BLIND EQUALIZATION OF I.I.R. SINGLE-INPUT MULTIPLE-OUTPUT CHANNELS WITH COMMON ZEROS USING SECOND-ORDER STATISTICS

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ABSTRACT

The problem of blind equalization of SIMO (single-input multiple-output) communications channels is considered using only the second-order statistics of the data. Such models arise when a single receiver data is fractionally sampled (assuming that there is excess bandwidth), or when an antenna array is used with or without fractional sampling. We focus on direct design of finite-length MMSE (minimum mean-square error) blind equalizers. Unlike the past work on this problem, we allow infinite impulse response (IIR) channels. Our approaches also work when the "subchannel" transfer functions have common zeros so long as the common zeros are minimum-phase zeros. Illustrative simulation examples are provided.

1. INTRODUCTION

Consider a discrete-time SIMO system with N outputs and one input. The i-th component of the output at time k is given by

$$y_i(k) \;=\; \mathcal{F}_i(z)w(k) + n_i(k)\,, \;\; i=1,2,\cdots,N, \qquad (1-1)$$

$$\Longrightarrow \quad \mathbf{y}(k) \;=\; \mathcal{F}(z)w(k) + \mathbf{n}(k) \;=\; \mathbf{s}(k) + \mathbf{n}(k), \; (1-2)$$

where $\mathbf{y}(k) = [y_1(k) \vdots y_2(k) \vdots \cdots \vdots y_N(k)]^T$, similarly for $\mathbf{s}(k)$ and $\mathbf{n}(k)$, and z is the $\mathcal{Z}-$transform variable as well as the backward-shift operator (i.e., $z^{-1}w(k) = w(k-1)$, etc.). The sequence $w(k)$ is the (single) input at sampling time k, $y_i(k)$ is the i-th noisy output, $s_i(k)$ is the i-th noise-free output, $n_i(k)$ is the additive measurement noise,

$$\mathcal{F}(z) \;:=\; \sum_{l=0}^{\infty} \mathbf{F}_l z^{-l} \qquad (1-3)$$

and $\mathcal{F}_i(z) = \sum_{l=0}^{\infty} f_i(l)z^{-l}$ is the scalar transfer function with $w(k)$ as the input and $y_i(k)$ as the output; it represents the $i-$th subchannel. We allow all of the above variables to be complex-valued.

Such models arise in several useful baseband-equivalent digital communications and other applications. A case of some interest is that of fractionally-spaced samples of a single baseband received signal leading to a SIMO model [1],[4],[8]. Alternatively, a similar model can be derived when we have a single signal impinging upon an antenna array with N elements [5]. A similar model arises if we have an antenna array coupled with fractional sampling at each array-element [5]. In these applications one of the objectives is to recover the inputs $w(k)$ given the noisy measurements but not given the knowledge of the system transfer function. An overwhelming number of papers (see [4],[5],[9]-[12] and references therein) have concentrated on a two-step procedure: first estimate the channel impulse response (IR) and then design an equalizer using the estimated channel. A fundamental restriction in these works is that the channel is FIR with no common zeros among the various subchannels. A few (see [1]and [13], e.g.) have proposed direct design of the equalizer bypassing channel estimation. Still they assume FIR channels with no common zeros.

In this paper we allow IIR channels. We will also allow common zeros so long as they are minimum-phase. Finally, in the presence of nonminimum-phase common zeros, our proposed approach equalizes the spectrally-equivalent minimum-phase counterpart of $\mathcal{F}(z)$; it does not "fall apart" unlike quite a few existing approaches. We should note that our proposed approach is inspired by [1]. Unlike [1] our approach applies to antenna arrays since we do not require that $f_1(0) \neq 0$ but $f_i(0) = 0$ for $i = 2, 3, \cdots, N$ (as in [1]).

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2. PRELIMINARIES

2.1. FIR Inverses

Let $\mathcal{F}(z) = \mathcal{A}^{-1}(z)\mathcal{B}(z)$ where $\mathcal{A}(z) = 1 + \sum_{i=1}^{n_a} a_i z^{-i}$ is 1×1 and $\mathcal{B}(z) = \sum_{i=0}^{n_b} \mathbf{B}_i z^{-i}$ is $N \times 1$. Assume

- **(H1)** $N > 1$.
- **(H2)** Rank$\{\mathcal{B}(z)\} = 1$ $\forall z$ including $z = \infty$ but excluding $z = 0$, i.e., $\mathcal{B}(z)$ is irreducible [7, Sec. 6.3].
- **(H3)** $\mathcal{A}(z) \neq 0$ for $|z| \geq 1$.

