# AN EFFICIENT ARRAY CALIBRATION METHOD ON UNDERWATER HIGH RESOLUTION DIRECTION-FINDING

*Jianfeng Chen and Jianguo Huang*
College of Marine Engineering
Northwestern Polytechnical University
Xi'an, Shaanxi, 710072, China

## ABSTRACT

A novel and practical approach of array calibration in underwater high-resolution direction finding is presented to alleviate the effect of different errors generated in underwater array processing system in this paper. It is different from other algorithms in that array manifold used in subspace-based methods is obtained through automatic measurement instead of using theoretical value. It can efficiently calibrate the errors of gain, phase, mutual-coupling, position of sensors, and other reasons generated by the array and sensors even they are direction dependent. Spatial smoothing technique is employed and so the method is effective no matter the sources are correlated or uncorrelated. It is also proved by the experiment that spatial smoothing is beneficial to reduce some errors. At last, several experimental results are provided to verify the efficiency of the new calibration method.

## 1. INTRODUCTION

Advanced techniques for array processing based on eigen-decomposition of the covariance matrix of received signals have been discussed extensively in the literature since the beginning of this decade[1]. The popularity of these algorithms is due to their generality--they are applicable to arrays of arbitrary but known configuration and response. These methods are proved to be sensitive to modeling errors and require detailed knowledge of the array response (the array manifold)[3]. Their performances decrease drastically with a bad knowledge on the array manifold. However, in practice, the array response is always different from assumed one.

The performance degradation of these algorithms due to the errors have received some attention. In [2], J. Pierre and M. Kaveh had constructed an ultrasonic experiment testbed and attempted to solve this problem. In [3], A. J. Weiss and B. Friedlander critically proved that modeling mismatch may seriously result in the performance descent of high-resolution DOA methods and examined the resolution threshold of the MUSIC algorithm when the array response is perturbed from its assumed model. Also, they pointed out that preprocessing on the array output may alleviate the problem. In [4], N. Fistas and A. Manikas presented a general global array calibration method. They considered position, phase, and gain error of sensors simultaneously. By their method, all the error parameters can be worked out and then the array can be calibrated at last. In [5], A. P. C. Ng provided a method considering the case when wavelength is unknown or imprecisely known and showed that an inherent ambiguity prevented a linear array from resolving wavelength and DOA simultaneously. In [6], B. C. Ng proposed a calibration method based on a maximum likelihood approach. It can evaluate the calibration matrix consisting of the unknown gain, phase, and mutual-coupling coefficients as well as the positions of sensors using a set of calibration sources in known locations.

Through Monte-Carlo simulation we find that the methods list above all can get properly good results (assuming all the errors are direction-independent). In order to apply the high resolution DOA methods on practicality, we construct an underwater high resolution array processing experimental system.

In section 2, the experimental system is described and the error factors in each link are analyzed. In section 3, an efficient calibration method is presented and several experimental results are shown in section 4. Section 5 gives a concise conclusion.

## 2. UNDERWATER HIGH-RESOLUTION ARRAY PROCESSING EXPERIMENTAL SYSTEM

The experimental system frame is shown in Figure 1. The entire system consists of a $20\times7\times8m^3$ water tank, two transmitting transducers, a linear array with 14 sensors, multi-channel filter and amplifier, a data acquisition system, and a super signal processing system which can give real-time estimation results. The linear array is fixed in water tank 10 meters far from the array.

The transmitters generate a narrowband signal. The wall of water tank is specially processed so as to minimize the reflection.

From figure 1 we can see that the acoustic wave emitted from transducers must pass many steps and then the DOA estimates of sources can be got at last. It is started with the electrical-sonic transformation, propagation in water, sonic-electrical transformation, A/D sampling, and then the DOA estimates can be obtained. Because the electrical equipment, physical devices, water environment, positions and responses of array elements are all with uncertainty, a variety of errors are embedded. We summarize most of errors as follows:

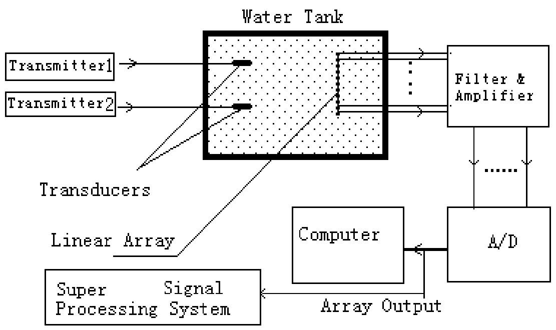


**Figure 1.** Block diagram of the experimental system

(1)Original signal error: The transmitters' center frequency may be slightly different from the designed value.
(2)The inconsistency of multi-channel in the filter and amplifier.
(3)The difference between each channel of A/D.
(4)The different response of each sensor.
(5) Position error of elements.
(6) Far field condition is unsatisfied.
(7)The mutual-coupling of each two sensors.
(8)The difference of local acoustic field of each sensor.

