REVERSIBLE DISCRETE COSINE TRANSFORM

Kunitoshi KOMATSU and Kaoru SEZAKI

Institute of Industrial Science, The University of Tokyo
7-22-1 Roppongi, Minato-ku, Tokyo 106, JAPAN
E-mail: komatsu@pipi.iis.u-tokyo.ac.jp

ABSTRACT

In this paper a reversible discrete cosine transform (RDCT) is presented. *N*-point reversible transform is firstly presented, then 8-point RDCT is obtained by substituting the 2 and 4-point reversible transforms for 2 and 4-point transforms which compose 8-point discrete cosine transform (DCT), respectively. Integer input signal can be losslessly recovered, although the transform coefficients are integer numbers. If the floor functions are ignored in RDCT, the transform is exactly the same as DCT with determinant = 1. RDCT is also normalized so that we can avoid the problem that dynamic range is nonuniform. Simulation on continuous-tone still images shows that the lossless and lossy compression efficiencies of RDCT are comparable to those obtained with reversible wavelet transform.

1. INTRODUCTION

Unified lossless and lossy image coding system is useful for various applications, since we can reconstruct lossy and lossless images from a part and the whole of the encoded data, respectively. This coding system can be realized by using reversible transform. Reversible wavelet transform (RWT) [1][2][3][4], Lossless-DCT (LDCT) [5] and reversible Walsh-Hadamard transform (RWHT) [6] have been proposed as reversible transforms. LDCT is especially promising, since it is compatible with JPEG and MPEG. In this paper we propose a new reversible discrete cosine transform.

When integer input signal is transformed by DCT, the transform coefficients become real. Therefore the quantization step must be reduced in order to reconstruct input losslessly. This results in low compession efficiency. Ohta *et al*. approximately expressed transform bases of DCT by integer numbers, and designed reversible transform, LDCT, by using reversible quantization table. Although the determinant of LDCT is approximately equal to 1, the transform coefficients are not easily obtained because of the reversible quantization table. In this paper we propose a reversible discrete cosine transform, RDCT, of which forward and inverse transforms are represented by simple equations.

The organization of this paper is as follows. In section 2, *N*-point reversible transform is derived. In section 3, 8-point RDCT is derived. The lossless and lossy compression efficiencies of RDCT are shown in section 4. Finally, conclusions are drawn in the last section.

2. *N*-POINT REVERSIBLE TRANSFORM

2.1 2-point Reversible Transform

We begin with the following 2-point transform.

$$\begin{bmatrix} \theta_0 \\ \theta_1 \end{bmatrix} = \begin{bmatrix} x_0 + \lfloor c_0 x_1 + 0.5 \rfloor \\ x_1 + \lfloor c_1 x_0 + 0.5 \rfloor \end{bmatrix} \tag{1}$$

where $\lfloor \cdot \rfloor$ corresponds to downward truncation, θ_0 and θ_1 are integer transform coefficients and x_0 and x_1 are integer inputs. If the real numbers c_0 and c_1 satisfy $c_0 c_1 \le 0$, this transform becomes reversible [6]. If the floor functions are deleted, the determinant of the transform matrics becomes $1 - c_0 c_1$. Therefore redundancy occurs in transform domain, when $c_0 c_1 < 0$.

This problem is avoided by using the following transform instead.

$$\begin{bmatrix} \theta_0 \\ \theta_1 \end{bmatrix} = \begin{bmatrix} x_0 + \lfloor c_0 x_1 + 0.5 \rfloor \\ x_1 + \lfloor c_1 \theta_0 + 0.5 \rfloor \end{bmatrix} \tag{2}$$

It is obvious that this transform is reversible for any c_0 and c_1. If the floor functions are deleted, the coefficients of x_0 and x_1 of θ_1 become c_1 and $1 + c_1 c_2$, respectively, that is, the determinant becomes 1. However, the problem that the dynamic range is nonuniform remains.

