EIGENSTRUCTURE BEAMSPACE ROOT ESTIMATOR BANK WITH INTERPOLATED ARRAY

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ABSTRACT

A beamspace root modification of PseudoRandom Joint Estimation Strategy (PR-JES) [1] is developed. The essence of PR-JES is to generate the eigenstructure-based estimator bank for given sample covariance or data matrix. Combining the results of "parallel" underlying estimators, PR-JES removes the outliers and improves the threshold performance. In the case of non-uniform array, the interpolated array approach is used to enable the application of root underlying estimators. Simulations and results of real ultrasonic data processing show that the proposed beamspace root implementation significantly outperforms spectral elementspace PR-JES and achieves the performance similar or better than that of stochastic ML method.

1. PRELIMINARIES

Spectral eigenstructure estimators have excellent performance at high SNR. However, these properties are achieved at a significant computational cost. The performances of eigenstructure estimators may degrade as the SNR goes down below a certain threshold. In recent decade, computationally efficient search-free eigenstructure methods have been elaborated, e.g. root MUSIC [2]. Root MUSIC is known to have significant computational advantages as well as better threshold performance relative to spectral MUSIC [3]. Unfortunately, root MUSIC cannot be applied to non-uniform arrays. To overcome this problem, a promising *interpolated* root MUSIC approach has been proposed [4].

Another approach improving the performance of direction finding techniques is the beamspace transformation [5]. It offers lower computational cost and better threshold and asymptotic performances than elementspace approach. Recently, there has been a promising trend to combine root and beamspace approaches into one scheme [6].

Let an array of n sensors receives q narrowband plane waves. The $n \times 1$ vector of sensor outputs is given by [1]-[4]

$$\boldsymbol{x}(i) = \boldsymbol{A}\boldsymbol{s}(i) + \boldsymbol{n}(i)\ , \tag{1}$$

where $\boldsymbol{A} = [\boldsymbol{a}(\theta_1), \ldots, \boldsymbol{a}(\theta_q)]$ is the $n \times q$ direction matrix, $\theta_1, \theta_2, \ldots, \theta_q$ are the signal DOA's, $\boldsymbol{a}(\theta)$ is the $n \times 1$ direction vector, $\boldsymbol{s}(i)$ is the $q \times 1$ vector of random source waveforms, and $\boldsymbol{n}(i)$ is the $n \times 1$ vector of sensor noise. The source waveforms and noise are assumed to be stationary zero-mean independent Gaussian processes. The array covariance matrix $\boldsymbol{R} = \mathrm{E}[\boldsymbol{x}(i)\boldsymbol{x}^H(i)] = \boldsymbol{A}\boldsymbol{S}\boldsymbol{A}^H + \sigma^2 \boldsymbol{I}_n$, where $\boldsymbol{S} = \mathrm{E}[\boldsymbol{s}(i)\boldsymbol{s}^H(i)]$, σ^2 is the noise variance, $\boldsymbol{I}_n$ is the $n \times n$ identity matrix, $\mathrm{E}[\cdot]$ and $(\cdot)^H$ denote the expectation operator and the Hermitian transpose, respectively. Consistent estimates of eigenvectors and eigenvalues of $\boldsymbol{R}$ are given by the eigendecomposition of the sample covariance matrix [1]-[4]

$$\hat{\boldsymbol{R}} = \frac{1}{M}\sum_{i=1}^{M} \boldsymbol{x}(i)\boldsymbol{x}^H(i) = \hat{\boldsymbol{U}}_S \hat{\boldsymbol{\Lambda}}_S \hat{\boldsymbol{U}}_S^H + \hat{\boldsymbol{U}}_N \hat{\boldsymbol{\Lambda}}_N \hat{\boldsymbol{U}}_N^H\ , \tag{2}$$

where the $\hat{q} \times \hat{q}$ and $(n-\hat{q}) \times (n-\hat{q})$ diagonal matrices $\hat{\boldsymbol{\Lambda}}_S$ and $\hat{\boldsymbol{\Lambda}}_N$ contain the $\hat{q}$ and $n-\hat{q}$ signal and noise subspace eigenvalues, respectively; the columns of the $n \times \hat{q}$ and $n \times (n-\hat{q})$ matrices $\hat{\boldsymbol{U}}_S$ and $\hat{\boldsymbol{U}}_N$ contain the signal and noise subspace eigenvectors, and $\hat{q}$ is any consistent estimate of q.

