# A NEW ADAPTIVE NOTCH FILTER WITH CONSTRAINED POLES AND ZEROS USING STEIGLITZ-MCBRIDE METHOD

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ABSTRACT

In this paper we present a new adaptive notch filter (ANF) using the well-known Steiglitz-McBride method (SMM) [1] for an IIR filter with the constrained poles and zeros. The proposed ANF, termed as SMM-ANF, converges to the unbiased solution, has fast convergence speed, and requires less computational complexity than existing recursive maximum likelihood adaptive notch filters (RML-ANF) [2]. In the stationary environments, we analyze SMM-ANF convergence properties using the ordinary differential equation (ODE) [4] technique; we derive conditions for the SMM-ANF convergence solution unbiased. Simulations further display that SMM-ANF has better resolution in identifying frequencies of multiple sine waves than RML-ANF. In the nonstationary environments, we also show that SMM-ANF and RML-ANF have approximately identical tracking performance. Simulations are also done to verify the theoretically derived results.

## 1. INTRODUCTION

Adaptive notch filters (ANF) can be used to estimate frequencies of sinusoidal signals in noise and eliminate sinusoidal disturbances with unknown time-varying frequencies. Early development of ANFs used the structure of FIR filters. Since IIR filters are computationally more efficient than FIR filters for characterizing sinusoidal signals, recently several new IIR ANFs have been proposed. One of the most popular structure was proposed by Nehorai [2, 3], and Ng [5] which constrains that the poles and zeros of IIR filters are not only identical but also symmetric with respect to the real axis; such structure is of the minimum number of filter coefficients because $m$ coefficients are required to model a signal with $m$ frequencies of sine waves. The proposed ANF in this paper is also developed on this filter structure.

Several ANF algorithms using the IIR filter with minimum number of coefficients were developed: Nehorai [2] presented an ANF algorithm, called the recursive maximum likelihood method, which is termed RML-ANF here. Ng [5] presented two ANF adaptation algorithms called stochastic Gauss-Newton (SGN) and approximate maximum likelihood (AML) algorithms. The comparison of SGN, RML, and AML ANFs was reported in [7]: RML-ANF yields the most accurate parameter estimate but its computational complexity is high; AML-ANF requires the least computational burden but obtains a biased solution.

In this paper we derive a new ANF algorithm, called SMM-ANF, by modifying the well-known system identification approach, Steiglitz-McBride method (SMM). Since SMM is well-known for its simple realization, fast convergence speed, and proven convergence to the unbiased solution for off-line identification and on-line adaptive IIR filters [8, 9]. The proposed ANF algorithm inherits such advantages: the algorithm requires about the same computational complexity as AML but converges to an unbiased solution; the convergence speed is comparable to that of SGN and RML ANFs; the tracking performance is approximately identical to that of RML-ANF. Simulations further show that SMM-ANF has better resolution in discerning frequencies of multiple sine waves.

This paper is structured as follows: In Section 2 we derive the formulation and algorithm of SMM-ANF. In Section 3, we list the main results of the analytic convergence and tracking properties of SMM-ANF. The simulation results are discussed in Section 4. Section 5 concludes the paper.

## 2. SMM FOR ADAPTIVE NOTCH FILTERS

Here we derive SMM-ANF by modifying SMM for the structure of adaptive line enhancement(ALE). SMM is used to identify the notch filter; the key is to use delayed measured signal as the input for SMM. We also show that the resulting block diagram of SMM-ANF, if used for off-line frequency estimation and the noise is assumed white, is equivalent to the generalized least-squares (GLS) method [14].

### 2.1. Steiglitz-McBride Method

SMM [1] block diagram for system identification is shown in Fig. 1, where $D_k(q^{-1})$ and $N_k(q^{-1})$ are computed to minimize the mean square error of $e_s(n)$ under which the coefficients of all-pole prefilters $1/D_{k-1}(q^{-1})$ are fixed. Note that $q^{-1}$ is a unit-delay operator. It was shown in [8] that if the orders of $D_k(q^{-1})$ and $N_k(q^{-1})$ are sufficient and the measurement noise $e(n)$ is white, SMM will converge asymptotically to the unbiased solution.