The dynamic range can become uniform by using the following transform instead.

$$\begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} x_0 + \lfloor c_0 x_1 + 0.5 \rfloor \\ x_1 + \lfloor c_1 \theta_0 + 0.5 \rfloor \\ \theta_0 + \lfloor c_2 \theta_1 + 0.5 \rfloor \end{bmatrix} \tag{3}$$

where the transform coefficients are θ_1 and θ_2. If the floor functions are deleted, the coefficients of x_0 and x_1 of θ_1 become c_1 and $1 + c_0 c_1$, respectively, and those of θ_2 be-

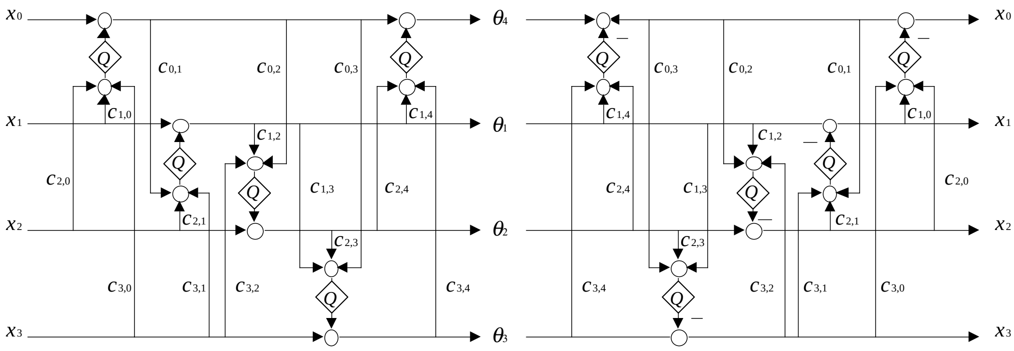


Fig. 1. 4-point reversible transform and inverse transform.

come $1+c_1c_2$ and $c_0+c_2+c_0c_1c_2$, respectively. Therefore, the determinant becomes 1. The inverse transform is as follows.

$$\begin{bmatrix}\theta_0\\ x_1\\ x_0\end{bmatrix}=\begin{bmatrix}\theta_2-\lfloor c_2\theta_1+0.5\rfloor\\ \theta_1-\lfloor c_1\theta_0+0.5\rfloor\\ \theta_0-\lfloor c_0x_1+0.5\rfloor\end{bmatrix}. \tag{4}$$

For example, Eq. 3 with $c_0=c_2=1-\sqrt{2}$ and $c_1=1/\sqrt{2}$ becomes normalized 2-point WHT, if the floor functions are deleted. The dynamic range of this normalized RWHT is uniform, unlike the conventional RWHT

$$\begin{bmatrix}\theta_0\\ \theta_1\end{bmatrix}=\begin{bmatrix}\lfloor x_0/2+x_1/2\rfloor\\ x_0-x_1\end{bmatrix}. \tag{5}$$

2.2 *N*-point Reversible Transform

Eq. 3 can be easily generalized into *N*-point reversible transform as follows.

$$\begin{aligned}\theta_j&=x_j+\left\lfloor\sum_{i=0}^{j-1}c_{i,j}\theta_i+\sum_{i=j+1}^{N-1}c_{i,j}x_i+0.5\right\rfloor,(0\le j\le N-1)\\ \theta_N&=\theta_0+\left\lfloor\sum_{i=1}^{N-1}c_{i,N}\theta_i+0.5\right\rfloor\end{aligned} \tag{6}$$

where $\sum_{i=0}^{-1}=\sum_{i=N}^{N-1}\equiv 0$ and the transform coefficients are $\theta_1,\theta_2,\ldots,\theta_N$. Even if a quantization with any quantization step is used instead of floor function, the transform is reversible. However, we use floor function, since the entropy of the transform coefficients must be small in image coding.