The dimension of array observations can be reduced by the so-called beamspace preprocessing [5] $\boldsymbol{y}(i) = \boldsymbol{T}^H\boldsymbol{x}(i)$ where $\boldsymbol{y}(i)$ is the $p \times 1$ vector of beamspace observations, $\boldsymbol{T}$ is the $n \times p$ beamspace matrix satisfying $\boldsymbol{T}^H\boldsymbol{T} = \boldsymbol{I}_p$.

The $p \times p$ covariance matrix of beamspace observations reads

$$\boldsymbol{R}_{\mathrm{B}} = \mathrm{E}[\boldsymbol{y}(i)\boldsymbol{y}^H(i)] = \boldsymbol{T}^H\boldsymbol{A}\boldsymbol{S}\boldsymbol{A}^H\boldsymbol{T} + \sigma^2\boldsymbol{I}_p\ . \tag{3}$$

Similarly to (2), consistent estimates of eigenvectors and eigenvalues of $\boldsymbol{R}_{\mathrm{B}}$ can be found from the eigendecomposition of the $p \times p$ sample beamspace covariance matrix:

$$\hat{\boldsymbol{R}}_{\mathrm{B}} = \frac{1}{M}\sum_{i=1}^{M} \boldsymbol{y}(i)\boldsymbol{y}^H(i) = \hat{\boldsymbol{E}}_S \hat{\Gamma}_S \hat{\boldsymbol{E}}_S^H + \hat{\boldsymbol{E}}_N \hat{\Gamma}_N \hat{\boldsymbol{E}}_N^H\ . \tag{4}$$

Using (3) and (4), elementspace algorithms can be easily reformulated in beamspace domain. For example, the beamspace root MUSIC polynomial can be expressed as [6]

$$f_{\mathrm{B-MUSIC}}(z) = \boldsymbol{a}^T(z^{-1})\boldsymbol{T}\hat{\boldsymbol{E}}_N\hat{\boldsymbol{E}}_N^H\boldsymbol{T}^H\boldsymbol{a}(z)\ , \tag{5}$$

where $\boldsymbol{a}(z) = (1, z, \ldots, z^{n-1})^T$. The source DOA's can be found from the appropriately selected roots of (5) [3], [4].

2. BEAMSPACE ROOT ESTIMATOR BANK

The concept of estimator bank was introduced in [1]. Given the $n \times M$ data matrix $\boldsymbol{X} = [\boldsymbol{x}(1), \boldsymbol{x}(2), \ldots, \boldsymbol{x}(M)]$, the idea is to generate multiple underlying DOA estimators and then to combine appropriately their results in the final DOA estimate. Given arbitrary beamspace root estimators $f_i(z)$, $i = 1, \ldots, K$ (which are computed using the matrix $\boldsymbol{X}$), let us say that these estimators form the estimator bank $\mathcal{F} = \{f_i(z), i = 1, \ldots, K\}_{\boldsymbol{X}}$ of the

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dimension K. It is very suitable to generate the estimator bank pseudorandomly [1]. This allows one to choose the dimension K dynamically, based on the required compromise between the computational cost and threshold performance [1]. For this purpose, weighted beamspace root MUSIC estimators can be exploited:

$$f_{\boldsymbol{W}}(z) = \boldsymbol{a}^T(z^{-1})\boldsymbol{T}\hat{\boldsymbol{E}}_N\boldsymbol{W}\hat{\boldsymbol{E}}_N^H\boldsymbol{T}^H\boldsymbol{a}(z) \,, \tag{6}$$

where $\boldsymbol{W}$ is the $(p-\hat{q})\times(p-\hat{q})$ non-negative definite weighting matrix. Now, the underlying estimators can be generated via withdrawal of the rank-one weighting matrices $\boldsymbol{W}_i, i = 1,\ldots,K$ from the complex Gaussian random generator:

$$\boldsymbol{W}_i = \boldsymbol{w}_i\boldsymbol{w}_i^H, \quad \boldsymbol{w}_i \sim \mathcal{CN}(\boldsymbol{0}, \boldsymbol{I}_{p-\hat{q}}), \quad i = 1,\ldots,K \,. \tag{7}$$

