ADAPTIVE BLIND SEPARATION OF CONVOLUTIVE MIXTURES OF INDEPENDENT LINEAR SIGNALS

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ABSTRACT

This paper is concerned with the problem of blind separation of independent signals (sources) from their linear convolutive mixtures. The various signals are assumed to be linear non-Gaussian but not necessarily i.i.d. Recently an iterative, normalized higher-order cumulant maximization based approach was developed using the fourth-order normalized cumulants of the "beamformed" data. A byproduct of this approach is a decomposition of the given data at each sensor into its independent signal components. In this paper an adaptive implementation of the above approach is developed using a stochastic gradient approach. Some further enhancements including a Wiener filter implementation for signal separation and adaptive filter reinitialization are also provided. A computer simulation example is presented.

1. INTRODUCTION

Given noisy measurements $y_i(k)$, $(i = 1, 2, \cdots, N)$, at time k at N sensors, let these measurements be a linear convolutive mixture of M source signals $x_j(k)$, $(j = 1, 2, \cdots, M)$:

$$y_i(k) = \sum_{j=1}^{M} U_{ij}(z)x_j(k) + n_i(k), \quad i = 1, 2, \cdots, N, \tag{1-1}$$

$$\Longrightarrow \quad \mathbf{y}(k) = \mathcal{U}(z)\mathbf{x}(k) + \mathbf{n}(k), \tag{1-2}$$

where $ij-$th element of $\mathcal{U}(z)$ is $U_{ij}(z)$, $\mathbf{y}(k) = [y_1(k) \vdots y_2(k) \vdots \cdots \vdots y_N(k)]^T$, similarly for $\mathbf{x}(k)$ and $\mathbf{n}(k)$, z^{-1} is both the backward-shift operator (i.e., $z^{-1}x(k) = x(k-1)$, etc.) as well as the complex variable in the $\mathcal{Z}-$transform, $x_j(k)$ is the j-th input at sampling time k, $y_i(k)$ is the i-th output, $n_i(k)$ is the additive Gaussian measurement noise, and $U_{ij}(z) := \sum_{l=0}^{\infty} u_{ij}(l)z^{-l}$ is the scalar transfer function with $x_j(k)$ as the input and $y_i(k)$ as the output. We allow all of the above variables to be complex-valued.

Suppose that we design a MIMO dynamic system $\mathcal{E}(z)$ with N inputs and M outputs such that the overall $M \times M$ system

$$\mathcal{T}(z) := \mathcal{E}(z)\mathcal{U}(z) \tag{1-3}$$

decouples the source signals. Following the 2×2 case considered in [4], this implies that we must have ($T_{ij}(z)$ denotes the $ij-$th element of $\mathcal{T}(z)$)

$$T_{ij}(z) \begin{array}{ll} = 0 & \text{for} \quad i \neq i_j \\ \neq 0 & \text{for} \quad i = i_j \end{array} \tag{1-4}$$

where $i = 1, 2, \cdots, M$; $j = 1, 2, \cdots, M$ and $i_j \in \{1, 2, \cdots, M\}$ such that $i_j \neq i_l$ for $j \neq l$. That is, in every column and every row of $\mathcal{T}(z)$ there is exactly one non-zero entry. In a blind separation problem, the nonzero entries of $\mathcal{T}(z)$ are allowed to be a scalar linear system (shaping filter), unlike the equalization problems where they must be constant gains and/or pure delays.

The problem considered above arises in a wide variety of applications: array processing, speech enhancement ("cocktail party" problem), and noise cancellation, see [1]-[12] and references therein. The prior work done can be classified into two broad categories based upon the underlying propagation model: instantaneous mixtures and convolutive mixtures. The general model (1-2) represents a linear convolutive mixture. The work reported in [4], [7] and [11] (and references therein) deals with linear convolutive mixture (dynamic mixing) models. Past work on separation of convolutive mixtures may be categorized into several classes: time-domain approaches ([7], [8], [9], [10]), frequency-domain approaches ([4],[11]), adaptive (recursive) approaches ([7], [9], [10]) and non-recursive (batch) approaches ([4], [8], [11]). In this paper we present time-domain adaptive approaches. Quite a few of existing approaches are limited either to $M = N = 2$ ([4], [9]) or to $M = N$ ([7]). Although [11] treats a general case, their analysis is restricted to the case of two sources ($M = 2$). In this paper we consider a general case of $N \geq M$ with M arbitrary.