It has been shown in [6] (using some results from [2]) that under **(H1)-(H3)** there exists a finite degree left-inverse (not necessarily unique) of $\mathcal{F}(z)$:

$$\mathcal{G}(z)\mathcal{F}(z) \;=\; 1 \qquad (2-1)$$

where $\mathcal{G}(z)$ is $1 \times N$ given by

$$\mathcal{G}(z) \;=\; \sum_{l=0}^{L_e} \mathbf{G}_l z^{-l} \;\text{ for any }\; L_e \geq n_a + n_b - 1. \quad (2-2)$$

Remark 1: The left-inverse $\mathcal{G}(z)$ of $\mathcal{F}(z)$ consists of two parts: $\mathcal{G}(z) = \mathcal{G}_B(z)\mathcal{A}(z)$ where $\mathcal{G}_B(z)\mathcal{B}(z) = 1$ so that $\mathcal{G}(z)\mathcal{F}(z) = \mathcal{G}_B(z)\mathcal{A}(z)\mathcal{A}^{-1}(z)\mathcal{B}(z) = \mathcal{G}_B(z)\mathcal{B}(z) =1$. Finite length left-inverses of FIR SIMO channels have been subject of intense research activities [4]-[6],[8]-[13].

2.2. Linear Innovations Representations

Assume further the following:

- **(H4)** $\{w(k)\}$ is zero-mean, white. Take $E\{|w(k)|^2\} = 1$.

Lemma 1. Under **(H1)**-**(H4)**, $\{\mathbf{s}(k)\}$ may be represented as

$$\mathbf{s}(k) \;=\; -\sum_{i=1}^{M} \mathbf{D}_i \mathbf{s}(k-i) \;+\; I_s(k) \qquad (2-3)$$

where $M = n_a + n_b - 1$, $\mathbf{D}_i$'s are some $N \times N$ matrices such that $\det(\mathcal{D}(z)) \neq 0$ for $|z| \geq 1$, $\mathcal{D}(z) = I + \sum_{i=1}^{M} \mathbf{D}_i z^{-i}$ and $\{I_s(k)\}$ is a zero-mean white $N \times 1$ random sequence (linear innovations for $\{\mathbf{s}(k)\}$) with

$$E\{I_s(k)I_s^{\mathcal{H}}(k)\} = \mathbf{F}_0\mathbf{F}_0^{\mathcal{H}} \;\text{ and }\; \|\mathbf{F}_0\|^{-2}\mathbf{F}_0^{\mathcal{H}} I_s(k) = w(k). \qquad (2-4)$$

Proof: Consider the process

$$\mathbf{s}'(k) := \mathcal{A}(z)\mathbf{s}(k) = \mathcal{B}(z)w(k). \qquad (2-5)$$

By [9] and [14], under (**H1**), (**H2**) and (**H4**), we have

$$\mathbf{s}'(k) = -\sum_{i=1}^{n_b-1} \mathbf{D}_i'\mathbf{s}'(k-i) + I_s'(k) \qquad (2-6)$$

where $\mathbf{D}_i'$s are some $N \times N$ matrices such that $\det(\mathcal{D}'(z)) \neq 0$ for $|z| \geq 1$, $\mathcal{D}'(z) = I + \sum_{i=1}^{M} \mathbf{D}_i' z^{-i}$ and $\{I_s'(k)\}$ is a zero-mean white $N \times 1$ random sequence with

$$E\{I_s'(k)I_s'^{\mathcal{H}}(k)\} = \mathbf{F}_0\mathbf{F}_0^{\mathcal{H}} \text{ and } \|\mathbf{F}_0\|^{-2}\mathbf{F}_0^{\mathcal{H}} I_s'(k) = w(k). \qquad (2-7)$$

Since $\mathbf{s}(k) = \mathcal{A}^{-1}(z)\mathbf{s}'(k)$, it follows from (2-6) that (2-3) holds true with $I_s(k) \equiv I_s'(k)$ such that $\mathcal{D}(z) = \mathcal{A}(z)\mathcal{D}'(z)$. This completes the proof. □