In the case of non-uniform array, the interpolated ULA should be exploited to allow the application of root estimators. The interpolated approach [4] is based on the idea that the manifold of a virtual ULA can be obtained via linear interpolation of the real non-uniform array within a limited angular sector. In elementspace, the $n\times r$ interpolation matrix $\boldsymbol{B}$ obeys $\boldsymbol{B}^H\boldsymbol{a}(\theta)\simeq\breve{\boldsymbol{a}}(\theta)$ for any angle θ within the sector $[\theta_{\min},\theta_{\max}]$, where $\boldsymbol{a}(\theta)$ and $\breve{\boldsymbol{a}}(\theta)$ are the $n\times 1$ and $r\times 1$ steering vectors of real and interpolated array manifold, respectively. The interpolation matrix $\boldsymbol{B}$ is computed by a least squares fit [4].

The $r\times r$ covariance matrix of the virtual array is given by

$$\boldsymbol{R}_{\rm I} = \boldsymbol{B}^H\boldsymbol{R}\boldsymbol{B} = \boldsymbol{B}^H\boldsymbol{A}\boldsymbol{S}\boldsymbol{A}^H\boldsymbol{B} + \sigma^2\boldsymbol{B}^H\boldsymbol{B} \,. \tag{8}$$

For $r\le n$ the interpolated sample covariance matrix should be "prewhitened" as [4]

$$\hat{\boldsymbol{R}}_{\rm I} = (\boldsymbol{B}^H\boldsymbol{B})^{-1/2}\boldsymbol{B}^H\hat{\boldsymbol{R}}\boldsymbol{B}(\boldsymbol{B}^H\boldsymbol{B})^{-1/2} \,. \tag{9}$$

Applying the $r\times p$ $(q<p\le r)$ beamspace transformation $\boldsymbol{T}$ to the $r\times 1$ data vectors $\breve{\boldsymbol{x}}(i) = (\boldsymbol{B}^H\boldsymbol{B})^{-1/2}\boldsymbol{B}^H\boldsymbol{x}(i)$, we obtain the beamspace interpolated observations $\breve{\boldsymbol{y}}(i) = \boldsymbol{T}^H\breve{\boldsymbol{x}}(i) = \boldsymbol{T}^H(\boldsymbol{B}^H\boldsymbol{B})^{-1/2}\boldsymbol{B}^H\boldsymbol{x}(i)$ with the $p\times p$ covariance matrix $\boldsymbol{R}_{\rm B-I} = {\rm E}[\breve{\boldsymbol{y}}(i)\breve{\boldsymbol{y}}^H(i)]$ given by

$$\boldsymbol{R}_{\rm B-I} = \boldsymbol{T}^H(\boldsymbol{B}^H\boldsymbol{B})^{-1/2}\boldsymbol{B}^H\boldsymbol{R}\boldsymbol{B}(\boldsymbol{B}^H\boldsymbol{B})^{-1/2}\boldsymbol{T} \,. \tag{10}$$

The structure of (10) enables the application of root MUSIC and weighted root MUSIC algorithms. Define the $p\times p$ beamspace interpolated sample covariance matrix

$$\hat{\boldsymbol{R}}_{\rm B-I} = \frac{1}{M}\sum_{i=1}^{M}\breve{\boldsymbol{y}}(i)\breve{\boldsymbol{y}}^H(i) = \boldsymbol{T}^H\hat{\boldsymbol{R}}_{\rm I}\boldsymbol{T}$$
$$= \boldsymbol{T}^H(\boldsymbol{B}^H\boldsymbol{B})^{-1/2}\boldsymbol{B}^H\hat{\boldsymbol{R}}\boldsymbol{B}(\boldsymbol{B}^H\boldsymbol{B})^{-1/2}\boldsymbol{T} \,. \tag{11}$$

Similarly to (2) and (4), write the eigendecomposition of (11) as

$$\hat{\boldsymbol{R}}_{\rm B-I} = \hat{\boldsymbol{V}}_S\hat{\Pi}_S\hat{\boldsymbol{V}}_S^H + \hat{\boldsymbol{V}}_N\hat{\Pi}_N\hat{\boldsymbol{V}}_N^H \,. \tag{12}$$