Because the electrical equipment used in our experiment are advanced and accurate enough, (1) (2) (3) can be ignored.

The output of each sensor unified by the first sensor before fixed on array (response 1) and after fixed on array (response 2) is shown in Table 1. The divergence between them is due to several reasons, such as reflection, mutual-coupling, error of sensors response and some other reasons. It is hard to establish a model to calibrate.

As for (4), there have been many papers presented on this problem, such as [4][6], etc.. But because of complex effect of these different errors, it is difficult to improve the performance of high resolution algorithms by using the parameters evaluated only based on observed data from several directions. It has been tested that the accurate DOA estimates can only be obtained in the directions around the position of known sources.

As for (6), we find that because of unknown phase error of sensors, mutual-coupling and 'border effect' caused by the pedestal of the array, only geometrical compensation is not effective.

After the failure of attempt to calibrate the array by using the methods presented above and in [2]-[6], we propose a new calibration method. Considering the requirement of applications, our algorithm is simple, efficient, adaptive to correlated and uncorrelated sources, and operating easily.

## 3. CALIBRATION ALGORITHM

In order to calibrate most of the errors in our experimental system, we propose the following algorithm. Omit the tedious derivation, we give the clear steps of the method directly.

**Step 1: Unify the output of each sensor**

Based on the assumption that sensors of the array are all the same and isotropic, the power of each channel should be identical. From Table 1 we can find that the amplitude of each sensor is much different from each other and even up to 1.6 times. So, before dealing with other errors, the output of each sensor is unified at first.

Assume $x_i[n]$ is the output of *i*th sensor at snapshot n, $P_i$ is the average output power of *i*th channel, N is the number of used snapshots and M is the number of sensors. We preprocess the data by

$$\tilde{x}_i[n] = \frac{x_i[n]}{P_i}, \quad i=1,2,...,M,\ n=1,2,...,N \qquad (1)$$

where $\tilde{x}_i[n]$ is the unified array output of *i*th sensor. After this, the power of all sensors are all the same. Most of gain error of the sensors and inconsistency of channels are reduced.

**Step 2: Spatial Smoothing**

Because of the limited snapshots used in estimation, the bandwidth of narrowband signals and other reasons, it is inevitable that the sources are correlated between each other. Here the forward/backward spatial smoothing method[9] is employed to deal with the problem.

For linear array with M sensors, it can be divided into P subarrays. Each subarray has L sensors. Then P=M-L+1. Forward smoothing covariance matrix of *k*th subarray is defined as

$$R_k^f = E\left[\vec{\tilde{x}}_k[n]\vec{\tilde{x}}_k^H[n]\right], \quad k=1,2,...,P \qquad (2)$$

where $\vec{\tilde{x}}_k[n] = \left[\tilde{x}_k[n] \quad \tilde{x}_{k+1}[n] \quad \dots \quad \tilde{x}_{k+L-1}[n]\right]^T$, $(\cdot)^H$ denotes conjugate transpose operator, $(\cdot)^T$ denotes transpose operator.

Forward/backward smoothing covariance matrix is given by

$$\hat{R} = \frac{1}{2P}\sum_{i=1}^{P}(\hat{R}_i^f + \hat{R}_i^b) = \frac{1}{2}(\hat{R}^f + J(\hat{R}^f)^H J) \qquad (3)$$

where $J = \begin{bmatrix} 0 & & 1 \\ & \cdot\cdot & \\ 1 & & 0 \end{bmatrix}$

Then covariance matrix $\hat{R}_{(L\times L)}$ defined in (3) is used in high resolution method instead of original $\hat{R}_{(M\times M)}$.

