COMPUTING OPTICAL FLOW FOR MOTION IMAGES SEQUENCES BY APPLYING MULTI-CONSTRAINTS TO MULTI-LOCATIONS

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ABSTRACT

Computing optical flow is one of the most fundamental problem to the motion image analysis. Many methods have been proposed for computing optical flow, among them gradient-based methods are the most well-known and most used. In the paper, a new gradient-based method for the computation of optical flow is proposed. In this method, optical flow is computed by minimizing a weighted least-squares error estimator for a constant motion vector model in a local spatial neighborhood, where the weight of each image location in the neighborhood is determined by its multiple constraints. Several experiments on real and synthetic image sequences have been carried out to verify the efficacy and the reliability of the new method.

1. INTRODUCTION

Early days' computer-based vision systems had been constrained to static image analysis due to the memory and processing limitations of the computers. However, significant advances in computer hardware have enabled the computer systems to analyze motion image sequences and interpret complex three-dimensional(3D) scenes[1]. Motion image analysis has become one of the most important and most studied subject in the field of computer vision during the last two decades because it has many advantages comparing to static image analysis. Motion image analysis systems are much more comparable to human-vision systems because man is always in a moving environment or in the motion of his own; A motion image sequence provides much more information than a single picture and some lost information in static images, such as objects' moving speed and 3D structure, etc., may be recovered through motion image analysis.

Optical flow, the two-dimensional motion vectors field, arises from the relative motion between an observer and objects in the environment. Once determined, optical flow can give useful information about 3D motion structure of the objects and spatial arrangements of the environment[2]. The computation of optical flow is one of the most fundamental and important subject in the field of motion image analysis and a wide range of applications, such as robot vision, auto-navigation can be its potential beneficiaries. However, computing optical flow is still far from complete and remains challenging because the computation is intrinsically affected by such difficulties as motion discontinuities, noise in image intensity and occlusion between different moving objects and between moving objects and stationary background[3].

Many methods for computing optical flow have been proposed and others continue to appear. Of them, gradient-based methods, which compute optical flow from image intensity's gradients, are the most well known and most used[4]. In this paper, a new method for computing optical flow has been proposed. The new method is the combination of two gradient-based methods, one is multiple constraints method and the other is weighted local spatial optimization method. In the proposed method, multiple constraints are firstly derived for each image location, then the constraints are applied to multiple image locations by implementing weighted least-squares fit to a constant model of motion vector in each spatial neighborhood.

The organization of the paper is as follows. In section 2, we will present multiple constraints method and investigate its advantages and disadvantages. In section 3, we will explain weighted local spatial optimization method briefly and point out its problems. The new method and its algorithm will be presented in section 4. To verify our method, several experiments have been carried out and the results are shown in section 6. Finally, the paper will be concluded in section 7.

2. MULTIPLE CONSTRAINTS METHOD

Gradient-based methods are based on *gradient constraint equation*, which relates image intensity's gradients to the two components of motion vector

$$E_x u + E_y v + E_t = 0, \qquad (1)$$

where E_x, E_y and E_t are image intensity's spatial and temporal gradients; u and v are motion vector's two components along x and y axes. Differencing equation (1) with respect to x and y we obtain two new equations,

$$
\begin{aligned}
E_{xx} u + E_{yx} v &= -E_{tx} \\
E_{xy} u + E_{yy} v &= -E_{tx}.
\end{aligned}
\qquad (2)
$$

Added with equation (1), three constraint equations forming an over-determined set can be obtained for two unknowns u and v [5],

$$A \begin{pmatrix} u \\ v \end{pmatrix} = \vec{b} \qquad (3)$$

$$A = \begin{pmatrix} E_x & E_y \\ E_{xx} & E_{xy} \\ E_{yx} & E_{yy} \end{pmatrix} \qquad \vec{b} = - \begin{pmatrix} E_t \\ E_{tx} \\ E_{ty} \end{pmatrix}.$$

Therefore, motion vector $\vec{v} = (u, v)^t$ at image location (x, y) can be solved as

$$\vec{v} = (A^t A)^{-1} A^t \vec{b}. \tag{4}$$

Multiple constraints method has many advantages comparing to other gradient-based methods. For each image location, the number of constraints is enough for determining motion vector uniquely, so optical flow can be computed with high spatial resolution; Multiple constraints method is able to provide robust computation of optical flow because it is based on a noise-insensitive over-determined system.

