NONSTATIONARY ARRAY SIGNAL DETECTION USING TIME-FREQUENCY AND TIME-SCALE REPRESENTATIONS

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ABSTRACT

Quadratic time-frequency representations (TFRs) and time-scale representations (TSRs) have been shown to be very useful for detecting nonstationary signals in the presence of nonstationary noise. The theory developed thus far is only for the single observation case; however, in many situations involving signal detection, there are advantages in using an array of receiving sensors. Sensor arrays allow for target or source localization and can provide a large gain in the SNR. We show that time-frequency and time-scale representations provide a natural structure for the detection of a large class of nonstationary signals in the presence of nonstationary noise using an array of sensors. That is, time-frequency and time-scale provide a detection structure that is both optimal and allows for efficient implementation. In developing the TFR/TSR-based optimal quadratic array processor, we consider several types of array environments including those with full, partial, and no coherence.

1. INTRODUCTION

The detection of signals in noise is a classical hypothesis testing problem. In many situations, the signal may include unknown time and frequency or time and scale offsets. Such situations include the well known delay-doppler situation in radar/sonar detection problems. Time-frequency representations (TFRs) and time-scale representations (TSRs), which describe the signal jointly in terms of both time and frequency or time and scale, are powerful tools for designing the optimal detector in such situations. Recently it has been shown that the optimal quadratic detector for the detection of nonstationary Gaussian signals with unknown time and frequency or time and scale offsets in the presence of noise can be implemented naturally within Cohen's class of TFRs or TSRs [1]. In this paper, we will extend this idea to detection using an array of sensors.

In many situations involving signal detection and estimation, there are many advantages in using a sensor array instead of a single element, directionality and SNR gain are the most important. In active sensing situations such as radar and sonar, a known waveform is generated which in turn propagates through a medium and is reflected by some target back to the array. That transmitted signal undergoes a delay and frequency shift in the narrowband case and a delay and scale offset in the wideband case. In addition, the angle at which the return signal arrives may be unknown. Moreover, changing target and environmental characteristics, combined with other types of disturbances, cause the signals that arrive at the array to be regarded as random, and at times the physical phenomena responsible for the randomness in the signal make it plausible to assume that the signals are Gaussian (perhaps nonstationary) random processes. Though the signal structure is known, it may still contain unknown parameters such as a delay, frequency shift, scale offset, or angle of arrival. These uncertainties, combined with the nonstationary nature of the signal and noise processes, make TFRs and TSRs potentially powerful tools in designing the optimal detector in the array environment. Traditional array processing schemes assume that the signal and noise processes are stationary and the use of TFRs/TSRs in array processing has been limited to simple, generally suboptimal, matched-field beamforming techniques.

In this paper we consider the problem of detecting arbitrary nonstationary second-order signals [1] with unknown time and frequency or time and scale offsets arriving in a linear array with an unknown angle of arrival. We explicitly show how the optimal detector for such a problem can be implemented naturally and efficiently in the time-frequency or time-scale domain. This paper is organized as follows: Section 2 briefly reviews TFRs and TSRs, followed by a discussion of the detection problem that we consider in Section 3. In Section 4, quadratic detection in a partially coherent linear array is explored. In Section 5, the array detection problem will be shown to be naturally suited to time-frequency or time-scale representations whereby we develop the TFR/TSR-based optimal quadratic array processors. The model of partial coherence used includes as special cases the coherent and non-coherent environments, for which the optimal detector simplifies considerably. In Section 6 we present an example illustrating the benefits of the TFR-based quadratic array processor and in Section 7 we conclude the paper.

2. TIME-FREQUENCY AND TIME-SCALE REPRESENTATIONS

TFRs have become increasingly popular tools for analyzing and processing signals with time-varying spectral content. A one dimensional signal $x(t)$ is mapped by a TFR into a two-dimensional distribution, $P_x(t, f)$, which is a function of both time and frequency. This joint representation exploits the nonstationary char-

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[1] Second-order signals are those which are completely characterized by their second-order statistics; Gaussian signals are an example.

acteristics of a signal and therefore can be very useful in detecting nonstationary signals. Similarly, a TSR analyzes a signal jointly in terms of time and scale.