For a uniform virtual array $\breve{\boldsymbol{a}}(\theta) = \boldsymbol{a}(z)$, where $\boldsymbol{a}(z)$ has the same structure as before but another dimension $r\times 1$. Using (10)-(12), express the polynomial of weighted interpolated beamspace root MUSIC as

$$f_{\boldsymbol{W}}(z) = \boldsymbol{a}^T(z^{-1})(\boldsymbol{B}^H\boldsymbol{B})^{-1/2}\boldsymbol{T}\hat{\boldsymbol{V}}_N\boldsymbol{W}\hat{\boldsymbol{V}}_N^H \cdot\boldsymbol{T}^H(\boldsymbol{B}^H\boldsymbol{B})^{-1/2}\boldsymbol{a}(z) \,. \tag{13}$$

For $\boldsymbol{W} = \boldsymbol{I}_{p-\hat{q}}$, (13) yields the non-weighted interpolated beamspace root MUSIC polynomial

$$f_{\rm B-I-MUSIC}(z) = \boldsymbol{a}^T(z^{-1})(\boldsymbol{B}^H\boldsymbol{B})^{-1/2}\boldsymbol{T} \cdot\hat{\boldsymbol{V}}_N\hat{\boldsymbol{V}}_N^H\boldsymbol{T}^H(\boldsymbol{B}^H\boldsymbol{B})^{-1/2}\boldsymbol{a}(z) \,. \tag{14}$$

3. BEAMSPACE ROOT PR-JES

Elementspace spectral PR-JES [1] exploits the preliminary knowledge of source localization sectors Θ_S. However, the same knowledge is required for the design of array interpolation and beamspace matrices. It is very suitable to estimate these sectors once, and then to exploit the same estimate of Θ_S in PR-JES, as well as for the design of the matrices $\boldsymbol{T}$ and $\boldsymbol{B}$. In multiple sector case, different beamspace and interpolation preprocessing are necessary within each sector. Each underlying estimator is represented now as a polynomial set of the dimension L, where L is the total number of such sectors. Hence, PR-JES has more complicated structure than given in [1], because is applied now to the underlying polynomial sets rather than to the underlying estimators.

Consider the polynomial set $\{f_i(z)\}_{i=1}^L$ associated with an arbitrary estimator $f(z)$. Let the ith polynomial $f_i(z)$ from this set has N_i selected roots [4] associated with the angles localized within an arbitrary interval $[\theta_{a,i},\theta_{b,i}]$. Define $N = \sum_{i=1}^L N_i$. The following hypothesis will be used for sorting out the "successful" estimators:

$$\mathcal{H}: \;\; N\ge\hat{q} \,. \tag{15}$$

(15) can be interpreted as the test on the presence of more than $\hat{q}-1$ sources in $\hat{\Theta}_S$.

Now, the proposed beamspace root implementation of PR-JES [1] can be summarized for given data matrix $\boldsymbol{X}$ and estimate $\hat{q}$ as follows:

Step 1: Specify the estimate of source localization sectors as L non-overlapping intervals $\hat{\Theta}_S = [\theta_{a,1},\theta_{b,1}]\cup[\theta_{a,2},\theta_{b,2}]\cup\cdots\cup[\theta_{a,L},\theta_{b,L}]$.

Step 2: For each interval $[\theta_{a,i},\theta_{b,i}]$ compute the interpolation and beamspace matrices and the beamspace root MUSIC polynomial (14). Denote this polynomial $f_i(z)$. As a result of this step, L different polynomials $f_i(z)$, $i = 1,\ldots,L$ are available for different intervals $[\theta_{a,i},\theta_{b,i}]$, $i = 1,\ldots,L$.