Inverse transform is as follows.

$$\begin{aligned}\theta_0&=\theta_N-\left\lfloor\sum_{i=1}^{N-1}c_{i,N}\theta_i+0.5\right\rfloor\\ x_j&=\theta_j-\left\lfloor\sum_{i=0}^{j-1}c_{i,j}\theta_i+\sum_{i=j+1}^{N-1}c_{i,j}x_i+0.5\right\rfloor,\ (N-1\ge j\ge 0)\end{aligned} \tag{7}$$

$$\boldsymbol{A}=\begin{bmatrix}a&b\\ c&d\end{bmatrix}=\begin{bmatrix}1&0\\ (d-1)/b&1\end{bmatrix}\begin{bmatrix}1&b\\ 0&1\end{bmatrix}\begin{bmatrix}1&0\\ (a-1)/b&1\end{bmatrix}$$

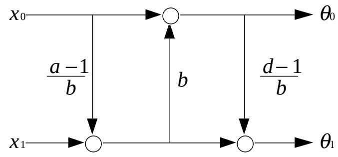


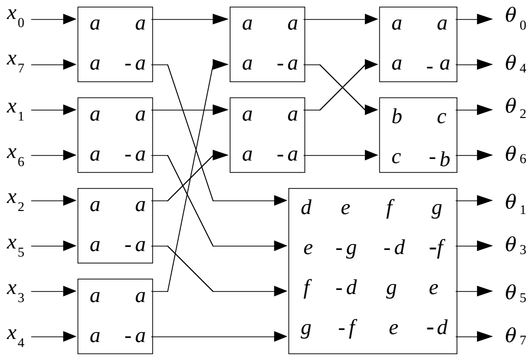
Fig. 2 PLUV decomposition of matrics ***A*** and Ladder Network realizing matrics ***A***.

Fig. 1 shows a 4-point reversible transform and the inverse transform.

Bruekers *et al.* proposed a reversible transform using Ladder Network [7]. A transform with determinant =1 can be represented by Ladder Network based on PLUV decomposition (Fig. 2). Even if the coefficients and the intermediate results are quantized, the input can be reconstructed losslessly. They showed Ladder Networks realizing 4-point DCT and 6-point DFT as example. While their method is based on decomposition of transform matrics, we don't use decomposition of transform matrics. We firstly obtain the generalized transform matrics from Eq. 6 and then obtain the coefficients of Eq. 6 by comparing it with a desired transform matrics, for example, DCT.

3. 8-POINT REVERSIBLE DISCRETE COSINE TRANSFORM

A normalized 8-point RDCT is obtained by comparing Eq. 6 with *N* =8 with normalized 8-point DCT. However, we can obtain RDCT more easily, since the 8-point DCT can be decomposed into 2-point and 4-point transforms as shown in Fig.3. It is obvious that the whole transform is reversible, when reversible transforms are substituted for every transform in Fig.3.



$$a = \frac{1}{\sqrt{2}} \qquad b = \cos\frac{\pi}{8} \qquad c = \cos\frac{3\pi}{8}$$
$$d = \frac{1}{\sqrt{2}}\cos\frac{\pi}{16} \qquad e = \frac{1}{\sqrt{2}}\cos\frac{3\pi}{16}$$
$$f = \frac{1}{\sqrt{2}}\cos\frac{5\pi}{16} \qquad g = \frac{1}{\sqrt{2}}\cos\frac{7\pi}{16}$$

Fig. 3 8-point DCT.

The coefficients $c_{i,j}$ are obtained by comparing Eq. 6 with N =4 with the 4-point transform in Fig.3 as follows.

$$\begin{array}{ll} c_{1,0} = -c_{3,4} = -0.5942 & c_{2,0} = c_{2,4} = -4.025 \\ c_{3,0} = -c_{1,4} = -1.435 & c_{2,1} = c_{2,3} = 3.185 \\ c_{3,1} = -c_{1,3} = 1.133 & c_{0,1} = -c_{0,3} = 0.6935 \\ c_{3,2} = c_{1,2} = 0.2114 & c_{0,2} = 0.4413 \end{array} \tag{8}$$

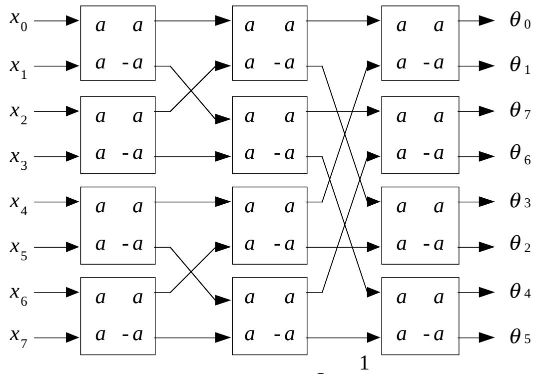
Eq.3 with $c_0 = c_2 = \left(\cos(3\pi/8) - 1\right)/\cos(\pi/8)$ and $c_1 = \cos(\pi/8)$ is reversible version of the 2-point transform into θ_2 and θ_6 in Fig.3.