Any bilinear TFR from Cohen's class can be expressed as [3]

$$P_x(t,f;\Phi) = \int\int W_x(u,v)\Phi(u-t,v-f)dudv \ , \quad (1)$$

where W_x is the auto-Wigner distribution (WD) of x, defined as [3]

$$W_x(t,f) = \int x(t+\tau/2)x^*(t-\tau/2)e^{-j2\pi f\tau}d\tau \ , \ (t,f) \in \mathbb{R}^2 \ , \quad (2)$$

and Φ is a two-dimensional kernel that completely characterizes the TFR $P_x(t,f;\Phi)$. Similarly, any bilinear TSR from the affine class can be expressed as

$$C_x(t,a;\Pi) = \int\int W_x(u,v)\Pi((u-t)/a, av)dudv \ , \quad (3)$$

where $(t,a) \in \mathbb{R} \times (0,\infty)$ and again the kernel Π completely characterizes the TSR $C_x(t,a;\Pi)$. Cross-TFRs $P_{xy}(t,f;\Phi)$ and cross-TSRs $C_{xy}(t,a;\Pi)$ will prove to be useful tools when multiple signals are being processed. In this case, the auto-WD is replaced with a cross-WD defined as

$$W_{xy}(t,f) = \int x(t+\tau/2)y^*(t-\tau/2)e^{-j2\pi f\tau}d\tau \ , \ (t,f) \in \mathbb{R}^2 \ . \quad (4)$$

3. THE DETECTION PROBLEM

Consider the following composite hypothesis testing problem in continuous time:

$$\begin{aligned} H_0 : x(t) &= n(t) \\ H_1 : x(t) &= s^{(\tau,\alpha)}(t) + n(t) \end{aligned} \quad (5)$$

where $t \in T$, the time interval of observation, x is the observed signal, n is arbitrary zero-mean complex Gaussian noise with correlation function $R_n(t_1,t_2) = E[n(t_1)n^*(t_2)]$, and s is a zero-mean complex arbitrary second-order signal with correlation function $R_s(t_1,t_2)$. The parameters (τ,α) represent certain nuisance parameters that are assumed to be unknown. The parameter τ represents a time shift and the parameter α represents either a scale offset (c) or frequency shift (ν) [2]. TFRs and TSRs provide a natural detection framework for such hypothesis testing problems for two main reasons: first, detecting a second-order signal (such as a Gaussian signal) in the presence of Gaussian noise involves a quadratic function of the observations [4], and bilinear TFRs and TSRs are quadratic in the observations; second, TFRs and TSRs posses additional degrees of freedom provided by the TFR and TSR parameters (time and frequency for Cohen's class and time and scale for the affine class).

Because of our assumptions, the dependence of s on (τ,α) is only through the correlation function, which we denote by $\bar{R}_s^{(\tau,\alpha)}$. In the case of time and frequency shifts,

$$\bar{R}_s^{(\tau,\nu)}(t_1,t_2) = R_s(t_1-\tau, t_2-\tau)e^{j2\pi\nu t_1}e^{-j2\pi\nu t_2} \quad (6)$$

[2] Actually, we can start by arbitrarily parameterizing the composite hypothesis H_1, but from [1] we know time-frequency and time-scale detectors are naturally suited to deal with time and frequency shifts or time and scale offsets.

for some correlation function R_s. In the case of time and scale offsets,

$$\bar{R}_s^{(\tau,c)}(t_1,t_2) = cR_s(c(t_1-\tau), c(t_2-\tau)) \quad (7)$$

again for some correlation function R_s. Note that (6) corresponds to $s(t;\tau,\nu) = s_{(\tau,\nu)}(t-\tau)e^{j2\pi\nu t}$ in (5), where, for each (τ,ν), $s_{(\tau,\nu)}$ is any second-order signal with correlation function R_s. Similarly, (7) is equivalent to $s(t;\tau,c) = \sqrt{c}s_{(\tau,c)}(c(t-\tau))$ for any second-order signal $s_{(\tau,c)}$ with correlation function R_s. We will consider the parameters (τ,α) to be deterministic but unknown. In statistical hypothesis testing, for each observation, x, a real-valued test statistic, $L(x)$, is compared to a threshold to decide in favor of H_0 or H_1; that is, to decide whether the signal is present or not. We assume that both the signal and the noise processes are independent of each other and are completely characterized by their correlation functions.