The realization of SMM is simple and the convergence speed is fast. Several SMM-related algorithms were developed for off-line frequency estimation from the disturbed sinusoidal signals such as iterative quadratic maximum like-

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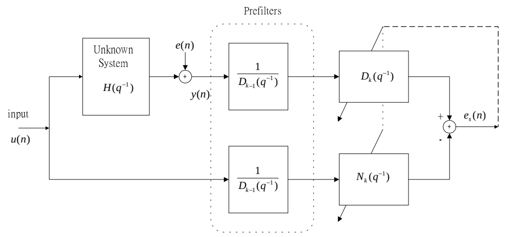


Figure 1: Block diagram of SMM for system identification.

lihood (IQML) [11], GLS, and iterative filtering algorithm (IFA) [10]. SMM is also modified for on-line adaptive IIR filters [9] and is used successfully for various applications. However, as far as we know, on-line SMM for adaptive notch filters is new.

### 2.2. Notch Filter

Consider a measured stationary data $y(n)$ which comprises a known number of sine waves and a measurement noise $e(n)$,

$$y(n) = \sum_{i=1}^{m} c_i \cos(w_i n + \phi_i) + e(n) \tag{1}$$

where the amplitudes $\{c_i\}$, phases $\{\phi_i\}$, and frequencies $\{w_i\}$ are unknown constants. It is known that $y(n)$ can be represented by an ARMA model [10],

$$A(q^{-1})y(n) = A(q^{-1})e(n) \tag{2}$$

where $A(q^{-1})$ is a monic polynomial of order $2m$ with $m$ coefficients and its roots are on the unit circle with arguments equal to $\{w_i\}$.

Let $\rho \in (0,1)$. Define

$$\epsilon(n) = \frac{A(q^{-1})}{A(\rho q^{-1})} y(n). \tag{3}$$

The parameter $\rho \in (0,1)$ is a contraction factor which enables the notch filter $A(q^{-1})/A(\rho q^{-1})$ stable. It is known that $\epsilon(n)$ in (3) approximates the noise $e(n)$ to an order $o(\sqrt{1-\rho})$, where $o(x)$ is defined such that $|o(x)/x|$ is bounded as $x \to 0$. If the notch filter is identified, the signal frequencies can be obtained by solving $A(q^{-1})$. Therefore, the frequency estimation of the signal can be formulated as the notch filter identification.

### 2.3. SMM for Notch Filter Estimation

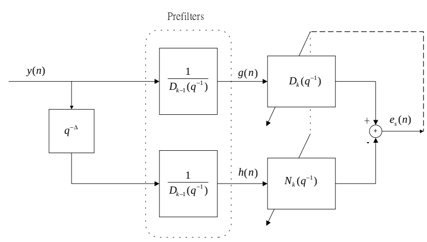


Figure 2: SMM using ALE for notch filter identification.

To apply SMM for identifying the notch filter we adopt ALE structure and use the delayed $y(n)$ as input; the resulting block diagram is depicted in Fig. 2. The delay parameter $\Delta$ discussed in [13] is better chosen to be larger than the correlation length of noise $e(n)$, but smaller than the correlation length of sine wave signals. In particular, if the noise is white, $\Delta$ is chosen as 1. If the noise $e(n)$ is colored, then $\Delta$ can be selected to decorrelate the signals between $g(n)$ and $h(n)$ of prefilter outputs in the upper and lower paths in Fig. 2. Such flexibility enables the proposed algorithm to identify frequencies of measured signals in the colored noise environments.