Step 3: For each interval $[\theta_{a,i},\theta_{b,i}]$, $i = 1,\ldots,L$, find the roots $\{z_{i,1},z_{i,2},\ldots,z_{i,N_i}\}$ of $f_i(z)$ associated with the angles localized within $[\theta_{a,i},\theta_{b,i}]$. Test the hypothesis (15). If (15) is accepted then estimate source DOA's from the $\hat{q}$ closest to the unit circle roots selected from the roots $\{z_{1,1},z_{1,2},\ldots,z_{1,N_1},z_{2,1},z_{2,2},\ldots,z_{2,N_2},\ldots,z_{L,1},z_{L,2},\ldots,z_{L,N_L}\}$. Then, the algorithm is terminated (go to step 6). If (15) is not accepted, go to the next step.

Step 4: Generate K pseudorandom vectors $\boldsymbol{w}_l$, $l = 1,\ldots,K$ using (7) and compute K underlying polynomials (13) for each interval $[\theta_{a,i},\theta_{b,i}]$ using the previously computed interpolation and beamspace matrices for these intervals. Denote these polynomials $f_i^{(l)}(z)$, $i = 1,\ldots,L$, $l = 1,\ldots,K$. As a result, K polynomial sets $\{f_i^{(l)}(z)\}_{i=1}^L$, $l = 1,\ldots,K$ are available.

Step 5: For each interval $[\theta_{a,i},\theta_{b,i}]$, $i = 1,\ldots,L$, and each underlying polynomial $f_i^{(l)}(z)$ corresponding to this interval, find the roots $\{z_{i,1}^{(l)},z_{i,2}^{(l)},\ldots,z_{i,N_i^{(l)}}^{(l)}\}$ associated with the angles localized within $[\theta_{a,i},\theta_{b,i}]$. Here $N_i^{(l)}$ is the number of such roots of the polynomial $f_i^{(l)}(z)$. For each polynomial set $\{f_i^{(l)}(z)\}_{i=1}^L$, set $N_i = N_i^{(l)}$, and and test the hypothesis (15). If (15) is accepted for any J $(0<J\le K)$ polynomial sets, say, $\{\tilde{f}_i^{(l)}(z)\}_{i=1}^L$, $l = 1,\ldots,J$ from the total number of K polynomial sets $\{f_i^{(l)}(z)\}_{i=1}^L$, $l = 1,\ldots,K$ then for each polynomial $\tilde{f}_i^{(l)}(z)$ find the roots $\{\tilde{z}_{i,1}^{(l)},\tilde{z}_{i,2}^{(l)},\ldots,\tilde{z}_{i,\tilde{N}_i^{(l)}}^{(l)}\}$ associated with the angles localized within

$[\theta_{a,i}, \theta_{b,i}]$. Here $\tilde{N}_i^{(l)}$ is the number of such roots of the polynomial $\tilde{f}_i^{(l)}(z)$. Estimate the kth DOA as

$$\hat{\theta}_k = \text{med}\,\{\tilde{\theta}_k^{(1)}, \tilde{\theta}_k^{(2)}, \ldots, \tilde{\theta}_k^{(J)}\}\,, \qquad k = 1, \ldots, \hat{q}\,, \tag{16}$$

where $\tilde{\theta}_1^{(l)} < \tilde{\theta}_2^{(l)} < \cdots < \tilde{\theta}_{\hat{q}}^{(l)}$ is the ordered set of angles associated with the $\hat{q}$ closest to the unit circle roots selected from the roots $\{\tilde{z}_{1,1}^{(l)}, \tilde{z}_{1,2}^{(l)}, \ldots, \tilde{z}_{1,\tilde{N}_1^{(l)}}^{(l)}, \tilde{z}_{2,1}^{(l)}, \tilde{z}_{2,2}^{(l)}, \ldots, \tilde{z}_{2,\tilde{N}_2^{(l)}}^{(l)}, \ldots, \tilde{z}_{L,1}^{(l)}, \tilde{z}_{L,2}^{(l)}, \ldots, \tilde{z}_{L,\tilde{N}_L^{(l)}}^{(l)}\}$. In (16), the median averaging is defined as

$$\text{med}\,\{b_1, \ldots, b_h\} = \begin{cases} (c_{\frac{h}{2}} + c_{\frac{h}{2}+1})/2, & \text{even } h \\ c_{\frac{h+1}{2}}, & \text{odd } h \end{cases}\,, \tag{17}$$