Lemma 2. Let $\mathcal{R}_{ssL_e}$ denote a $[N(L_e+1)] \times [N(L_e+1)]$ matrix with its ij-th block element as $\mathbf{R}_{ss}(j-i) = E\{\mathbf{s}(k+j-i)\mathbf{s}^{\mathcal{H}}(k)\}$. Then under (**H1**)-(**H4**), $\rho(\mathcal{R}_{ssL_e}) \leq NL_e + 1$ for $L_e \geq n_a + n_b - 1$ where $\rho(A)$ denotes the rank of A. •

Sketch of proof: It follows from Lemma 1 and (2-3) that

$$\begin{bmatrix} I & \mathbf{D}_1 & \cdots & \mathbf{D}_{n_a+n_b-1} & 0 & \cdots & 0 \end{bmatrix} \mathcal{R}_{ssL_e} = \begin{bmatrix} \mathbf{F}_0\mathbf{F}_0^{\mathcal{H}} & 0 & \cdots & 0 \end{bmatrix}. \qquad (2-8)$$

Apply Sylvester's inequality [7, p. 655] to (2-8) to deduce the desired result. □

3. BLIND EQUALIZATION: NO COMMON ZEROS

Assume that (**H1**)-(**H4**) hold true. In addition assume the following regarding the measurement noise:

(**H5**) $\{\mathbf{n}(k)\}$ is zero-mean with $E\{\mathbf{n}(k+\tau)\mathbf{n}^{\mathcal{H}}(k)\} = \sigma_n^2 I_{N\times N}$ where $I_{N\times N}$ is the $N \times N$ identity matrix.

3.1. Zero-Delay Zero-Forcing Equalizer

Using (1-3), (2-1) and (2-2), we have

$$\sum_{l=0}^{\infty} \mathbf{G}_{m-l}\mathbf{F}_l = \begin{cases} 1, & m = 0 \\ 0, & m = 1, 2, \cdots, \end{cases} \qquad (3-1)$$

leading to

$$\begin{bmatrix} \mathbf{G}_0 & \mathbf{G}_1 & \cdots & \mathbf{G}_{L_e} \end{bmatrix} \overline{\mathcal{S}} = \begin{bmatrix} 1 & 0 & \cdots & \cdots \end{bmatrix} \qquad (3-2)$$

where $\overline{\mathcal{S}}$ is the $(N(L_e+1)) \times \infty$ matrix given by

$$\overline{\mathcal{S}} = \begin{bmatrix} \mathbf{F}_0 & \mathbf{F}_1 & \mathbf{F}_2 & \mathbf{F}_3 & \cdots & \cdots & \cdots \\ 0 & \mathbf{F}_0 & \mathbf{F}_1 & \mathbf{F}_2 & \cdots & \cdots & \cdots \\ \vdots & \vdots & \vdots & \vdots & & & \\ 0 & 0 & \cdots & 0 & \mathbf{F}_0 & \mathbf{F}_1 & \cdots \end{bmatrix}. \qquad (3-3)$$

Let $\overline{\mathcal{S}}^{\#}$ denote the pseudoinverse of $\overline{\mathcal{S}}$. By [15, Prop. 1], $\overline{\mathcal{S}}^{\#} = \overline{\mathcal{S}}^{\mathcal{H}}(\overline{\mathcal{S}}\,\overline{\mathcal{S}}^{\mathcal{H}})^{\#}$. Then the minimum norm solution to the FIR equalizer is given by [15, Sec. 6.11]

$$\begin{bmatrix} \mathbf{G}_0 & \mathbf{G}_1 & \cdots & \mathbf{G}_{L_e} \end{bmatrix} = \begin{bmatrix} \mathbf{F}_0^{\mathcal{H}} & 0 & \cdots & 0 \end{bmatrix} (\overline{\mathcal{S}}\,\overline{\mathcal{S}}^{\mathcal{H}})^{\#}. \qquad (3-4)$$