Since the resulting transfer function at convergence is desired to be the notch filter, the following equation should be satisfied

$$\lim_{k\to\infty} [1 - q^{-\Delta} \frac{N_k(q^{-1})}{D_k(\rho q^{-1})}] = \frac{A(q^{-1})}{A(\rho q^{-1})} \tag{4}$$

Therefore, the polynomials $D_k(\rho q^{-1})$ and $N_k(q^{-1})$ in Fig. 2 can be defined as

$$D_k(q^{-1}) = \hat{A}_k(\rho q^{-1}), \quad N_k(q^{-1}) = q^{\Delta}[\hat{A}_k(\rho q^{-1}) - \hat{A}_k(q^{-1})] \tag{5}$$

where $\hat{A}_k(q^{-1}) = 1 + q^{-2m} + \sum_{k=1}^{m-1} \hat{a}_k(q^{-k} + q^{k-2m}) + \hat{a}_m q^{-m}$.

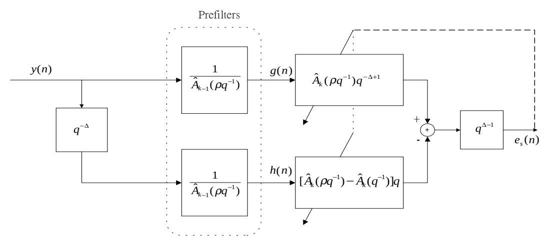


Figure 3: Block diagram using SMM in an ALE structure. It is unrealizable due to the advance operator for $\Delta > 1$.

Using (5) for Fig. 2 we obtain the block diagram shown in Fig. 3. Note that in Fig. 3 we extract the factor $q^{\Delta-1}$ to the output because $N_k(q^{-1})$ in (5) is not realizable except $\Delta = 1$. If the additive noise is white, then $\Delta$ is set to 1; the new adaptive notch filter can be directly derived from Fig. 3. However, if $\Delta > 1$, then Fig. 3 is not realizable due to the advance operator $q^{\Delta-1}$. Since the advance operation does not affect the magnitude of frequency response, here we neglect the advance operator and obtain the new realizable SMM-ANF shown in Fig. 4.

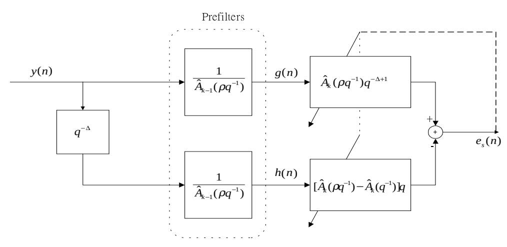


Figure 4: Block diagram of SMM-ANF

### 2.4. Adaptive Algorithm

The adaptive algorithm can be derived directly from Fig. 4. Let the estimated coefficient vector $\hat{\boldsymbol{\theta}}(k) = [\hat{a}_{k,1}, \cdots, \hat{a}_{k,m}]^T$ where the superscript $T$ denotes the transpose operation. Using the standard recursive least square (RLS) procedure, we derive the detailed adaptive algorithm of SMM-ANF as below:

Design Variables: $m, \Delta, \lambda_1, \lambda_r, \lambda_\infty, \rho_1, \rho_r, \rho_\infty, \kappa$.
Initialization:

$$\begin{aligned}\hat{\boldsymbol{\theta}}(0) &= \hat{\boldsymbol{\theta}}(1) = [0, \cdots, 0]^T, \quad P_0 = \kappa I \\ g(-i) &= 0 \quad \text{for } i = 1, \cdots, 2m\end{aligned}$$

Nominal Values:

$$\begin{aligned}m &= \text{no. of sinusoidal frequencies} = \text{no. of parameters} \\ \kappa &= I(\text{identity matrix}) \\ \lambda_1 &= 0.7, \quad \lambda_r = 0.99, \quad \lambda_\infty = 1 \\ \rho_1 &= 0.7, \quad \rho_r = 0.99, \quad \rho_\infty = 0.995\end{aligned}$$