4. QUADRATIC DETECTION IN A LINEAR ARRAY

In the linear array configuration, the signal comes in to the array of M sensors with spacing d at angle θ, where θ is assumed to be unknown. Figure 1 illustrates the linear array configuration. We

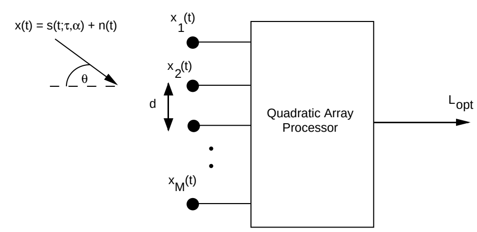


Figure 1: Linear Array Configuration

will denote the signal at the ith sensor by $x_i(t)$. Due to the linear array configuration, the signal at the ith sensor is a delayed version of the signal at the first sensor, and the value of the delay depends on the unknown angle of arrival, θ. That is, $x_i(t) = x_1(t-iD)$, where $D = \frac{d}{c}sin(\theta)$ and c is the velocity of propagation in the medium. Note that $x_i(t) = s(t-iD) + n(t)$ when the signal is present and $x_i(t) = n(t)$ when only noise is present. A useful quantity in the derivation of the optimal detector will be the array cross-correlation matrix, denoted by $\mathbf{P}$, where the (i,j)th element is given by the cross-correlation function between $x_i(t)$ and $x_j(t)$; that is,

$$\mathbf{P}_{ij} = E[x_i(t_1)x_j^*(t_2)] \equiv R_{x_ix_j}(t_1,t_2) \quad (8)$$

Let $\mathbf{P}_s$ denote the array cross-correlation matrix when the signal alone is present, and let $\mathbf{P}_n$ denote the array cross-correlation matrix when the noise alone is present. It is customary to assume that the noise between sensors is uncorrelated , which implies that $\mathbf{P}_n$ is diagonal and given by $\mathbf{P}_n = diag(R_n, R_n, \ldots R_n)$.

A concern arises when considering the use of a very large array in order to achieve high array gain; the signal received at widely separated sensors may have reduced coherence due to the complexity in the propagation of the signal from the source to spatially separated receivers [2]. Since we are only considering the second-order statistics of the signal, the model for partial coherence used will be given in terms of the correlation function. An exponential power law model [2] will be used whereby the cross-correlation function between the ith and jth sensors will be scaled by the coefficient $c_{ij} = e^{-\frac{|i-j|}{L}}$, where L is a dimensionless characteristic correlation length. We may arrange the decorrelation coefficients in matrix form as $\mathbf{C} = \{c_{ij}\}$. Proceeding with the formulation of the optimal detector, for purposes of simplification, it will be convenient to deal with the aligned sensor outputs; that is, let $y_i(t) = x_i(t + iD)$. It will also be convenient to arrange the aligned sensor outputs in vector form as $\mathbf{Y}^\theta = [y_1(t)\ y_2(t)\ \ldots\ y_M(t)]^T$ where the superscript θ denotes the dependence of aligning the sensor signals on the unknown angle of arrival.

With the assumed decorrelation structure, the array cross- correlation matrix when the signal alone is present, as applied to the aligned sensor outputs, is given by the following Kronecker product:

$$\mathbf{P}_s^{(\tau,\alpha)}(t_1, t_2) = \mathbf{C} \otimes R_s^{(\tau,\alpha)}(t_1, t_2) \tag{9}$$

where $R_s^{(\tau,\alpha)}(t_1, t_2)$ is the autocorrelation function of the signal component at a particular value of time shift τ and scale or frequency offset α. Since the parameters τ, α, and θ are unknown, we use the analogue of a generalized likelihood ratio test (GLRT) [4] in which an estimate of the parameters is formed and used to obtain the optimal test statistic. The optimal test statistic based on the deflection[3] criterion [4] is given by

$$L_{opt} = \max_{(\tau,\alpha,\theta)} \langle \tilde{\mathbf{P}}_n^{-1} \left(\mathbf{C} \otimes \mathbf{R}_s^{(\tau,\alpha)} \right) \tilde{\mathbf{P}}_n^{-1} \mathbf{Y}^\theta, \mathbf{Y}^\theta \rangle, \tag{10}$$

where $\mathbf{R}_s$ denotes the linear operator defined by the corresponding correlation function R_s as

$$(\mathbf{R}_s x)(t) = \int R_s(t, \tau) x(\tau) d\tau \ , \tag{11}$$

and $\tilde{\mathbf{P}}_n$ is a matrix of linear operators corresponding the correlation functions in the matrix $\mathbf{P}_n$; observe that $\tilde{\mathbf{P}}_n$ is diagonal and given by $\tilde{\mathbf{P}}_n = diag(\mathbf{R}_n, \mathbf{R}_n, \ldots \mathbf{R}_n)$ where $\mathbf{R}_n$ is the linear operator corresponding to the correlation function of the noise process. Upon expanding the inner product in (10), we obtain

$$L_{opt} = \max_{(\tau,\alpha,\theta)} \sum_{i=1}^{M} \sum_{j=1}^{M} c_{ij} \langle \mathbf{R}_n^{-1} \mathbf{R}_s^{(\tau,\alpha)} \mathbf{R}_n^{-1} y_i^\theta(t), y_j^\theta(t) \rangle. \tag{12}$$