In a fashion similar to $\mathcal{R}_{ssL_e}$ in Lemma 2, let $\mathcal{R}_{yyL_e}$ denote a $[N(L_e+1)] \times [N(L_e+1)]$ matrix with its ij-th block element as $\mathbf{R}_{yy}(j-i) = E\{\mathbf{y}(k+j-i)\mathbf{y}^{\mathcal{H}}(k)\}$; define similarly $\mathcal{R}_{nnL_e}$ pertaining to the additive noise. Carry out an eigendecomposition of $\mathcal{R}_{yyL_e}$. Then the smallest $N-1$ eigenvalues of $\mathcal{R}_{yyL_e}$ equal σ_n^2 because under (**H1**)-(**H4**), $\rho(\mathcal{R}_{ssL_e}) \leq NL_e + 1$ whereas $\rho(\mathcal{R}_{nnL_e}) = NL_e + N = \rho(\mathcal{R}_{yyL_e})$. Thus a consistent estimate $\widehat{\sigma}_n^2$ of σ_n^2 is obtained by taking it as the average of the smallest $N-1$ eigenvalues of $\widehat{\mathcal{R}}_{yyL_e}$, the data-based consistent estimate of $\mathcal{R}_{yyL_e}$.

Under (**H4**) and (**H5**),

$$(\overline{\mathcal{S}}\,\overline{\mathcal{S}}^{\mathcal{H}}) = \mathcal{R}_{ssL_e} = \mathcal{R}_{yyL_e} - \mathcal{R}_{nnL_e} = \mathcal{R}_{yyL_e} - \sigma_n^2 I. \qquad (3-5)$$

Thus, $(\overline{\mathcal{S}}\,\overline{\mathcal{S}}^{\mathcal{H}})$ can be estimated from noisy data. However, we don't know $\mathbf{F}_0$. To this end, we seek an $N \times N$ FIR filter $\mathcal{G}_a(z) := \sum_{i=0}^{L_e} \mathbf{G}_{ai} z^{-i}$ satisfying

$$\begin{bmatrix} \mathbf{G}_{a0} & \mathbf{G}_{a1} & \cdots & \mathbf{G}_{aL_e} \end{bmatrix} = \begin{bmatrix} I_{N\times N} & 0 & \cdots & 0 \end{bmatrix} \mathcal{R}_{ssL_e}^{\#}. \qquad (3-6)$$

Comparing (3-4) and (3-6) it follows that

$$\begin{bmatrix} \mathbf{G}_0 & \mathbf{G}_1 & \cdots & \mathbf{G}_{L_e} \end{bmatrix} = \mathbf{F}_0^{\mathcal{H}} \begin{bmatrix} \mathbf{G}_{a0} & \mathbf{G}_{a1} & \cdots & \mathbf{G}_{aL_e} \end{bmatrix} \qquad (3-7)$$

leading to

$$\sum_{i=0}^{L_e} \mathbf{G}_i z^{-i} =: \mathcal{G}(z) = \mathbf{F}_0^{\mathcal{H}} \mathcal{G}_a(z). \qquad (3-8)$$

In practice, therefore, we apply $\mathcal{G}_a(z)$ to the data leading to

$$\mathbf{v}(k) := \mathcal{G}_a(z)\mathbf{y}(k) = \mathbf{v}_s(k) + \mathcal{G}_a(z)\mathbf{n}(k) \qquad (3-9)$$

such that

$$\mathbf{F}_0^{\mathcal{H}}\mathbf{v}_s(k) = w(k) \qquad (3-10)$$

where

$$\mathbf{v}_s(k) := \mathcal{G}_a(z)[\mathbf{y}(k) - \mathbf{n}(k)] = \mathcal{G}_a(z)\mathbf{s}(k). \qquad (3-11)$$

In (3-10) $\{w(k)\}$ is a white scalar sequence (by assumption (**H4**)), however, $\{\mathbf{v}_s(k)\}$ is not necessarily a white vector sequence. Given the second-order statistics of $\{\mathbf{v}_s(k)\}$, how does one estimate $\mathbf{F}_0$ so that $\{w(k)\}$ satisfying (**H4**) is recovered? We need to have $R_{ww}(\tau) := E\{w(k+\tau)w^*(k)\} = 0$ for $|\tau| \neq 0$. By (3-9), $R_{ww}(\tau) = \mathbf{F}_0^{\mathcal{H}} R_{v_sv_s}(\tau)\mathbf{F}_0$. Define ($L > 0$ is some large integer)

$$\overline{R}_{v_sv_s} := \begin{bmatrix} R_{v_sv_s}^T(-1) & R_{v_sv_s}^T(-2) & \cdots & R_{v_sv_s}^T(-L) \end{bmatrix}^T \qquad (3-12)$$

where $R_{v_sv_s}(\tau) := E\{\mathbf{v}_s(k+\tau)\mathbf{v}_s^{\mathcal{H}}(k)\}$.