Main Loop:

$$\begin{aligned}g(n) &= [\frac{1}{\hat{A}_{k-1}(q^{-1})}]y(n) \\ h(n) &= [\frac{1}{\hat{A}_{k-1}(q^{-1})}]y(n-\Delta) \\ \psi_i(n) &= \begin{cases} -\rho_n^i g(n-i-\Delta+1) + (\rho_n^i - 1)h(n-i+1) \\ -\rho_n^{2m-i} g(n-2m+i-\Delta+1) + (\rho_n^{2m-i} - 1) \\ h(n-2m+i+1), \qquad i = 1, \cdots, m-1 \\ -\rho_n^m g(n-m-\Delta+1) + (\rho_n^m - 1) \\ h(n-m+1), \qquad i = m \end{cases} \\ \boldsymbol{\psi}(n) &= [\psi_1(n), \psi_2(n), \cdots, \psi_m(n)]^T \\ e_s(n) &= g(n-\Delta+1) + \rho_n^{2m} g(n-2m-\Delta+1) \\ &\quad -(\rho_n^{2m} - 1)h(n-2m+1) - \boldsymbol{\psi}^T(n) \cdot \hat{\boldsymbol{\theta}}_{n-1} \\ P_n &= \frac{1}{\lambda_n}\left[P_{n-1} - \frac{P_{n-1}\boldsymbol{\psi}(n)\boldsymbol{\psi}^T(n)P_{n-1}}{\lambda_n + \boldsymbol{\psi}^T(n)P_{n-1}\boldsymbol{\psi}(n)}\right] \\ \hat{\boldsymbol{\theta}}_n &= \hat{\boldsymbol{\theta}}_{n-1} + P_n\boldsymbol{\psi}(n)e_s(n) \\ \lambda_{n+1} &= \lambda_r\lambda_n + (1-\lambda_r)\lambda_\infty \\ \rho_{n+1} &= \rho_r\rho_n + (1-\rho_r)\rho_\infty\end{aligned}$$

Some comments on the algorithm are listed as follows:

- The presented algorithm is in the form of RLS and is similar to that of RML, SGN and AML ANFs. The differences between these algorithms are on the choice of the regression vector $\boldsymbol{\psi}(n)$ and error $e_s(n)$. It is well known that different convergence and tracking performances arise from the choice of the regression vector and error.
- The parameter $\rho$ is often set to a small initial value and is gradually increased to a value close to 1. Such strategy has been used in previous ANF algorithms such as RML, SGN and AML. The purpose for $\rho$ with a small initial value is to widen the notch of the prefilter frequency response to prevent from the slow convergence due to poor estimate of initial filter coefficients; conversely, the reason for $\rho_\infty$ set close to 1 is to narrow the notch for reducing the bias of the frequency estimate after convergence. We also show that for SMM-ANF such setting of the parameter $\rho$ not only increases the convergence speed but also helps ensure the convergence of the algorithm. Note that $\rho_\infty$ is always less than 1 in order to stabilize the prefilters.

### 2.5. Relation between SMM-ANF and RGLS

The adaptive algorithm can be further simplified if the prefilter coefficients are assumed to vary slowly because the prefilter output $h(n)$ can be approximated by the delayed $g(n)$. Theoretically, if the prefilters are time-invariant, then the prefilter and the delay commute; therefore, $h(n) = g(n-\Delta)$.

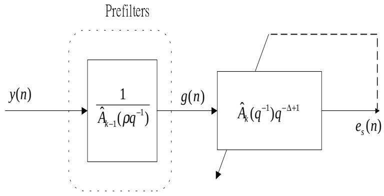


Figure 5: Equivalent block diagram of SMM-ANF for off-line frequency estimation.

The simplified block diagram of SMM-ANF is shown in Fig. 5; the adaptive algorithm for Fig. 5 is obviously much simplified. If $\Delta = 1$ and if the block diagram of Fig. 5 is used for off-line frequency estimation, then the algorithm is equivalent to GLS [14] and is equivalent to IFA [10] as $\rho = 1$. Therefore, if the delay parameter $\Delta$ is 1, then the simplified SMM-ANF derived for Fig. 5 is equivalent to recursive generalized least square (RGLS) [15] method.