2. MODEL ASSUMPTIONS

We impose the following conditions on model (1-1)-(1-2):

(AS1) $N \geq M$ (at least as many outputs as inputs).

(AS2) The vector sequence $\{\mathbf{x}(k)\}$ is stationary, its various components are mutually independent, and $\mathcal{U}(z)$ is stable. Moreover, $\{\mathbf{x}(k)\}$ is linear, i.e.

$$\mathbf{x}(k) = \mathcal{V}(z)\mathbf{w}(k), \tag{2-1}$$

where $\{\mathbf{w}(k)\}$ is a zero-mean, $M-$vector stationary non-Gaussian process, temporally i.i.d. and spatially independent, with nonzero fourth cumulants. Because of the mutual independence of the components of $\mathbf{x}(k)$, we take $\mathcal{V}(z)$ to be diagonal.

(AS3) Consider the composite system

$$\mathbf{y}(k) = \mathcal{F}(z)\mathbf{w}(k) + \mathbf{n}(k), \quad \text{with} \quad \mathcal{F}(z) := \mathcal{U}(z)\mathcal{V}(z). \tag{2-2}$$

Assume that $\text{rank}\{\mathcal{F}(z)\} = M$ for any $|z| = 1$.

(AS4) Since the composite system is causal, we have

$$\mathcal{F}(z) = \sum_{l=0}^{\infty} \mathbf{F}_l z^{-l} \approx \sum_{l=0}^{L} \mathbf{F}_l z^{-l}. \tag{2-3}$$

(AS5) The noise $\{\mathbf{n}(k)\}$ is a zero-mean, stationary Gaussian sequence independent of $\{\mathbf{w}(k)\}$.

Let $\mathcal{F}^{(i)}(z)$ denote the $i-$th column of $\mathcal{F}(z)$. In blind convolutive signal separation we are interested in decomposing the observations at the various sensors into its independent components. That is, our objective is to estimate $\mathcal{F}^{(i)}(z)w_i(k)$ for $i = 1, 2, \cdots, M$ given $\{\mathbf{y}(k)\}$ without having a prior knowledge of $\mathcal{F}(z)$. Denote the $ij-$th element of $\mathcal{F}(z)$ as $F_{ij}(z)$.

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3. A BATCH SOLUTION [8]

In this section we briefly discuss the batch (non-recursive) approach of [8]; its adaptive version is developed in Sec. 4. Let $\mathrm{CUM}_4(w)$ denote the fourth-order cumulant of a complex-valued scalar zero-mean random variable w, defined as

$$\mathrm{CUM}_4(w) = E\{|w|^4\} - 2[E\{|w|^2\}]^2 - |E\{w^2\}|^2. \quad (3-1)$$

Consider an $1 \times N$ row-vector polynomial equalizer (filter) $\mathcal{C}^T(z)$, with its j-th entry denoted by $\mathcal{C}_j(z)$, operating on the data vector $\mathbf{y}(k)$. Let the equalizer output be denoted by $e(k)$:

$$e(k) \;=\; \sum_{i=1}^{N} \mathcal{C}_i(z) y_i(k). \quad (3-2)$$

Following [6] consider maximization of the cost

$$J \;:=\; \frac{|\mathrm{CUM}_4(e(k))|}{[E\{|e(k)|^2\}]^2} \quad (3-3)$$

for designing a linear equalizer to recover one of the inputs. It is shown [6] that when (3-3) is maximized w.r.t. $\mathcal{C}(z)$, then (3-2) reduces to

$$e(k) \;=\; d w_{j_0}(k-k_0), \quad (3-4)$$

where d is some complex constant, k_0 is some integer, j_0 indexes some input out of the given M inputs.

An source-iterative solution is given by [8]:

Step 1. Maximize (3-3) w.r.t. $\mathcal{C}(z)$ to obtain (3-4).