5. TFR/TSR-BASED ARRAY DETECTION

The connection to TFRs and TSRs is made through the use of the Weyl correspondence which relates inner products, positive definite linear operators, and the Wigner distribution. Using the fact that the Weyl correspondence involves a covariance to time, frequency, and scale offsets, using the methods in [1] it can be verified that the test statistic in (12) may conveniently be expressed in terms of TFRs or TSRs, allowing for a natural and efficient implementation of the optimal detector. In the case where α corresponds to a frequency shift ν, the optimal test statistic is given by

$$L_{opt} = \max_{(t,f,\theta)} \sum_{i=1}^{M} \sum_{j=1}^{M} c_{ij} P_{y_i y_j}^\theta (t, f; \Phi = WS_{R_n^{-1} R_s R_n^{-1}}). \tag{13}$$

[3] Deflection-optimal detectors can be interpreted as "maximum SNR" detectors because deflection is a measure of SNR.

We use the superscript θ here to denote the fact that the TFR must be formed for each hypothesized angle of arrival. When α corresponds to a scale offset c, the optimal test statistic is given by

$$L_{opt} = \max_{(t,c,\theta)} \sum_{i=1}^{M} \sum_{j=1}^{M} c_{ij} C_{y_i y_j}^\theta (t, \frac{1}{c}; \Pi = WS_{R_n^{-1} R_s R_n^{-1}}). \tag{14}$$

Observe that in (13) and (14) we must form the sum of all weighted cross-TFRs/TSRs; we will refer to this quantity as a matrix TFR or TSR. Figure 2a illustrates the partially coherent detection structure. Because the detector involves forming TFRs or TSRs of signals that are aligned to examine different spatial directions, we may think of this detection structure in terms of time-frequency-space or time-scale-space. Since we have not assumed any spatial statistical characteristics of the signal, the kernel is the same regardless of the angle of arrival being analyzed.

If the array environment is perfectly coherent, then we have that $c_{ij} = 1 \forall i, j$. Using fundamental properties of bilinear TFRs and TSRs, it can be verified that the optimal test statistic will involve first summing the sensor observations and then applying the TFR/TSR (with the kernel as before) for each hypothesized angle of arrival and choosing the maximum value. If the array environment is noncoherent, then $\mathbf{C} = \mathbf{I}$ and it can be verified that the optimal test statistic will include first taking the TFR/TSR of each sensor observation (again, with kernel as before), and then summing the resulting TFRs/TSRs for each hypothesized angle of arrival and choosing the maximum value. Figure 2b and figure 2c illustrate the coherent and noncoherent detection structures, respectively.

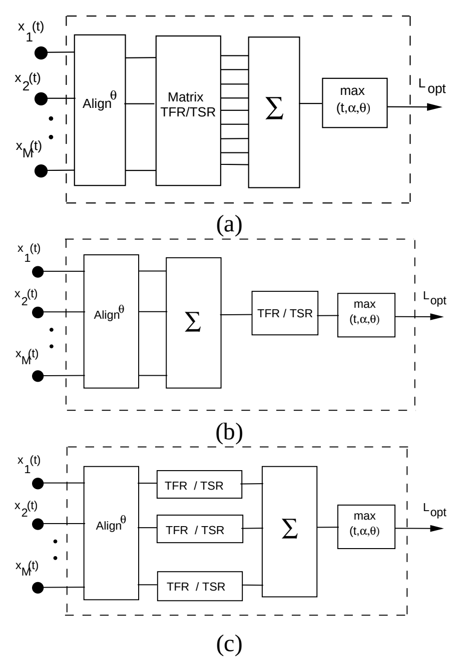


Figure 2: TFR/TSR-Based Optimal Quadratic Array Processors: (a) Partially Coherent Detector (b) Coherent Detector (c) Non-Coherent Detector

6. SIMULATION

In this section we present an example illustrating the performance of the proposed optimal quadratic array processor. Let us suppose that the signal of interest is characterized by the following random process:

$$S(t) = \sum_{k=1}^{3} Z_k e^{-\beta_k t^2}, \tag{15}$$

where $Z_k \sim \mathcal{N}(0,1)$ and each β_k is a fixed complex number. Note that $S(t)$ is the sum of three chirp signals[4] with random amplitude scaling factor Z_k; each of the chirps is characterized by β_k whose real part determines the variance of the Gaussian envelope and imaginary part determines the chirp rate.