Lemma 3. $\overline{R}_{v_sv_s}$ is rank deficient for any $L \geq 1$ such that $\overline{R}_{v_sv_s}\mathbf{F}_0 = 0$. •

Proof: We have

$$R_{wv_s}(\tau) = E\{w(k+\tau)\mathbf{v}_s^{\mathcal{H}}(k)\} = 0 \quad \forall \tau \geq 1 \qquad (3-13)$$

because $\mathbf{v}_s(k)$ is obtained by causal filtering of $\mathbf{y}(k)$, hence of $w(k)$. Using (3-10) in (3-13) it then follows that there exists a $N \times 1$ $\mathbf{F}_0 \neq 0$ such that $\mathbf{F}_0^{\mathcal{H}} R_{v_sv_s}(\tau) = 0$ $\forall \tau \geq 1$. Equivalently (since $R_{v_sv_s}(-\tau) = R_{v_sv_s}^{\mathcal{H}}(\tau)$)

$$R_{v_sv_s}(-\tau)\mathbf{F}_0 = 0 \quad \forall \tau \geq 1. \qquad (3-14)$$

The desired result is then immediate. □

Pick a $N \times 1$ column-vector $\mathbf{H}_0$ to equal the rightmost right singular vector in a singular-value decomposition

(SVD) $\overline{R}_{v_s v_s} = U\Sigma V^{\mathcal{H}}$, i.e. the right singular vector corresponding to the smallest singular value. In other words, pick $\mathbf{H}_0$ to equal the last column of V. Then since ideally the smallest singular value of $\overline{R}_{v_s v_s}$ is zero, we have $\mathbf{H}_0^{\mathcal{H}} R_{v_s v_s}(\tau)\mathbf{H}_0 = 0$ for $\tau = 1, 2, \cdots, L$. Since the overall system with $w(k)$ as input and $\mathbf{H}_0^{\mathcal{H}}\mathbf{v}_s(k)$ as output is ARMA$(n_a, n_b + L_e)$, it follows that $\mathbf{H}_0^{\mathcal{H}}\mathbf{v}_s(k)$ is zero-mean white if $L \geq n_b + L_e$, hence, a scaled version of $w(k)$. Therefore, we have ($\alpha \neq 0$)

$$\mathbf{H}_0^{\mathcal{H}}\mathbf{v}_s(k) \;=:\; w'(k) = \alpha w(k) \qquad (3-15)$$

(because $\overline{R}_{v_s v_s}\mathbf{H}_0 = 0$). Thus, once $\mathbf{H}_0$ is found, one has the complete inverse filter to recover a scaled version of $w(k)$ via a zero-forcing filter.

Remark 2: $\mathbf{F}_0$ can also be estimated (up to a scale factor as unit norm $\mathbf{H}_0$) using the prediction error method of [9],[14] (even though [9] and [14] restrict their discussion to FIR models and real-valued data). Using (2-3) we obtain ($L_e \geq n_a + n_b - 1$)

$$\left[\ \mathbf{D}_1 \quad \cdots \quad \mathbf{D}_{L_e}\ \right] \mathcal{R}_{ssL_e} = -\left[\ \mathbf{R}_{ss}(1) \quad \cdots \quad \mathbf{R}_{ss}(L_e)\ \right] \qquad (3-16)$$

leading to the minimum norm solution

$$\left[\ \mathbf{D}_1 \quad \cdots \quad \mathbf{D}_{L_e}\ \right] = -\left[\ \mathbf{R}_{ss}(1) \quad \cdots \quad \mathbf{R}_{ss}(L_e)\ \right] \mathcal{R}_{ssL_e}^{\#}. \qquad (3-17)$$

Note that if $L_e > n_a + n_b - 1$, then $\mathbf{D}_i = 0$ for all $i > n_a + n_b - 1$ by Lemma 2. By (2-3)–(2-4) we have

$$\mathbf{R}_{II}(0) = \mathbf{F}_0\mathbf{F}_0^{\mathcal{H}} = \mathbf{R}_{ss}(0) + \sum_{i=1}^{L_e} \mathbf{D}_i \mathbf{R}_{ss}(-i). \qquad (3-18)$$