where $\{c_1, \ldots, c_h\} = \text{sort}\,\{b_1, \ldots, b_h\}$ and $\text{sort}\,\{\cdots\}$ denotes the operator of sorting in ascending (descending) order. If (15) is not accepted for all K polynomial sets $\{f_i^{(l)}(z)\}_{i=1}^{L}, l = 1, \ldots, K$ then estimate the kth DOA as

$$\hat{\theta}_k = \text{med}\,\{\theta_k^{(1)}, \theta_k^{(2)}, \ldots, \theta_k^{(K)}\}\,, \qquad k = 1, \ldots, \hat{q}\,, \tag{18}$$

where $\theta_1^{(l)} < \theta_2^{(l)} < \cdots < \theta_{\hat{q}}^{(l)}$ is the ordered set of angles localized in the whole array field of view $[-90°, 90°]$, and associated with the $\hat{q}$ closest to the unit circle roots selected from the overall number of $L(r-1)$ roots of the polynomial set $\{f_i^{(l)}(z)\}_{i=1}^{L}$.
Step 6: End of algorithm. □

4. SIMULATION RESULTS

In our simulations, we assumed a linear array of ten omnidirectional sensors. Two array geometries were used (with sensor positions in lambdas): $[0.00, 0.50, 1.00, 1.50, 2.00, 2.50, 3.00, 3.50, 4.00, 4.50]$ (ULA), and $[0.00, 0.14, 0.46, 1.92, 2.17, 2.26, 3.32, 3.46, 3.62, 4.37]$ (non-uniform). The first (ULA) geometry corresponds to the real array in simulations with non-interpolated algorithms, whereas the second, non-uniform geometry corresponds to the real array in simulations with interpolated techniques. In the latter case, the virtual array was always assumed to have the ULA geometry. Two uncorrelated equi-power Gaussian sources from $\theta_1 = 20°$ and $\theta_2 = 22°$, white Gaussian noise, and $M = 100$ were assumed. The comparison of performance is given in terms of DOA estimation RMSE (averaged over both sources and 100 simulation runs). The stochastic CRB was also plotted. We assumed $\hat{q} = 2$, $\hat{\Theta}_S = [13.5°, 28.5°]$, and $p = 7$. Fig. 1 shows the RMSE's of non-interpolated methods versus SNR. The dimension of estimator bank was $K = 20$. Beamspace root PR-JES is observed to perform better than elementspace spectral PR-JES and both MUSIC algorithms. In particular, it has much smaller asymptotic RMSE misadjustment (relative to CRB) than elementspace spectral PR-JES and MUSIC. The asymptotic performances of beamspace root PR-JES and beamspace root MUSIC are similar but the former has the lower SNR threshold. The performance of beamspace root PR-JES is nearly identical to that of elementspace and beamspace stochastic ML. Fig. 2 compares the RMSE's of interpolated and non-interpolated beamspace root versions of MUSIC and PR-JES versus SNR for the fixed $K = 20$. The interpolated and non-interpolated versions of each algorithm are observed to perform very similarly, and interpolated PR-JES has better threshold performance than interpolated MUSIC.

Figs. 3 shows the RMSE's versus K for the fixed $\text{SNR} = 0$ dB. Again, interpolated and non-interpolated beamspace root

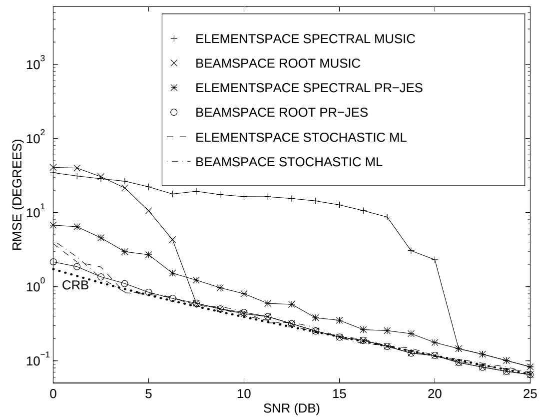

Figure 1: RMSE's of non-interpolated methods vs. SNR.