Step 2. Cross-correlate $\{e(k)\}$ (of (3-4)) with the given data (2-2) and define a possibly scaled and shifted estimate of $f_{ij_0}(\tau)$ as

$$\widehat{f}_{ij_0}(\tau) \;:=\; \frac{E\{y_i(k)e^*(k-\tau)\}}{E\{|e(k)|^2\}} \quad (3-5)$$

where $F_{ij}(z) = \sum_{l=-\infty}^{\infty} f_{ij}(l) z^{-l}$. Consider now the reconstructed contribution of $e(k)$ to the data $y_i(k)$ $(i = 1, 2, \cdots, N)$, denoted by $\widetilde{y}_{i,j_0}(k)$:

$$\widetilde{y}_{i,j_0}(k) := \sum_l \widehat{f}_{ij_0}(l) e(k-l). \quad (3-6)$$

Step 3. Remove the above contribution from the data to define the outputs of a MIMO system with N outputs and $M-1$ inputs. These are given by

$$y_i'(k) \;:=\; y_i(k) - \widetilde{y}_{i,j_0}(k). \quad (3-7)$$

Step 4. If $M > 1$, set $M \leftarrow M-1$, $y_i(k) \leftarrow y_i'(k)$, and go back to **Step 1**, else quit.

It has been shown in [6],[8] that

$$\widetilde{y}_{i,j_0}(k) = \sum_l f_{ij_0}(l) w_{j_0}(k-l), \quad (3-8)$$

i.e., we have decomposed the observations at the various sensors into its independent components: $\widetilde{y}_{i,j_0}(k)$ in (3-8) represents the contribution of $\{w_{j_0}(k)\}$ to the $i-$th sensor achieving **blind signal separation**. It has been shown in [6] that under the conditions (**AS1**)-(**AS4**) and no noise, the proposed iterative approach is capable of blind identification of a MIMO transfer function $\mathcal{F}(z)$ up to a time-shift, a scaling and a permutation matrix provided that we allow doubly-infinite equalizers.

4. ADAPTIVE ALGORITHM

In this section we develop a stochastic gradient-based "recursification" of all of the batch optimization steps discussed in Sec. 3. We will use the superscript (m) to denote the various quantities pertaining to stage m of the batch algorithm of Sec. 3 (i.e. m-th execution of Steps 1-4 therein). Let the length of the equalizer $\mathcal{C}(z)$ be L_e and let

$$\mathcal{C}_i(z) \;=\; \sum_{l=0}^{L_e-1} c_i(l) z^{-l}. \quad (4-1)$$

Initialization: Define

$$Y_i(k) = [\; y_i(k) \quad \cdots \quad y_i(k-L_e+1) \;]^T, \quad (4-2)$$

$$\mathbf{Y}^{(1)}(k) = \left[\; Y_1^T(k) \quad \cdots \quad Y_N^T(k) \;\right]^T, \quad (4-3)$$

$$\mathbf{y}^{(1)}(k) \;=\; \mathbf{y}(k). \quad (4-4)$$

DO FOR $m = 1, 2, \cdots, M$**:**

$$\widetilde{\mathbf{C}}^{(m)}(k) \;=\; \mathbf{C}^{(m)}(k-1) \;+\; \mu_1 \; \nabla_{\mathbf{C}^*} \, J_k^{(m)}(\mathbf{C}^{(m)}(k-1)) \quad (4-5)$$

$$\mathbf{C}^{(m)}(k) \;=\; \widetilde{\mathbf{C}}^{(m)}(k)/\|\widetilde{\mathbf{C}}^{(m)}(k)\| \quad (4-6)$$

where

$$\begin{aligned}\nabla_{\mathbf{C}^*} J_k^{(m)}(\mathbf{C}^{(m)}(k)) \;&=\; \mathrm{sgn}(\gamma_{4k}^{(m)}) \frac{2}{m_{2k}^{(m)3}} \\ &\times \left\{ \left[m_{2k}^{(m)} \left(e^{(m)2}(k) - \widetilde{m}_{2k}^{(m)} \right) e^{(m)*}(k) \right.\right. \\ &\left.\left. - \left(m_{4k}^{(m)} - |\widetilde{m}_{2k}^{(m)}|^2 \right) e^{(m)}(k) \right] \mathbf{Y}^{(m)*}(k) \right\}, \end{aligned} \quad (4-7)$$

$$m_{2k}^{(m)} \;=\; (1-\mu_2) m_{2(k-1)}^{(m)} \;+\; \mu_2 |e^{(m)}(k)|^2, \quad (4-8)$$

$$\widetilde{m}_{2k}^{(m)} \;=\; (1-\mu_2) \widetilde{m}_{2(k-1)}^{(m)} \;+\; \mu_2 e^{(m)2}(k), \quad (4-9)$$

$$m_{4k}^{(m)} \;=\; (1-\mu_2) m_{4(k-1)}^{(m)} \;+\; \mu_2 |e^{(m)}(k)|^4, \quad (4-10)$$