By using spatial smoothing technique, the correlation between sources is resolved. Meanwhile, the effect generated by the errors of gain, phase, position of sensors are also reduced. It was verified in our experiments.

**Step 3: Calibrate the array manifold**

Through measurement we found that the directivities of sensors are inconsistent and so the errors are seriously direction-dependent. It is an efficient way[2] to degrade this problem by precisely measuring the array manifold over interesting range of directions.

Assuming the searching range is [-α,α], we fix a source under water and its angle to the array center is β. The array can be rotated automatically with a step of Δ, then β can change from -α to α, that is,

$$\beta_j = -\alpha + (j-1)\Delta\ , \quad j=1,2,...,q\ , \quad q=2\alpha/\Delta$$

When the array is in *j*th position, let $x_i^{(j)}[n]$ be the output of *i*th sensor of snapshot *n*(i=1,2,...,M, n=1,2, ...,N). Then the covariance matrix of *j*th position is given by

$$\hat{R}^{(j)} = \frac{1}{N}\sum_{n=1}^{N}\vec{x}^{(j)}[n]\vec{x}^{(j)}[n]^H \qquad (4)$$

where $\vec{x}^{(j)}[n] = [x_1^{(j)}[n], x_2^{(j)}[n], \ldots, x_M^{(j)}[n]]^T$.

Therefore there is a one-to-one correspondence between the principal eigenvector $\vec{v}_1^{(j)}$ of $\hat{R}^{(j)}$ and the direction-vector a(θ). We can also use spatial smoothing to reduce the dimension of $\vec{v}_1^{(j)}$ from M×1 to L×1. By automatic measuring, we can get *q* principal vectors, $\vec{v}_1^{(j)}$ (*j*=1,2,...,*q*), which are the real array manifold.

**Step 4: Estimate the DOAs**

Now we represent the MUSIC method using real array manifold as

$$\hat{P}_{MUSIC}(\theta[j]) = \frac{1}{\left|\sum_{i=m+1}^{L}\vec{v}_1^{(j)H}\vec{e}_i\right|^2} \quad j=1,2,...,q \qquad (5)$$

where *m* is the number of sources, $\vec{e}_i$ (i=m+1,...,L) is the noise subspace of covariance matrix $\hat{R}$, $\theta[j]$ (j=1,2,...,q) is the *j*th direction.

This step accumulates all the information over the searching range and so each point of estimation result is based on the correspondent prior knowledge.

In order to simplify the procedure of operation, we have designed an auto measurement and storage system. Before the test or application, the array manifold is automatically measured and stored in an EEPROM or disk of PC. It is very convenient to obtain the direction vector by looking up the list. So the computational load is not increased.

## 4. EXPERIMENTAL RESULTS

This section contains several experimental results. We apply the calibration method proposed before in the experiment and employ the MUSIC and Mini-Norm method to estimate the DOAs of test data. The results before calibration and after calibration is compared. Furthermore, statistical results of separation probability of the MUSIC and Mini-Norm methods are also list in Table 2,3.

In the first test, two uncorrelated sources are placed at 0° and -2.4° (about 1/3 beamwidth apart). Test results are shown in figure 2. It is very clear that before calibration, the Mini-Norm can not get the effective estimation. The spectrum peak deviate from true signal direction and two sources can not be recognized. After calibration, we can see, two sharp peaks in spatial spectrum show super performance of high-resolution ability of the Mini-Norm. It is proved in [8] by a statistical approach that Mini-Norm has super separation ability over the MUSIC. Here, in Table 2, it is also verified by our test.

**Table 2:** Resolution probability comparison
(Two uncorrelated sources, 1/3 Beamwidth apart)

| | Mini-Norm | MUSIC |
|---|---|---|
| No calibration | 0% | 0% |
| After Calibration | 100% | 67% |

In the second test , two full coherent sources are emitted, which are located in 0.3° and -3.7° (nearly 3/5 beamwidth apart). Figure 3 and Table 3 show the estimated spatial spectrum and statistical results. In this case, the method can also get high-resolution result. But the resolution ability is less than uncorrelated case.

**Table 3:**Resolution probability comparison
(Two coherent sources, 1/3 Beamwidth apart)

| | Mini-Norm | MUSIC |
|---|---|---|
| No calibration | 0% | 0% |
| After calibration | 81% | 45% |

## 5. CONCLUSION

In this paper an underwater high resolution array processing system is described and an efficient array calibration method is presented. Three features of this method are very important in application. At first, it is very simple. It nearly does not increase the computation load in array processing. Secondly, it can deal with correlated sources. The third one is that it can calibrate most types of array errors, especially the direction-dependent errors. It is more suited for engineering application.

