{A_s(t)e^{j\Phi_{b,a}(t_s)}1}{2a}\int_{-\infty}^{\infty} A_g(\frac{t-b}{a})e^{i\frac{(t-t_s)^2}{2}\Phi''_{b,a}(t_s)}dt \qquad (6)$$
$$= |WT(b,a|e^{i\Psi(b,a)}$$

The expression of amplitude and phase of the wavelet can be found jn[10][11].

As to FM signal, its energy is concentrated along a ridge on the time-scale plane of its wavelet transform. The ridge is defined by a set of points (b,a) such that the stationary point $t_s(b,a) = b$, so from the definition of stationary point we get

$$\Phi'_{b,a}(t_s) = \phi_s'(t_s) - \phi_g'(\frac{t_s - b}{a}) = 0 \qquad (7)$$

we can see on the ridge

$$a = a_r(b) = \frac{\phi_g'(0)}{\phi_s'(b)} \qquad (8)$$

thus the frequency modulation law (instantaneous frequency) is inversely proportional to the ridge . So the estimation of instantaneous frequency equals to the extraction of ridge.

4. RIDGE EXTRACTION AND INSTANTANEOUS FREQUENCY ESTIMATION

Next define the wavelet curve as the sets of points $(b_0, a_r(b_0))$ on time-scale plane with constant stationary points such that $t_s(b,a) = b_0$, it is uniquely determined by the central frequency of a set of shifted and dilated wavelets on this curve. The restriction of the wavelet transform to wavelet curve by setting $t_s = b_0$ in equation [6] can be interpreted as a superposition of wavelets localized on that curve.

The phase of wavelet coefficients on the wavelet curve has the property as

$$\left.\frac{d\Psi}{db}\right|_{t_s(b,a)=b_0} = \frac{\omega_0}{a} \qquad (9)$$

thus, the ridge can be extracted through the phase of transform coefficients by the fixed-point algorithm. Usually, the algorithm is implemented through an iteration way.

Given a signal sequence $s(t_k)$,k=0,1,2,…N-1. Let $a_0(t_0)$ be an arbitrary initial estimation of $a_r(t_0)$. Then the iteration can be described as

$$a_{i+1}(t_0 + kT) = \frac{\omega_0}{\Psi'(t_0 + kT, a_i)} \qquad (10)$$
$$a_0(t_0 + (k+1)T) = a_c(t_0 + kT)$$

where $a_c(t_0 + kT)$ is the converged solution at time $t_0 + kT$, and the convergence criterion is

$|a_{i+1} - a_i| < \varepsilon$, an arbitrary small positive number .

As soon as the ridge is extracted, the instantaneous frequency is obtained. Then, the amplitude modulation law can be calculated by substituting the estimated frequency in the wavelet transform.

***Some skills to improve the performance of the above algorithm*:**

1.*The determination of the initial value*

In practice, the algorithm is not always successful due to the non proper choice of initial value of a. The initial value effect the possibility of convergence and the velocity of convergence. If the initial value is proper in the sense that it is not too far from the real value with respect to the initial frequency of the signal, the algorithm will converge rapidly. Other wise, it may not converge at all or converge to fully wrong results.

Given a signal sequence, we may find its initial scale a_0 obtained through searching the first peak in its scalogram. This procedure can be found in [12].This is valid in case that the spectrum of signal is not merged in noise completely.

2. *Post-processing of Estimation Results*

In noisy conditions the instantaneous frequency estimation obtained by the above algorithm is fluctuating. To get the accurate estimation of some parameters such as the initial frequency f_0 and frequency rate k in the chirp signals, we apply data fitting technique to the estimated frequencies. After this, the effect of noise to the estimation is greatly reduced as proven by experimental results.

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5.EXPERIMENTAL RESULTS

In this section, we introduce some experimental results to demonstrate the excellent performance of the proposed method by applying it to several simulated signals and actual sonar echo data. We here give only simulation results for instantaneous frequency (IF) estimation for comparison with some other IF estimators.

1. Wideband FM signal

Consider the wideband FM signal $s(t) = \sin(10t + t^3/6)$, setting the time duration T=12s and sampling frequency fs=5f(T), the signal is shown in Fig1.a. We first testify the improvement to the algorithm by using the initial value determination.