$$\gamma_{4k}^{(m)} \;=\; m_{4k}^{(m)} - 2\, m_{2k}^{(m)2} - |\widetilde{m}_{2k}^{(m)}|^2 \quad (4-11)$$

and

$$e^{(m)}(k) \;=\; \mathbf{C}^{(m)T}(k) \mathbf{Y}^{(m)}(k). \quad (4-12)$$

Set

$$\widehat{\widetilde{\mathbf{y}}}^{(m)}(k) \;=\; \sum_{n=-L_1}^{L_2} \widetilde{\mathbf{F}}_n^{(m)}(k) e^{(m)}(k-n) \quad (4-13)$$

where $\widehat{\widetilde{\mathbf{y}}}^{(m)}(k)$ represents (cf. (3-6)) the contribution of the extracted source at the $m-$th stage to the measurements at time k, and where $(n = -L_1, -L_1+1, \cdots, L_2)$

$$\widetilde{\mathbf{F}}_n^{(m)}(k) \;=\; \mathbf{R}_n^{(m)}(k)/m_{ee}^{(m)}(k), \quad (4-14)$$

$$m_{ee}^{(m)}(k) \;=\; (1-\mu_3) m_{ee}^{(m)}(k-1) \;+\; \mu_3 |e^{(m)}(k)|^2, \quad (4-15)$$

$$\mathbf{R}_n^{(m)}(k) \;=\; (1-\mu_3)\mathbf{R}_n^{(m)}(k-1) \;+\; \mu_3 \mathbf{y}^{(m)}(k) e^{(m)*}(k-n) \quad (4-16)$$

and

$$\mathbf{y}^{(m+1)}(k) \;=\; \mathbf{y}^{(m)}(k) - \widehat{\widetilde{\mathbf{y}}}^{(m)}(k). \quad (4-17)$$

Define

$$\widetilde{Y}_i^{(m)}(k) = \left[\; \widetilde{y}_i^{(m)}(k) \quad \cdots \quad \widetilde{y}_i^{(m)}(k-L_e+1) \;\right]^T \quad (4-18)$$

where $\widetilde{y}_i^{(m)}(k)$ denotes the $i-$th component of $\widetilde{\mathbf{y}}^{(m)}(k)$. Set

$$\mathbf{Y}^{(m+1)}(k) = \left[\begin{array}{ccc} Y_1^{(m+1)T}(k) & \cdots & Y_N^{(m+1)T}(k) \end{array} \right]^T \quad (4-19)$$

where

$$Y_i^{(m+1)}(k) \;=\; Y_i^{(m)}(k) \;-\; \widetilde{Y}_i^{(m)}(k). \qquad (4-20)$$

ENDDO

The sequence $\{\widehat{\widetilde{\mathbf{y}}}^{(m)}(k)\}$ in (4-13) represents the contribution of the extracted source at the $m-$th stage to the measurements at time k. Variable $e^{(m)}(k)$ in (4-12) corresponds to (3-2), $J_k^{(m)}$ in (4-5) corresponds to (3-3), and $\nabla_{\mathbf{C}^*} J_k^{(m)}$ is the instantaneous gradient, all at time k and stage m. In (4-5) μ_1 is the update step-size and in (4-8)-(4-10) and (4-15)-(4-16), μ_2 and μ_3, respectively, are the forgetting factors (> 0, < 1).

Running Cost. To monitor the convergence of the equalizers in various stages of the algorithm, it is useful to calculate a running cost with the sign. Let $\tilde{J}_k^{(m)}$ denote the running cost for the $m-$th stage at time k, given by

$$\tilde{J}_k^{(m)} \;=\; \frac{m_{4k}^{(m)} - |\widetilde{m}_{2k}^{(m)}|^2}{m_{2k}^{(m)2}} - 2 \qquad (4-21)$$

where $m_{2k}^{(m)}$, $\widetilde{m}_{2k}^{(m)}$ and $m_{4k}^{(m)}$ are computed as in (4-8)-(4-10) but with a smaller μ_2.