However, there is a fatal problem in this method. When there are inconvenient or incorrect constraint equations in multi-constraints, it is necessary to detect and eliminate these "ill-posed" constraints from further computation otherwise the computed optical flow will tend to be in low reliability. In the method above, however, each constraint equation is equally used and no arguments exist to examine its "well-posedness".

3. WEIGHTED LOCAL SPATIAL OPTIMIZATION METHOD

Examining *gradient constraint equation* (equation (1)), we can see that the equation is in two unknowns (u, v). Therefore additional constraints are necessary for the gradient-based methods which use only *gradient constraint equation* for each image location.

Weighted local spatial optimization (WLSO) method introduced in another constraint that assumes optical flow is locally constant in a small spatial neighborhood. In Lucas and Kanade's[6], optical flow is computed by implementing a weighted least-squares fit to a constant motion vector in the spatial neighborhood Ω and minimizing

$$\sum_{i \in \Omega} w_i^2 \left(E_x u + E_y v + E_t \right)^2. \tag{5}$$

where w_i denotes the weight function which allocates different weights to different image locations in the neighborhood. Motion vector is then solved as

$$u = -\frac{1}{\Delta} \left(\sum w_i^2 E_y^2 \cdot \sum w_i^2 E_x E_t - \right.$$
$$\left. \sum w_i^2 E_x E_y \cdot \sum w_i^2 E_y E_t \right)$$
$$v = -\frac{1}{\Delta} \left(\sum w_i^2 E_x^2 \cdot \sum w_i^2 E_y E_t - \right.$$
$$\left. \sum w_i^2 E_x E_y \cdot \sum w_i^2 E_x E_t \right)$$
$$\Delta = \sum w_i^2 E_x^2 \cdot \sum w_i^2 E_y^2 - \left(\sum w_i^2 E_x E_y \right)^2 \tag{6}$$

WLSO method is famous for its simple structure and efficient computation because of its implementation of the least-squares method. However, the problem is that the least-squares method is computational efficient but non-robust. In this method the quadratic error term weighs heavily the contributions to the "optimal" solution from the data points which have large residual error[7]. Which means, whenever the gradient constraints are incorrectly estimated or there are motion discontinuities in the neighborhood, the WLSO method will always result in high-error-level optical flow.

4. APPLYING MULTIPLE CONSTRAINTS TO MULTIPLE IMAGE LOCATIONS

In this section, we will propose a new method for computing optical flow. The method applies multiple constraints to multiple image locations with the assumption of local constancy of optical flow. Firstly, for each image location in a spatial neighborhood, the "well-posedness" of its constraints is examined and image locations whose constraints are considered "ill-posed" will be eliminated from further computation; Secondly, WLSO method is implemented to compute optical flow using the remained image locations, and the weights are determined according to their computational reliabilities.

The rest of this section is consisted of two parts. Hessian matrix, which if used to examine the "well-posedness" of image locations' multi-constraint, will be firstly introduced. In the second part, we will present the algorithm of the proposed method.

4.1. Hessian Matrix

Rewrite equation (2) as

$$\begin{pmatrix} E_{xx} & E_{yx} \\ E_{xy} & E_{yy} \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = - \begin{pmatrix} E_{tx} \\ E_{ty} \end{pmatrix}, \tag{7}$$

where the coefficient matrix at the right side of the equation above is denoted as Hessian Matrix

$$H = \begin{pmatrix} E_{xx} & E_{yx} \\ E_{xy} & E_{yy} \end{pmatrix}. \tag{8}$$

Nagel and Enkelmann pointed out that the numerical stability of the computation of optical flow is guaranteed when the inversion of H is numerically stable (Nagel [8]). This condition is fulfilled when the determinant of H, d_H, is large and the conditioning number c_H of H is close to 1. Since H is symmetric ($E_{xy} = E_{yx}$), it follows that

$$c_H = \left| \frac{\lambda_{min}}{\lambda_{max}} \right|$$

where λ_{min} and λ_{max} are the two real eigenvalues of H with smaller and largest absolute value, respectively.

Consequently, it is evident that d_H large and $c_H \sim 1$ imply numerical stability in the computation of optical flow vectors.