By applying the proposed algorithm and the initial value determination method introduced above, we obtain the IF estimation of the signal plotted in Fig1.b as a solid line. It corresponds the real IF of the signal plotted as a dotted line exactly. While the estimation by TKA plotted as the dashed line shows bias at higher frequency.

If we choose the initial value arbitrarily , for example, as 50 or 0.01, the iteration will converge to wrong estimation as shown in Fig1.c and Fig1.d. This prove the significance of our method to determine the initial value from the signal.

Next the noise resistance of the algorithm is proved by adding a Gaussian white noise to the signal, letting SNR=9dB, the estimation results is shown in Fig1.e. Compared to the real IF (the dotted line), the estimation is almost the same except the first several ponits.

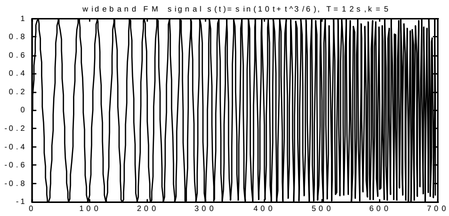

Fig1.a The wideband FM signal $s(t) = \sin(10t + t^3/6)$,T=12s,k=5

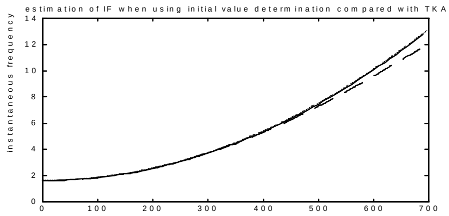

Fig 1.b Instantaneous frequency estimation of the above signal. The solid line is obtained by proposed method using the initial value determination, the dashed line corresponds to the actual instantaneous frequency of the signal, and the dashed line is obtained by TKA algorithm.

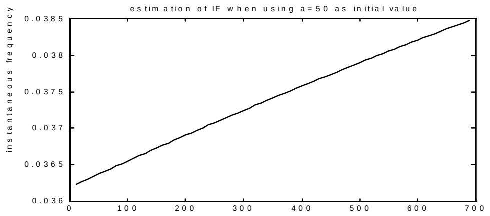

Fig 1.c The instantaneous frequency estimation obtained by the proposed method without the initial value determination, and a_0 is arbitrarily chosen as 50

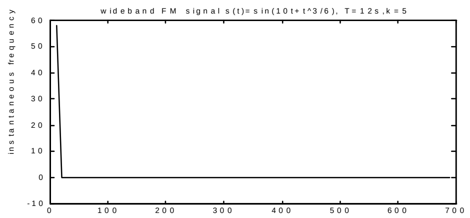

Fig1.d The instantaneous frequency estimation obtained by the proposed method without initial value determination, and a_0 is arbitrarily chosen as 0.01

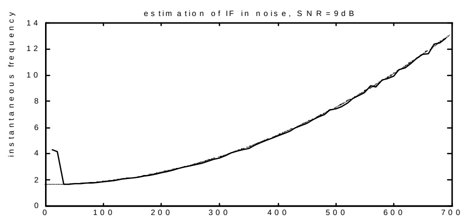

Fig 1.e The instantaneous frequency estimation of the noisy signal obtained by the proposed method represented by the solid line, the dashed line corresponds to real instantaneous frequency.

2. Chirp Signal

We now take for example the noisy chirp signal $s(t) = A\exp\left[j2\pi(f_0 t + \frac{k}{2} t^2) + \phi_0 \right] + n(t)$ where A is the amplitude of the signal and f_0, k, ϕ_0 are its initial frequency, frequency rate, and beginning phase, respectively. $n(t)$ is a zero-mean complex Guassian noise with variance $\sigma_n{}^2$, to investigate the parameter estimation performance of the proposed method. For the purpose of estimating initial frequency f_0 and frequency rate k, the post-processing method by data fitting is applied to the instantaneous frequency estimation obtained by above wavelet based iteration algorithm.

The IF estimation of a noisy chirp with the following: $A = 1, f_0 = 20Hz, k = 0.195Hz / s, \phi_0 = 0, f_s = 100Hz$, SNR=3 dB when defined as $10\log10(A^2 / \sigma^2)$ .is presented in Fig2.a. Although the IF estimation by the iteration algorithm is fluctuating as plotted as the solid line, the final estimation shown as dashed line by fitting the estimation to a 1-order polynomial only has little bias from the true instantaneous frequency of the signal as plotted with the dotted line.