5. FURTHER MODIFICATIONS

5.1. MMSE Signal Separation

5.1.1. Non-recursive Processing

A by-product of the solutions of Secs. 3 and 4 is the estimates of the system/channel impulse response. These estimates can be used to design MMSE estimators of $\mathcal{F}^{(i)}(z)w_i(k)$ with a controlled delay d to obtain an "optimum" performance (ignoring any effects of additive noise on the channel estimates). Let $\mathbf{F}_l^{(i)}$ denote the $i-$th column of $\mathbf{F}_l$. We wish to design a linear MMSE filter (equalizer) of length L_e+1 to estimate $\widetilde{\mathbf{y}}^{(j)}(k-d)$ as $\widehat{\widetilde{\mathbf{y}}}^{(j)}(k-d)$ given $\mathbf{y}(l)$ for $l = k, k-1, \cdots, k-L_e+1$ where $d \geq 0$,

$$\widetilde{\mathbf{y}}^{(j)}(k) \;:=\; \mathcal{F}^{(j)}(z)w_j(k) \;=\; \sum_{l=0}^{L} \mathbf{F}_l^{(j)} w_j(k-l), \quad (5-1)$$

$$\widehat{\widetilde{\mathbf{y}}}^{(j)}(k-d) \;:=\; \sum_{i=0}^{L_e-1} \mathbf{G}_i \mathbf{y}(k-i). \qquad (5-2)$$

Using the orthogonality principle, the desired solution is given by

$$\left[\begin{array}{ccc} \mathbf{G}_0 & \cdots & \mathbf{G}_{L_e-1} \end{array} \right] = \sigma_{wj}^2 \left[\begin{array}{ccc} \mathbf{H}_d & \cdots & \mathbf{H}_{d-L_e} \end{array} \right] \mathcal{R}_{yy}^{-1} \quad (5-3)$$

where $\mathcal{R}_{yy}$ denotes a $[NL_e] \times [NL_e]$ correlation matrix with $\mathbf{R}_{yy}(j-i)$ as its ij-th block element,

$$\mathbf{R}_{yy}(p) := E\{\mathbf{y}(t+p)\mathbf{y}^{\mathcal{H}}(t)\}, \quad \mathbf{H}_{d-p} := \sum_{k=0}^{L} \mathbf{F}_k^{(j)} \mathbf{F}_{k+d-p}^{(j)\mathcal{H}}. \quad (5-4)$$

In practice, we replace all the unknowns by their estimates. Also we design the equalizer only up to a scale factor by omitting σ_{wj}^2 from (5-3).

Remark 1. *Selection of Delay d:* In designing (5-2) the delay d was pre-determined. One may choose to select d via exhaustive optimization as detailed below. The MMSE when (5-2) is used can be expressed as

$$\mathcal{J}(d) = \mathrm{tr} E\left\{\widetilde{\mathbf{y}}^{(j)}(k-d)\widetilde{\mathbf{y}}^{(j)\mathcal{H}}(k-d)\right\} \;-\; \mathcal{J}'(d) \quad (5-5)$$

where

$$\mathcal{J}'(d) \;:=\; \sigma_{wj}^4 \;\mathrm{tr}\, \mathcal{H}\mathcal{R}_{yy}^{-1}\mathcal{H}^{\mathcal{H}}, \qquad (5-6)$$

$$\mathcal{H} \;:=\; \left[\begin{array}{cccc} \mathbf{H}_d & \mathbf{H}_{d-1} & \cdots & \mathbf{H}_{d-L_e} \end{array} \right]. \qquad (5-7)$$

Since the first term on the right-side of (5-5) is independent of d, minimizing $\mathcal{J}(d)$ w.r.t. d is equivalent to maximizing $\sigma_{wj}^{-4}\mathcal{J}'(d)$. $\square$

5.1.2. Adaptive Implementation

Note that $\mathcal{R}_{yy}^{-1}$ does not depend upon the stage m of the algorithm of Sec. 4. Its computation can easily be recursified by using the matrix inversion lemma: see Table 13.1 on p. 569 in [13]. Denote the data-based adaptive estimate of $\mathcal{R}_{yy}^{-1}$ at time k as $\mathcal{P}_{yy}(k)$. Let $\mathbf{H}_l^{(m)}(k)$ denote the estimate of $\mathbf{H}_l$ at stage m and time k of the multistage algorithm of Sec. 4. Note that $\widetilde{\mathbf{F}}_n^{(m)}(k)$ in (4-14) (see also (3-5)) denotes an estimate of $\mathbf{F}_n^{(i)}$ for some $i \in \{1, 2, \cdots, M\}$ (up to a scale factor and time shift). Therefore, from (5-2) and (5-5) we obtain the adaptive implementation at stage m; details are omitted.