Let us first illustrate the advantages of the TFR-based array detector over a traditional approach. Unknown time and frequency shifts as well as an unknown angle of arrival were introduced, and independent white Gaussian noise was added at each sensor such that the SNR was 0 dB (that is, $\mathbf{R}_n = \sigma \mathbf{I}$). In radar/sonar signal processing, the traditional approach in detecting the presence of a signal with unknown time and frequency offsets is to compute the cross-ambiguity function (point-by-point matched filter) of the transmitted and received signal and use the maximum value. Figure 3a displays the ROC curves for the single sensor matched filter and TFR-based detectors. Observe how the TFR-based detector clearly outperforms the matched filter technique because the signal of interest is random, not deterministic. Also displayed is the optimal TFR-based quadratic array processor for a coherent array environment with $M = 6$ sensors, which clearly outperforms the single sensor TFR detector.

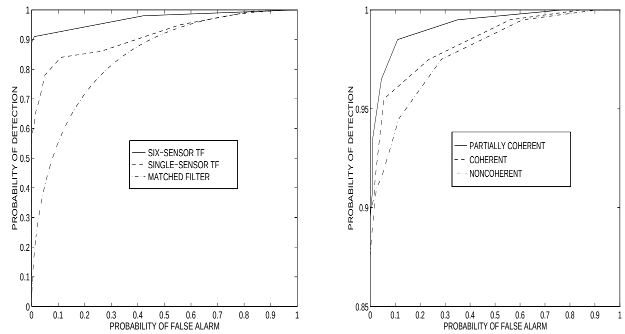
We now show how exploiting the partially coherent environment improves the performance of the quadratic array processor. We simulated coherence loss with small, random phase shifts between sensors and averaged over several experiments to obtain the decorrelation matrix $\mathbf{C}$. Using the experimentally obtained decorrelation coefficients, the partially coherent optimal quadratic array processor in (13) was implemented using $M = 6$ sensors with the signal process in (15) and the same noise process as before. Figure 3b displays the ROC curves for the partially coherent, coherent, and noncoherent quadratic array processors applied in this partially coherent environment. Note that the noncoherent detector has the poorest performance due to the fact that it does not exploit any of the coherence between sensors. The coherent detector performs better than the noncoherent detector, but not as well as the partially coherent detector which exploits the partially coherent characteristics of the environment. The partially coherent detector provides the benefits of both coherent and noncoherent combining.

7. SUMMARY

In this paper we showed that time-frequency and time-scale based detectors are naturally suited to quadratic detection in an array environment. In Section 2 we reviewed time-frequency and time-scale distributions and in Section 3 we set up the detection problem. In Section 4 we considered quadratic detection when using a linear array of receiving sensors in a partially coherent environment. The signal was assumed to contain an unknown time and frequency shift or unknown time and scale offset, and the angle of arrival into the array was assumed to be unknown. The deflection criterion we employed applied to arbitrary second-order signals in Gaussian noise.

By using a GLRT approach, the deflection-optimal test statistic was cast in the form of TFRs and TSRs in Section 5. The TFR/TSR-based structure allows for the optimal detector to be implemented naturally and efficiently by exploiting the many degrees of freedom available. In the general case of a partially coherent environment, the test statistic included a weighted sum of all cross-TFRs or cross-TSRs of the aligned sensor outputs for each value of hypothesized angle of arrival. Completely coherent and noncoherent cases were shown to be special cases of the partially coherent model. In the coherent case, the optimal test statistic simplified to include a single auto-TFR or auto-TSR of the sum of the aligned sensor outputs for each hypothesized angle of arrival. In the noncoherent case, the optimal test statistic simplified to include the sum of auto-TFRs or auto-TSRs of the aligned sensor outputs for each hypothesized angle of arrival. Section 6 illustrated the superior performance the the proposed TFR based quadratic array processor over traditional array detection approaches.

[4] Chirp waveforms are popular in radar and sonar signal processing.



(a) (b)

Figure 3: (a) ROC Curves for the Matched Filter Detector, Single-Sensor TFR Detector, and Six-Sensor TFR Coherent Processor (b) ROC Curves for the Partially Coherent, Coherent, and Noncoherent Detectors Applied in a Partially Coherent Environment

8. REFERENCES

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