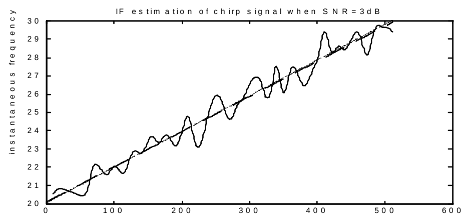

Fig 2.a Estimation of Instantaneous frequency of the chirp signal with parameters $A = 1, f_0 = 20Hz, k = 0.195Hz / s, \phi_0 = 0, f_s = 100Hz$. when SNR is 3dB

We also investigate the statistical performance of the estimator under different signal-to-noise ratios with fixed parameters A, f_0, k, ϕ_0 , Given a chirp signal sequence with parameter values as in Fig2.a, the sequence has 512 points. The estimates of f_0, k are presented in Fig2.b. On the y-axis . $10\log10(1/[\frac{1}{M}\sum_{i=1}^{M}(\hat{\theta} - \theta)^2]$ is plotted , where θ is the true value of parameters like f_0, k, ϕ_0, and $\hat{\theta}_i$ is its estimate at *i*th realization, M is the total number of realizations. The *x*.-axis is the signal-to-noise ratio defined as $10\log10(\frac{A^2}{\sigma_n{}^2})$ starting from 0dB to 30 dB in step of 1 dB. There are 100 realizations per SNR.

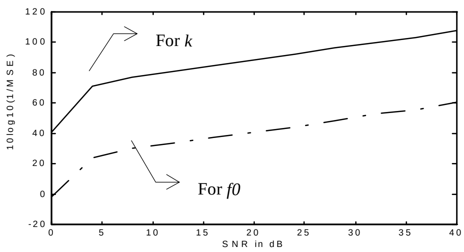

Fig 2.b Experimental obtained MSE(mean square errors) for initial frequency f_0 and modulation rate k of the chirp signal .

3. **Active sonar echo**

Last we apply our methodology to actual active sonar echo to extract the frequency modulation law embedded in it. The known emitted signal is is a linear chirp with frequency range 20 ~ 140kHz, sampling frequency 1MHz, and time duration 2ms. It is reflected back from the lake bottom and one sample of echoes is shown in Fig3.a. The instantaneous frequency estimation of the echo by proposed method is given in Fig3.b as the fluctuating curve. The frequency modulation embedded in echo gained by data fitting from the estimated instantaneous frequency is also shown in Fig3.b as that line.It is obvious that the estimation result is satisfactory.

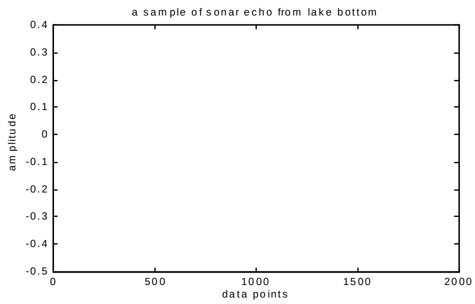

Fig 3.a A sample of echos reflected from lake bottom

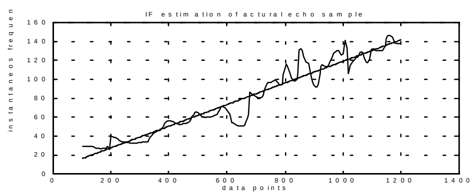

Fig 3.b Estimation of Instantaneous frequency of the echo(fluctuating curve and the modulation law calculated by data fitting(the line)

6. CONCLUSION

In this paper we introduced the wavelet based iteration algorithm to estimate the instantaneous parameters of FM signal. Two strategies to improve the performance of the algorithm are proposed. Compared to other IF estimation methods, our approach is more accurate, especially to some wideband signals, and less sensitive to noise. It has the superiority in terms of noise resistance by exploiting the time-scale locally filtering property of wavelet transform. The approach we proposed is shown to be very effective for improving the performance of the algorithm proposed in [10][11].

7. REFERENCES

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