Extensive simulations show that the simplified algorithm for Fig. 5 obtains convergence and tracking performances as close as SMM-ANF for Fig. 4 if the additive noise is white. However, if the noise is colored, the simplified algorithm is no longer useful and often results in incorrect convergence solution because the decorrelation effect due to $\Delta$ no longer exists.

## 3. CONVERGENCE AND TRACKING ANALYSES

The convergence and tracking analyses of the ANF are discussed in brief here. We apply the well-known ODE method to analyze SMM-ANF convergence properties in the stationary environments. We also analyze the ANF tracking performance following the analysis procedure of [12] for RML-ANF using the technique in [6].

Due to the limited space, here we only list the main result and skip the detailed derivation. We derive from the analysis a sufficient condition constraining the noise statistics under which SMM-ANF asymptotical convergence solution is unbiased. Such condition is used to show that SMM-ANF convergence solution is unbiased as the noise is white and the stabilizing parameter $\rho$ is infinitely close to 1. We also derive a necessary condition for SMM-ANF solution to be unbiased when the noise is colored. Since the condition is difficult to meet, the ANF convergence solution is generally biased. The ODE approach is also used to show that the setting of parameter $\rho$ in the algorithm not only increases the convergence speed but also helps ensure the convergence of SMM-ANF. From the tracking analysis we show that SMM-ANF performs as well as RML-ANF in the nonstationary environment. The excellent convergence and tracking properties enable the ANF suitable for ANF applications.

## 4. SIMULATION RESULTS

Here we present three simulations to illustrate SMM-ANF performance in the stationary and nonstationary environments. The first simulation demonstrates that SMM-ANF performs as well as RML-ANF in identifying the frequencies of sine wave signals in white noise environments. The second simulation illustrates that SMM-ANF has better resolution in frequency estimation than RML-ANF. The final simulation verifies that SMM-ANF has nearly the same tracking performance as RML-ANF.

### 4.1. Simulation 1

The signal in the numerical example consists of four sine waves, i.e., $y(n) = \sum_{k=1}^{4} c_k \sin 2\pi f_k n + e(n)$, where $e(n)$ is a zero-mean unit-variance white Gaussian noise. The signal frequencies are normalized and set to $f_1 = 0.1, f_2 = 0.2, f_3 = 0.3$, and $f_4 = 0.4Hz$.

We have done 100 independent experiments for signals of various lengths with various signal-to-noise ratios (SNRs). The results are presented in Table 1 where the variance of the estimated frequencies are listed. Note that the data in the parenthesis for SNR $= 0\ dB$ indicate the number of outlier occurring in 100 independent trials; here the trial will be classified as outlier occurring if the frequency estimate deviates from the truth frequency by 0.01. Comparing Table 1 with Table III in [2], we observe that SMM-ANF performs as well as RML-ANF for stationary signals in white noise environments.

Table 1: SMM-ANF simulation results of 100 independent experiments for in additive white noise environments.

| $N$ | SNR | $\hat{f}_1$ Var. | $\hat{f}_2$ Var. | $\hat{f}_3$ Var. | $\hat{f}_4$ Var. |
|---|---|---|---|---|---|
| | | $\times 10^{-4}$ | $\times 10^{-4}$ | $\times 10^{-4}$ | $\times 10^{-4}$ |
| 100 | 0 | 42.89(2) | 38.63(5) | 42.80(2) | 42.36(2) |
| | 8 | 19.11 | 17.86 | 18.77 | 19.25 |
| | 16 | 7.12 | 7.52 | 6.97 | 6.98 |
| | | $\times 10^{-5}$ | $\times 10^{-5}$ | $\times 10^{-5}$ | $\times 10^{-5}$ |
| 500 | 0 | 21.98 | 21.98 | 20.72 | 23.91 |
| | 8 | 9.30 | 9.33 | 8.98 | 9.34 |
| | 16 | 3.49 | 3.71 | 3.65 | 3.53 |
| | | $\times 10^{-6}$ | $\times 10^{-6}$ | $\times 10^{-6}$ | $\times 10^{-6}$ |
| 2000 | 0 | 10.32 | 11.46 | 12.44 | 10.49 |
| | 8 | 4.12 | 4.17 | 4.71 | 4.68 |
| | 16 | 1.62 | 1.96 | 1.91 | 1.66 |