Clearly $\rho(\mathbf{R}_{ss}(0)) = 1$. Carry out an eigendecomposition of $\mathbf{R}_{II}(0)$. Pick $\mathbf{H}_0$ as the unit norm eigenvector corresponding to the largest eigenvalue (ideally the only nonzero eigenvalue) of $\mathbf{R}_{II}(0)$. $\square$

Remark 3: It is worth noting that although $\mathbf{F}_0^{\mathcal{H}}\mathbf{v}_s(k) = w(k)$ (see (3-10)) and $\|\mathbf{F}_0\|^{-2}\mathbf{F}_0^{\mathcal{H}} I_s(k) = w(k)$ (see (2-4)), $\{I_s(k)\}$ is zero-mean white (linear innovations) whereas $\{\mathbf{v}_s(k)\}$ is in general colored. $\square$

3.2. MMSE Equalizer with Delay d

We wish to design an MMSE linear equalizer of a specified length. Using the orthogonality principle [16], the MMSE equalizer of length L_e+1 to estimate $w(k-d)$ ($d \geq 0$) based upon $\mathbf{y}(n)$, $n = k, k-1, \cdots, k-L_e$, satisfies

$$\left[\ \overline{\mathbf{G}}_{d,0} \quad \overline{\mathbf{G}}_{d,1} \quad \cdots \quad \overline{\mathbf{G}}_{d,L_e}\ \right] =$$
$$\left[\ \mathbf{F}_d^{\mathcal{H}} \quad \mathbf{F}_{d-1}^{\mathcal{H}} \quad \cdots \quad \mathbf{F}_0^{\mathcal{H}} \quad 0 \quad \cdots \quad 0\ \right] \mathcal{R}_{yyL_e}^{-1} \qquad (3-19)$$

where $\mathcal{R}_{yyL_e}$ has its $ij-$th block-element given by $\mathbf{R}_{yy}(j-i)$. Clearly one can obtain a consistent estimate of $\mathcal{R}_{yyL_e}$ from the given data. It remains to estimate $\mathbf{F}_l$'s to complete the design. Here the discussion of Sec. 3.1 becomes relevant. There we found a $\mathbf{H}_0$ to satisfy (3-15). From (3-9) and (3-15) we have

$$\mathbf{H}_0^{\mathcal{H}}\mathbf{v}_s(k) = \sum_{i=0}^{L_e} \mathbf{H}_0^{\mathcal{H}}\mathbf{G}_{ai}\mathbf{s}(n-i). \qquad (3-20)$$

Using (1-2), (3-15) and (3-20), we have

$$\mathbf{F}_\tau^{\mathcal{H}} = \alpha^{-1}\mathbf{H}_0^{\mathcal{H}} \sum_{i=0}^{L_e} \mathbf{G}_{ai}\mathbf{R}_{ss}^{\mathcal{H}}(\tau+i). \qquad (3-21)$$

Let $\mathcal{R}_{d,ssL_e}$ denote a $[N(L_e+1)] \times [N(L_e+1)]$ matrix with its ij-th block element as $E\{\mathbf{s}(k+d+j-i)\mathbf{s}^{\mathcal{H}}(k)\}$. Using (3-6) and (3-21) in (3-19) we obtain the desired solution

$$\left[\ \overline{\mathbf{G}}_{d,0} \quad \overline{\mathbf{G}}_{d,1} \quad \cdots \quad \overline{\mathbf{G}}_{d,L_e}\ \right]$$
$$= \alpha^{-1}\mathbf{H}_0^{\mathcal{H}}\left[\ I_{N\times N} \quad 0 \quad \cdots \quad 0\ \right] \mathcal{R}_{ssL_e}^{\#}\mathcal{R}_{d,ssL_e}^{\mathcal{H}}\mathcal{R}_{yyL_e}^{-1}. \qquad (3-22)$$

A scaled MMSE estimate of $w(t-d)$ is then given by

$$\widehat{w}(t-d) \;=\; \sum_{i=0}^{L_e} \alpha\overline{\mathbf{G}}_{d,i}\mathbf{y}(t-i). \qquad (3-23)$$

3.3. Summary of Algorithms

Given data $\mathbf{y}(k)$, $k = 1, 2, \cdots, T$. Pick the length L_e+1 and delay d of the MMSE equalizer. Estimate all correlation functions by sample averaging.