The normalized 8-point WHT can be decomposed into 2-point transforms as shown in Fig. 4. If the normalized 2-point RWHT is substituted for every 2-point transform in Fig.4, the normalized 8-point RWHT is obtained.

4. LOSSLESS AND LOSSY COMPRESSION EFFICIENCY

It is desirable that both lossless and lossy compression efficiency should be high in unified lossless and lossy image coding system. In this section we investigate lossless and lossy compression efficiency of RDCT.

Table 1 shows the entropy of the transform coefficients, when still images are transformed by RDCT where the transforms are applied in vertical direction after horizontal direction. The entropies of PCM, DPCM, Reversible SSKF5*3[1] (RSSKF), 8-point RWHT are also shown in Table 1, where the values of PCM are entropies of input images, DPCM is the method that predicts from the upper and left sides, and ten-band octave decomposition is used in RSSKF. The input images, GIRL, MOON, COUPLE and AERIAL are 256x256 with 8bit/pixel. It is seen from Table 1 that RDCT is superior to RWHT and comparable to RSSKF and DPCM.



$$a = \frac{1}{\sqrt{2}}$$

Fig. 4 8-point WHT.

Table 1. Entropy of lossless compression (bit/pel).

	GIRL	MOON	COUPLE	AERIAL
PCM	6.414	6.709	6.220	7.312
DPCM	4.794	5.049	4.573	5.894
RSSKF	4.674	5.033	4.355	5.782
RDCT	4.703	5.067	4.462	5.776
RWHT	4.967	5.180	4.705	6.120

Fig. 5 is SNR versus entropy of lossy reconstruction of RDCT, RWHT, RSSKF and DCT. It is seen from Fig. 5 that RDCT is superior to RWHT and comparable to RSSKF and DCT at low entropy. It seems that the compression efficiency of RDCT is low at high entropy because of the floor functions in Eq. 6.

5. CONCLUSIONS

In this paper N-point reversible transform is firstly presented, then 8-point RDCT is obtained by substituting reversible transform for every transform which composes 8-point DCT. The forward and inverse transforms of RDCT are represented by simple equations. If floor function is ignored in RDCT, it becomes the same as DCT. RDCT is also normalized so that we can avoid the problem that dynamic range is nonuniform. Simulation on continuous-tone still images shows that lossless and lossy compression efficiencies of RDCT are comparable to those obtained with reversible wavelet transform. RDCT is seem to be promising as the heart of unified lossless and lossy image coding system.

6. REFERENCES

[1] K. Komatsu and K. Sezaki, "Reversible subband coding of images," in *Proc. SPIE VCIP'95*, Taipei, Taiwan, vol. 2501, pp. 676-684, May 1995.

[2] A. Said and W. Perlman, "Reversible image compression via multiresolution representation and predictive coding," in *Proc. SPIE VCIP'93*, Cambridge, MA, vol. 2094, pp. 664-674, Nov 1993.

[3] A. Zandi, J. D. Allen, E L. Schwartz and M. Boliek, "CREW: Compression with reversible embedded wavelets," in *Proc. Data Compression Conf.*, Snowbird, UT, pp. 212-221, Mar. 1995.

[4] K. Irie and R. Kishimoto: "A study on perfect reconstructive subband coding", *IEEE Trans. Circuits & Syst. Video Technol.*, vol. 1, no.1. pp.42-48, March 1991.

[5] M. Ohta and R. Oami, "Lossless DCT picture coding," *Journal of the Institute of Television Engineers of Japan*, vol. 50, pp. 1162-1171, Aug. 1996.

[6] K. Komatsu and K. Sezaki, "Reversible transform coding of images," in *Proc. SPIE VCIP'96*, Orlando, FL, vol. 2727, pp.1094-1103, Mar. 1996.