5.2. Adaptive Filter Reinitialization

In the source-iterative (multistage) approaches of Secs. 3 and 4, any errors in cancelling the extracted sources from the preceding stages $l = 1, 2, \cdots, m-1$ affect the performance at stage m. The only stage that is immune to this phenomenon is stage $m = 1$. A possible solution to alleviate this error propagation from stage-to-stage is to use parallel stages where we still have M stages for M sources but they all operate directly on the given data record in parallel but with different initializations of the equalizers. The problem here is how to ensure that each stage converges to a distinct source. Here we propose to initialize the parallel stages using the results of the serial multistage implementation of Sec. 4 coupled with an MMSE solution similar to that of Sec. 5.1. For stage $m = 1$, there are no changes to the algorithm of Sec. 4. For stages $m \geq 2$, run the algorithm of Sec. 4 till the running cost (4-21) reaches a steady-state. Given the estimates of the subchannel impulse response at stage m, we can design an MMSE filter (in a fashion similar to Sec. 5.1.2) to estimate $w_j(k-d)$ given $\mathbf{y}(l)$ for $l = k, k-1, \cdots, k-L_e+1$. Let the extracted $w_j(k)$ at stage m be denoted by $w^{(m)}(k)$. Mimicking Sec. 5.1.2, a recursive MMSE solution at stage m and time k is given by

$$\widehat{w}^{(m)}(k-d) \;:=\; \sum_{i=0}^{L_e-1} \overline{\mathbf{G}}_i^{(m)}(k)\mathbf{y}(k-i) \qquad (5-8)$$

where

$$\left[\begin{array}{cccc} \overline{\mathbf{G}}_0^{(m)}(k) & \overline{\mathbf{G}}_1^{(m)}(k) & \cdots & \overline{\mathbf{G}}_{L_e-1}^{(m)}(k) \end{array} \right]$$
$$= \left[\begin{array}{cccccc} \widetilde{\mathbf{F}}_d^{(m)\mathcal{H}}(k) & \cdots & \widetilde{\mathbf{F}}_0^{(m)\mathcal{H}}(k) & 0 & \cdots & 0 \end{array} \right] \mathcal{P}_{yy}(k). \quad (5-9)$$

At stage m and time k, $\widehat{w}^{(m)}(k-d)$ is an MMSE estimate (with delay d) of $e^{(m)}(k)$ for the parallel implementation. Note that $\mathcal{C}(z) = \sum_{i=0}^{L_e-1} \overline{\mathbf{G}}_i^{(m)}(k) z^{-i}$ is the desired MMSE initializer.

6. SIMULATION EXAMPLE

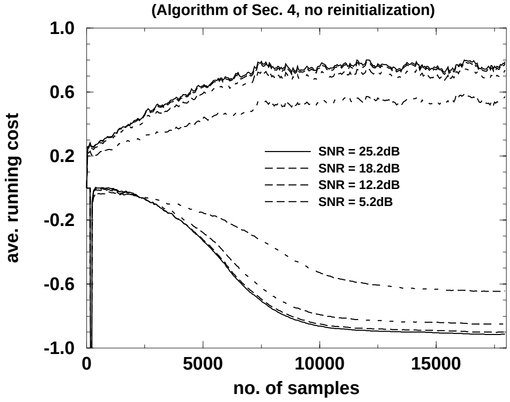
Take N=3 and M=2 in (2-2) with

$$\mathcal{F}^{(1)}(z) = \begin{bmatrix} 0.2 + 0.8z^{-1} + 0.4z^{-2} \\ 0.3z^{-1} - 0.6z^{-2} \\ 0. \end{bmatrix},$$

$$\mathcal{F}^{(2)}(z) = \begin{bmatrix} 0.5 - 0.3z^{-1} \\ -0.21z^{-1} - 0.5z^{-2} + 0.72z^{-3} + 0.36z^{-4} + 0.21z^{-6} \\ 0. \end{bmatrix}$$

Fig. 1.