### 4.2. Simulation 2

The measured signal in simulation is of the form $y(n) = c_1 \sin 2\pi f_1 n + c_2 \sin 2\pi f_2 n + e(n)$ where $e(n)$ is a zero-mean unit-variance white noise. For a signal with $f_1 = 0.1, f_2 = 0.11$, and SNR $= 10\ dB$ for each sine wave signal, the frequency estimates of SMM-ANF and RML-ANF are depicted in Fig. 6 which illustrate that while RML-ANF may fail to obtain correct frequency estimate ($\hat{f}_1 = 0.1, \hat{f}_2 = 0.2$) even the corresponding SNRs are large, SMM-ANF accurately converges to the correct estimate.

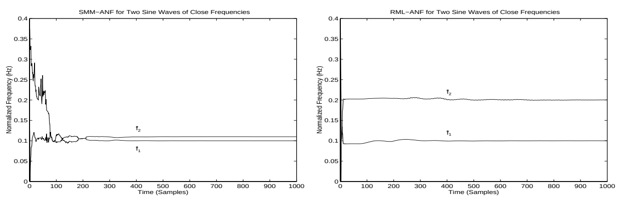


Figure 6: SMM-ANF and RML-ANF simulation results for a signal comprising 2 sine waves and a white noise.

We have tried 100 independent experiments using SMM-ANF and RML-ANF for the above setting. While RML-ANF fails to obtain correct frequency estimate in 63 experiments; SMM-ANF obtains correct frequency estimate for each trial. Qualitatively, it seems that the all-pole prefilters of SMM-ANF enforce the function to discern the sine waves of close frequencies, rendering SMM-ANF excellent in resolving the frequencies of multiple sine waves in the noise-contaminated signals.

### 4.3. Simulation 3

The measured signal is $y(n) = c_1 \cos nw(n) + e(n)$ and $w(n) = w(n-1) + \gamma v(n)$, where $v(n)$ is a white noise of unit variance. The signal for simulation is of $c = 1, w(0) = 0.3$ and of length 10,000. The same simulation in [12] is done for three cases with the following settings: (1) $\gamma = 0, \sigma_e^2 = 0.5$, (2) $\gamma = 0.001, \sigma_e^2 = 0$, (3) $\gamma = 0.001, \sigma_e^2 = 0.5$. In Table 2 simulation results for SMM-ANF and RML-ANF are presented. It shows that the simulation and theoretical results agree well, and SMM-ANF has nearly identical tracking performance to RML-ANF.

Table 2: Tracking performance of SMM-ANF and RML-ANF for three cases of different settings.

| | SMM-ANF | RML-ANF | $\hat{\Pi}_{w_1}$ |
|---|---|---|---|
| case 1 | $1.0567 \cdot 10^{-5}$ | $1.0173 \cdot 10^{-5}$ | $2.5 \cdot 10^{-5}$ |
| case 2 | $3.2647 \cdot 10^{-5}$ | $3.2488 \cdot 10^{-5}$ | $2.5 \cdot 10^{-5}$ |
| case 3 | $4.4995 \cdot 10^{-5}$ | $4.4070 \cdot 10^{-5}$ | $5.0 \cdot 10^{-5}$ |

## 5. CONCLUSIONS

A new ANF, called SMM-ANF is proposed in this paper. We study the convergence and tracking performances of the proposed ANF in the stationary and nonstationary environments by analysis using existed techniques and simulations. It is analytically shown that SMM-ANF has approximately the same tracking performance as RML-ANF, and the convergence solution of SMM-ANF is unbiased if the measurement noise is white as $\rho \to 1$. SMM-ANF displays excellent capability in resolving frequencies of signals with multiple sine waves. We believe that such excellent convergence and tracking properties will make the presented ANF useful for various applications.

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