## 6. REFERENCES


[1] R. Schmidt, Multiple Emitter Location and Signal Parameter Estimation, *RADC Spectrum Estimation Workshop Record*, pp.243~253, Oct. 1979

[2]J. Pierre, M. Kaveh, Experimental performance of calibration and direction finding algorithms, *IEEE T-ASSP*, vol. 2, pp. 114-118, 1991

[3]A. J. Weiss, B. Friedlander, Effects of modeling errors on the resolution threshold of the MUSIC algorithm, *IEEE Trans. Signal Processing*, vol. 42, no. 6, pp. 1519-1526, June, 1994

[4]N. Fistas and A. Manikas, A new general global array calibration method, *Proc. IEEE ICASSP* ,pp. IV-73-76, 1994

[5]A. P.C. Ng, Direction of arrival estimates in the presence of wavelength, gain, and phase errors, *IEEE Trans. Signal Processing*, vol. 43, no. 1, pp. 225-232, Jan. 1995

[6]B. C. Ng, etc., Sensor-array calibration using a Maximum-Likelihood approach, *IEEE T-AP*, vol. 44, no. 6, June 1996

[7]Q. T.(Keith) Zhang, Probability of resolution of the music algorithm, *IEEE Trans. Signal Processing*, vol. 43, no. 4, pp. 978-987, Apr. 1995

[8]Jianguo Huang, Jianfeng Chen, Statistical performance comparison of high-resolution DOA estimation methods, ICSP'96, Beijing, China, 505~508, 1996

[9]S. U. Pillai, B. H. Kwon, Forward/backward spatial smoothing techniques for coherent signal identification, *IEEE T-ASSP*, vol. 37, no. 1, pp. 8-15, Jan. 1989


**Table 1**: Response of each sensors

| Sensor ID | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Response 1 | 1 | 0.93 | 0.94 | 0.88 | 0.93 | 0.92 | 0.95 | 0.85 | 0.90 | 0.92 | 0.85 | 0.91 | 0.91 | 0.87 |
| Response 2 | 1 | 1.15 | 1.07 | 0.85 | 0.85 | 1.00 | 0.77 | 0.77 | 0.85 | 0.69 | 0.77 | 0.92 | 1.23 | 1.07 |

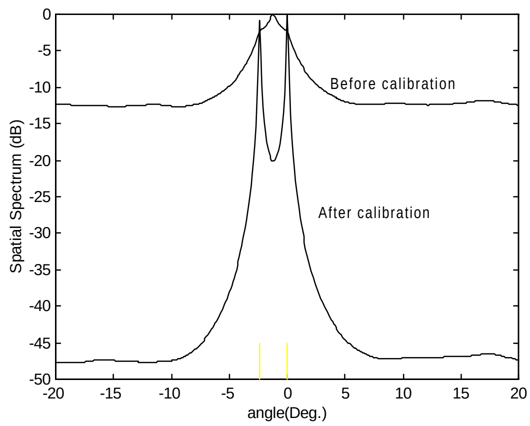


**Figure 2:** DOA estimates of Mini-Norm method
(Two uncorrelated sources, 1/3 beamwidth apart)

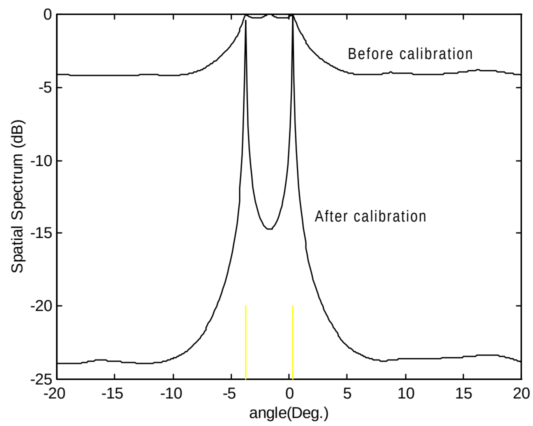


**Figure 3:** DOA estimates of Mini-Norm method
(Two full coherent sources, 3/5 beamwidth apart)