(Algorithm of Sec. 4, no reinitialization)



The input $\{w_1(k)\}$ is an i.i.d. complex Gaussian-mixture with 4th normalized cumulant as 0.7433 . The input $\{w_2(k)\}$ is an i.i.d. 4-QAM sequence with 4th normalized cumulant as -1. The additive noise is white, complex Gaussian. The powers of $\{w_j(k)\}$ were scaled so as to have $E\{\|\mathcal{F}^{(1)}(z)w_1(k)\|^2\} = E\{\|\mathcal{F}^{(2)}(z)w_2(k)\|^2\}$. The performance measure was taken to be the signal-to-interference-and-noise ratio (SINR) per source signal, defined as

$$\mathrm{SINR}_j = \frac{E\{\|\widetilde{\mathbf{y}}^{(j)}(k)\|^2}{E\{\|\widetilde{\mathbf{y}}^{(j)}(k) - \widehat{\alpha}\widehat{\widetilde{\mathbf{y}}}^{(j)}(k)\|^2\}} \qquad (6-1)$$

where $\widehat{\alpha}$ is that value of the scalar α which minimizes $E\{\|\widetilde{\mathbf{y}}^{(j)}(k) - \alpha\widehat{\widetilde{\mathbf{y}}}^{(j)}(k)\|^2\}$. The length of the inverse filters was 11 samples per sensor (output) for the approach of Sec. 4. The initial guess for the tap gains was: set $c_i(5) = 1$ for $i = m$ for the $m-$th stage equalizer ($m = 1, 2$) with the remaining tap gains set to zero. The algorithm step sizes and forgetting factors for each stage m were chosen as: $\mu_1 = 0.0005$, $\mu_2 = 0.015$ and $\mu_3 = 0.0005$ when $\gamma_{4k}^{(m)} \leq 0$ (see (4-11)), and $\mu_1 = 0.00025$, $\mu_2 = 0.0075$ and $\mu_3 = 0.0005$ when $\gamma_{4k}^{(m)} > 0$. For the running cost (4-21) computation we selected "μ_2"=0.002 in (4-8)-(4-10). The parameters L_1 and L_2 in (4-13) were selected as $L_1 = 15$ and $L_2 = 6$. To design the MMSE equalizers/filters we took $L_e = 11$ and d was optimized following Remark 1 of Sec. 5.1.1 over the range $[-15, 6]$.

Fig. 1 shows the evolution of the average running cost $J_k^{(m)}$ (see (4-21)), averaged over 100 Monte Carlo runs (after 'assigning' each equalizer cost to its corresponding extracted source) without using any filter reinitialization. Fig. 2 shows $J_k^{(m)}$ when reinitialization (after 12000 samples) of Sec. 5.2 is used. It turns out that source 1 ($w_1(k)$) is extracted first, so that reinitialization only affects source 2 (4-QAM). Table I shows the average SINR (based on 100 runs) for the two sources at the end of the run (i.e. at $k = 18000$) without and with filter reinitialization, for various SNR's. The SINR's were computed using the solution (4-13) as well as the MMSE solution of Sec. 5.1.2. It is seen that blind signal separation benefits from both, MMSE signal separation as well as filter reinitialization.

Fig. 2.

(Algorithm of Sec. 4 with reinitialization of Sec. 5.2)

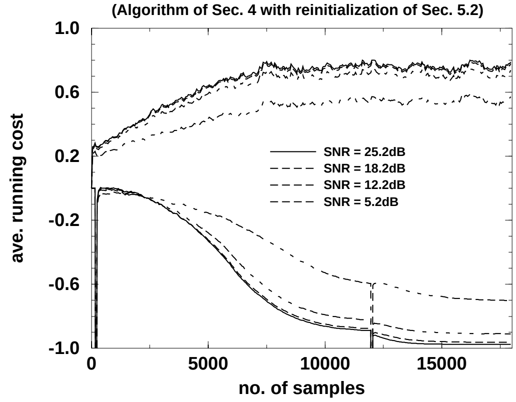


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TABLE I. Average SINR after blind separation. Serial: Algorithm of Sec. 4; Parallel: Algorithm of Sec. 4 + reinitialization of Sec. 5.2.

SNR	SOURCE 1 (Gaussian mixture)			
	serial		parallel	
	(4-13)	MMSE	(4-13)	MMSE
25.2 dB	8.653	10.667	8.653	10.667
18.2 dB	8.447	10.317	8.447	10.317
12.2 dB	7.807	9.253	7.807	9.253
5.2 dB	5.893	6.511	5.893	6.511

SNR	SOURCE 2 (4-QAM)			
	serial		parallel	
	(4-13)	MMSE	(4-13)	MMSE
25.2 dB	11.621	12.647	16.123	15.271
18.2 dB	11.198	12.134	15.078	14.351
12.2 dB	9.876	10.591	12.445	12.070
5.2 dB	6.505	6.862	7.746	7.626