3.3.1. ALGORITHM I :

Here $\mathbf{F}_0$ is estimated as the unit norm $\mathbf{H}_0$ that lies in the null space of $\overline{R}_{v_s v_s}$. Estimate noisefree correlations via (3-5). Use (3-22) and (3-23) for MMSE equalizer design.

3.3.2. ALGORITHM II :

Here $\mathbf{F}_0$ is estimated as in Remark 2. The rest is as in ALGORITHM I.

3.3.3. ALGORITHM III :

Here we will use (3-19) with $\mathbf{F}_i$ ($i = 0, 1, \cdots, d$) estimated using the basic approach of [9] and [14]. Although [9] and [14] derive all their results under the assumption of FIR channels with no common zeros, their results extend (with straightforward modifications) to models that satisfy (**H1**)-(**H5**) by virtue of Lemma 1.

4. BLIND EQUALIZATION: COMMON ZEROS

4.1. Minimum-Phase Zeros

Here the SIMO transfer function is

$$\mathcal{F}(z) \;=\; [\mathcal{B}_c(z)/\mathcal{A}(z)]\,\mathcal{B}(z) \qquad (4-1)$$

where $\mathcal{B}(z)$ satisfies (H2) and $\mathcal{B}_c(z)$ is a finite-degree scalar polynomial that collects all the common zeros of the subchannels. Assume that

(**H6**) Given model (4-1), $\mathcal{B}_c(z) \neq 0$ for $|z| \geq 1$.

Then while $\mathcal{A}^{-1}(z)\mathcal{B}(z)$ has a finite inverse, $\mathcal{B}_c^{-1}(z)$ is IIR though causal under (H6). Then (3-2) holds true approximately for "large" L_e, the approximation getting better with increasing L_e. Similarly Lemma 1 holds true approximately for "large" M and Lemma 2 also holds true approximately for $L_e \geq M$. It is then readily seen that the developments of Secs. 3.1, 3.2 and 3.3 are applicable.

4.2. Arbitrary Zeros

In this case (4-1) is true but $\mathcal{B}_c(z)$ does not necessarily satisfy (**H6**). We may rewrite (4-1) as

$$\mathcal{F}(z) \;=\; \overline{\mathcal{F}}(z)\mathcal{F}_{AP}(z) \qquad (4-2)$$

where $\mathcal{F}_{AP}(z)$ is an allpass (rational) function such that

$$\mathcal{B}_c(z)\mathcal{B}_c(z^{-1}) \;=\; \mathcal{F}_{AP}(z)\overline{\mathcal{B}}_{MP}(z) \qquad (4-3)$$

and $\overline{\mathcal{B}}_{MP}(z)$ is minimum-phase. Thus (within a scale factor) we have

$$\overline{\mathcal{F}}(z) \;=\; \left[\overline{\mathcal{B}}_{MP}(z)/\mathcal{A}(z)\right]\mathcal{B}(z). \qquad (4-4)$$

We may rewrite (1-2) as

$$\mathbf{y}(k) = \overline{\mathcal{F}}(z)w'(k) + \mathbf{n}(k) \text{ where } w'(k) := \mathcal{F}_{AP}(z)w(k). \quad (4-5)$$

Clearly $w'(k)$ satisfies (**H4**). Hence, (4-4)-(4-5) satisfy the requirements of Sec. 4.1. Therefore, one can "approximately" recover $w'(k)$ from the given data by applying the algorithms of Sec. 3.3. In order to recover $w(k)$ form $w'(k)$, one needs to exploit the higher-order statistics of $\{w'(k)\}$; see [2],[3] and references therein.