[7] F. Bruekers and A. Enden, "New Networks for Perfect Inversion and Perfect Reconstruction," *IEEE J. Sel. Areas Commun.*, vol. 10, no.1. pp.130-137, Jan. 1992.

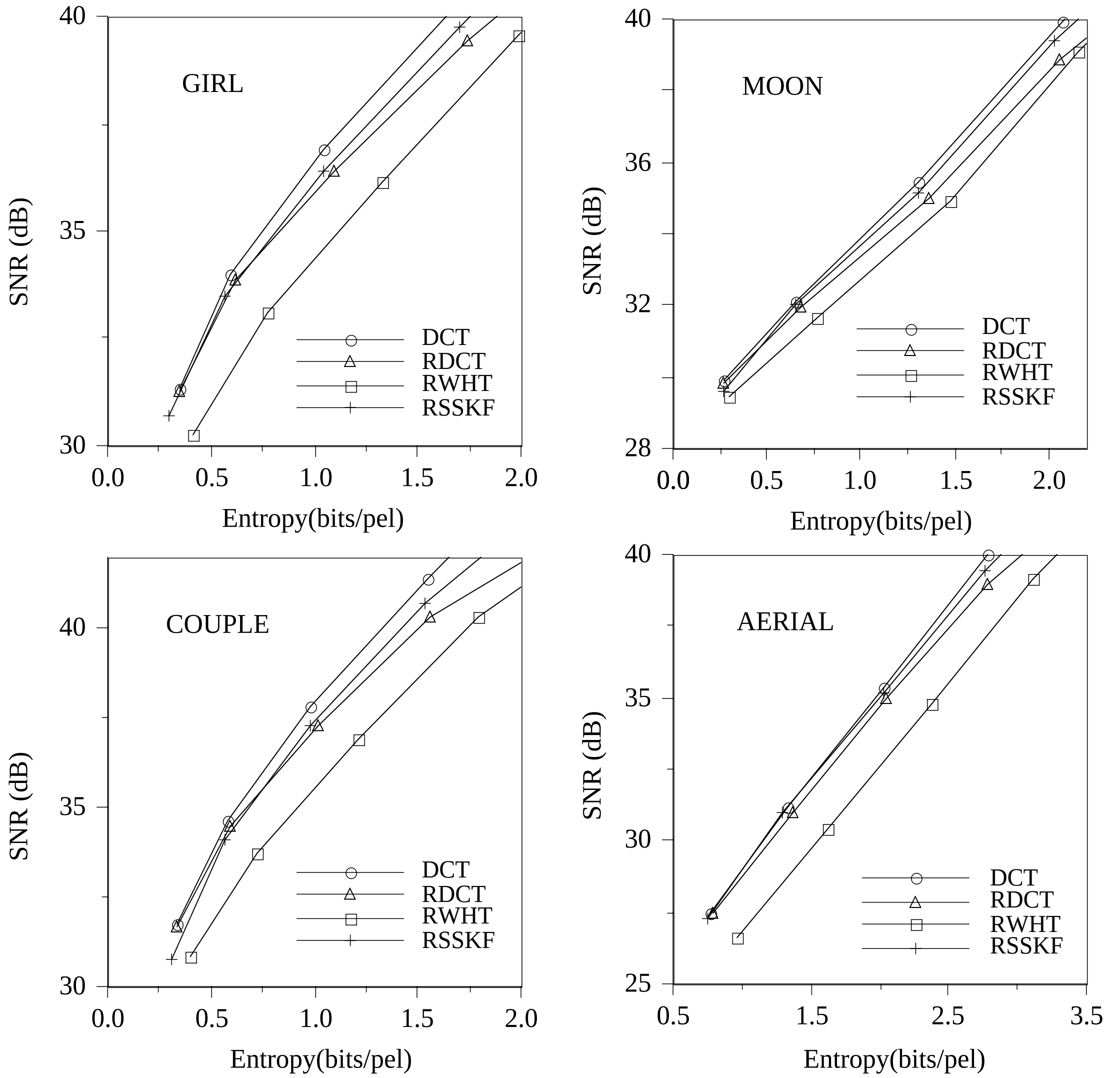


Fig. 5. Compression efficiency of lossy reconstruction