4.2. Combining Hessian Matrix with WLSO : Algorithm

The characteristics of Hessian matrix suggest us a new method that applies multiple constraints to multiple image locations

by combining Hessian matrix with Weighted Local Spatial Optimization method.

In this method, "well-posedness" of each image location's multiple constraints is firstly examined according to the value of the determinant of its Hessian Matrix, d_H; after the "ill-posed" being eliminated, optical flow is computed by using WLSO method, where the weight of each remained location is defined as the conditioning number, c_H, of its Hessian Matrix.

The algorithm of the new method is shown as follows:

1. Given a local spatial neighborhood (Ω) within which optical flow is assumed to be constant, for each image location in the neighborhood we examine its Hessian matrix:

   if
   $$d_H < t_1 \quad t_1 : \text{threshold of } d_H$$
   then
   　exclude the location from further computation.
   elseif
   $$\lambda_{max} > \lambda_{min} > t_2 \quad t_2 : \text{threshold of } \lambda$$
   then
   　use $c_H = \left| \frac{\lambda_{min}}{\lambda_{max}} \right|$ as the weight, w, of the location and proceed to step 2.
   else
   　set the weight of the location be zero, $w = 0.0$.

2. Repeat step 1. until all the image locations in the neighborhood have been examined;

3. Compute optical flow within the local neighborhood using WLSO method, the weight of each image location is defined as the value of c_H. □

Comparing to the original multiple constraints method and weighted local spatial optimization method, the new method has several advantages.

In the new method, Hessian matrix has been used as the criterion to evaluate the "well-posedness" or the reliability of multiple constraints. As Hessian matrix is highly related to the intensity distribution around the image location, it can be understood that the new method computes optical flow considering not only the motion constraints but also the image intensity distribution in the image.

Because of the exclusion of the "ill-posed" image locations, which always lead to large residual errors, WLSO's least-squares estimator will be affected less and the method is expected to be more robust. Moreover, instead of simply giving heavier weights to the center than the periphery within the local neighborhood implemented in the original WLSO method, the new method gives higher weights to the image locations whose gradients constraints have higher numerical reliability.

5. EXPERIMENTAL TECHNIQUES

To examine the performance of the new method, several experiments have been carried out on real image sequences and synthetic sequences for which optical flows were known. Before presenting the experimental results, we will describe briefly the image sequences used in the experiments.

5.1. Synthetic Image Sequences

The main advantage of synthetic inputs is that we have the access to the true optical flow and can therefore quantify the performance. Our synthetic image sequences include:

Sinusoidal Inputs : This is about a rotating sinusoidal plane and the rotating velocity is $\omega = 0.5$ degrees per frame.

Translating Planers : The sequence is consists of two different translating planers. The planar on the left is moving with velocity $v_1 = (0.0, -0.8)$ pixels/frame and the other planar is translating with velocity $v_2 = (0.0, 1.0)$ pixels/frame.

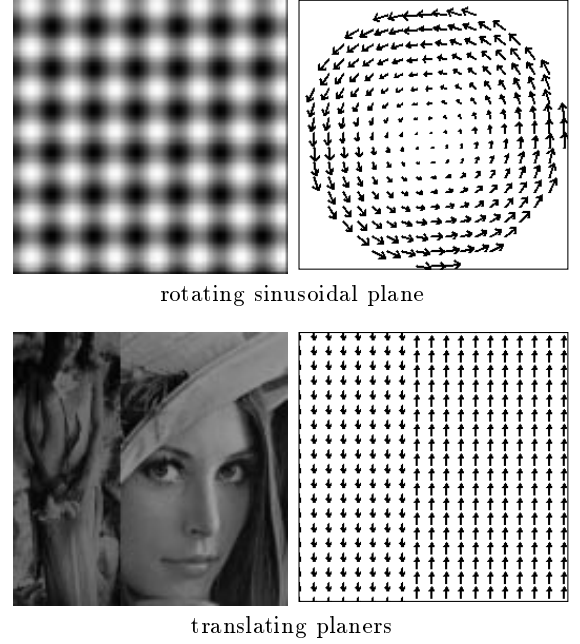rotating sinusoidal plane

translating planers

Figure 1: Frames from synthetic image sequences and their true optical flows

5.2. Real Image Sequences

Two real image sequences, shown in fig.2, were also used:

Dilational Sequence : The sequence was taken while the camera moving along it's line of sight toward the table near the center of the image. Image velocities are typically less than 1 pixel/frame.