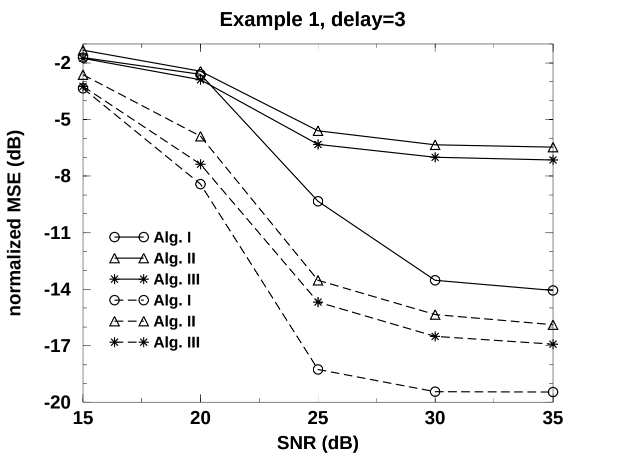

Fig. 1. Normalized MSE after MMSE equalization with $d = 3$. *Solid lines:* $T = 250$ *symbols, dashed lines:* $T = 1000$ *symbols.*

5. SIMULATION EXAMPLES

5.1. Example 1.

We have $N = 3$ in (1-2) with $\mathcal{F}(z) = \mathcal{A}^{-1}(z)\mathcal{B}(z)$ where

$$\mathcal{A}(z) = (1 - 0.5z^{-1})I_{3\times 3} \quad (5-1)$$

and $\mathcal{B}(z)$ is 3×1 MA(6) obtained from [10] as follows. Consider a raised cosine pulse $p_6(t, 0.1)$ with a roll-off factor 0.1, truncated to a length of $6T_s$ (T_s = symbol duration). As in [10], a two-ray multipath channel with (effective) impulse response $h(t) = p_6(t, 0.1) - 0.7p_6(t - T_s/3, 0.1)$ was sampled at intervals of $T_s/3$ (starting at $t = -3T_s$) to create the $\mathcal{B}(z)$ above. Transfer function $\mathcal{B}(z)$ satisfies (**H2**) [10], therefore, there exists a finite left inverse of length $L_e = 6$ (cf. Sec. 2.1). The scalar input $w(k)$ is 4-QAM. An MMSE equalizer of length $L_e = 8$ (9 taps per subchannel, totaling 27 taps –– overfitting) was designed with a delay d =3 (arbitrarily selected just for illustration). The Algorithms I–III were applied for record lengths $T = 250$ and 1000 symbols with varying SNR's. Fig. 1 shows the normalized MSE (MSE divided by $E\{|w(k)|^2\}$). It is seen that the proposed design approach can handle IIR channels with little difficulty. Algorithm I (newly proposed) performs the best.

5.2. Example 2.

Again we have $N = 3$ in (1-2) but with $\mathcal{F}(z) = \mathcal{B}_c(z)\mathcal{B}(z)$ where $\mathcal{B}(z)$ is as in Example 1 and $\mathcal{B}_c(z)$ is a scalar polynomial given by

$$\mathcal{B}_c(z) \ = \ 1 - 0.5z^{-1}. \quad (5-2)$$

Thus all three subchannels have a common zero at 0.5. The input $w(k)$ is 4-QAM as in Example 1. Note that in this example a finite left inverse does not exist. As in Example 1, an MMSE equalizer of length $L_e = 12$ was designed with a delay d =3. Fig. 2 shows the normalized MSE averaged over 100 Monte Carlo runs. It is seen that the proposed design approaches can handle subchannels with common minimum-phase zeros with little difficulty. As in Example 1, Algorithm I performs the best.

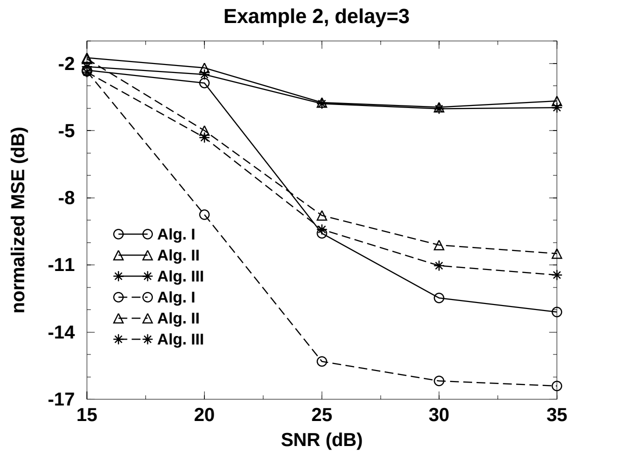

Fig. 2. Normalized MSE after MMSE equalization with $d = 3$. *Solid lines:* $T = 250$ *symbols, dashed lines:* $T = 1000$ *symbols.*

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