Hamburg Taxi Sequence : In this street scene there were four moving objects: 1) the taxi turning the corner; 2) a car in the lower left, driving from left to right; 3) a van in the lower right driving right to left; and 4) a pedestrian in the upper left. Image speeds of the four moving objects are approximately 1.0, 2.0, 3,0, and 0.3 pixels/frame respectively.

The dilational sequence is taken by the authors and the Hamburg Taxi sequence was provided courtesy of the University of Hamburg.
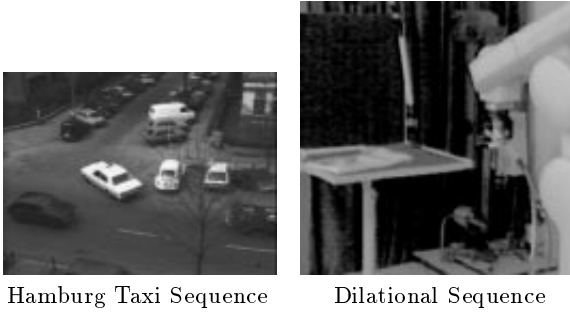
Hamburg Taxi Sequence   Dilational Sequence

Figure 2: Frames from real image sequences
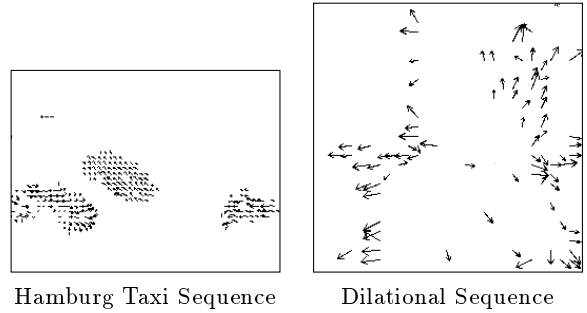

Hamburg Taxi Sequence   Dilational Sequence

Figure 3: Computed optical flows for real image sequences

5.3. Error Measurement

Following [4] we used an angular measure of error. Let velocities $\vec{v} = (u,v)^t$ be represented as 3D direction vectors, $\vec{V} \equiv \frac{1}{u^2+v^2+1}(u,v,1)^t$. The angular error between the true velocity $\vec{V_t}$ and an estimate $\vec{V_e}$ is

$$\phi_E = \arccos(\vec{V_t} \cdot \vec{V_e}). \qquad (9)$$

6. EXPERIMENTAL RESULTS

In comparing the performance of the proposed method and two original methods , we concentrate on computing time, error statistics and the density of computed optical flows.

rotating sinusoidal plaid

Technique	Cost(s)	Average Error	Density
Tretiak, Paster[5]	7.60	58.366°	0.974
Lucas, Kanade[6]	3.84	34.777°	0.991
Proposed method	5.06	19.798°	0.919

translating planers

Technique	Cost(s)	Average Error	Density
Tretiak, Paster[5]	7.57	4.462°	0.709
Lucas, Kanade[6]	3.81	0.655°	0.894
Proposed method	5.20	0.619°	0.857

Table 1: Summaries of the results on synthetic sequences

From Table 1 we can see that, although there is no significant difference of the processing time and the density among the three methods, the proposed method provides the measurements with the lowest average angular errors. That means, our method is able to provide more reliable measurements for the computation of optical flow.

For the real image sequences, we compute optical flows in three steps. The original sequences were firstly smoothed using a 3D Gaussian filter to reduce the existing noise; The proposed method was then applied to the smoothed sequences for the computation of optical flow; Finally, computed optical flows were again smoothed with a 2D Gaussian filter.

7. CONCLUSION

In this paper, we proposed a new method, which applies multiple constraints to multiple image locations, for computing optical flow of motion image sequences.In this method, optical flow was computed by minimizing a weighted least-squares error estimator for a constant motion vector model in a local spatial neighborhood, where the weight of each image locations in the neighborhood was determined by its multiple constraints.

Efficacy and reliability of the proposed method were verified through several experiments on real and synthetic image sequences.

8. REFERENCES

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