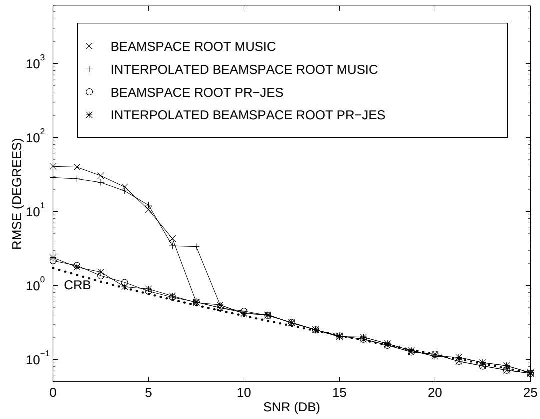

Figure 2: RMSE's of beamspace root MUSIC and PR-JES vs. SNR.

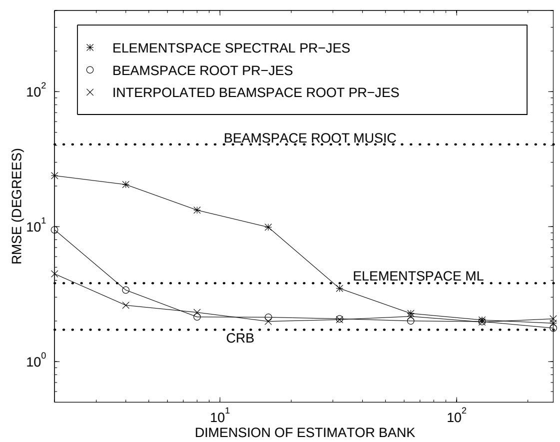

Figure 3: RMSE's vs. K for the fixed $\text{SNR} = 0$ dB.

PR-JES have very similar performance. Both these algorithms are observed to perform better than other techniques. They need lower dimension of estimator bank for convergence to CRB than elementspace spectral PR-JES. Moreover, PR-JES can perform even better than stochastic ML if the dimension of estimator bank is sufficiently large.

5. RESULTS OF REAL DATA PROCESSING

To test the algorithms, we used the experimental ultrasonic data recorded at University of Wyoming Source Tracking Array Testbed (UW STAT) [7]. These narrowband 6-element array data are available on the World Wide Web [7]. The dataset no. 3 with one stationary and one constant velocity source was used (corresponds to Fig. 2 in [7]). Similarly to [7], the forgetting factor was 0.97. Figs. 4 and 5 show the estimated source trajectories using beamspace root MUSIC and elementspace stochastic ML, respectively. Fig. 6 shows the estimated source trajectories using beamspace root PR-JES with $K = 20$. The spatial sectors Θ_S have been estimated using the conventional beamformer and $p = 5$ was assumed. Figs. 4-6 show that both root MUSIC and ML have serious problems that are manifested in multiple strong outliers, which are mainly concentrated between 300-th and 600-th snapshots (where the sources are closely spaced). The high variability of ML estimate can be explained by possible mismatch of underlying model and real data. We tried to avoid the convergence problems taking the parameters of the optimization routine (genetic algorithm) so that they provide full global convergence in similar simulated data examples. Beamspace root PR-JES is observed to overcome the problem of outliers, i.e. to have quite stable behavior.

6. CONCLUSIONS

Motivated by the superior performance and reduced computational complexity of beamspace and root implementations of eigenstructure techniques, the beamspace root modification of the PR-JES technique has been developed. Computer simulations and real ultrasonic data processing have demonstrated its superior performance.

7. REFERENCES

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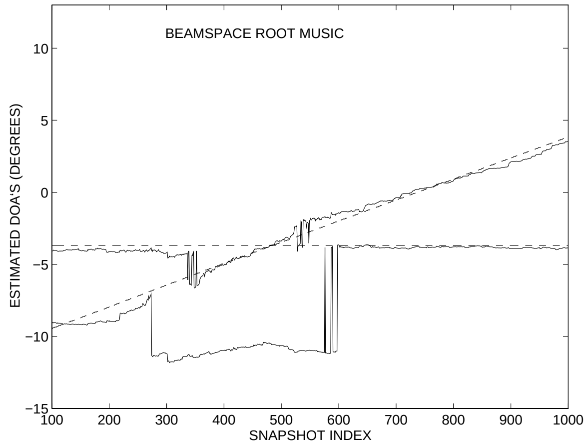

Figure 4: Results of real data processing.

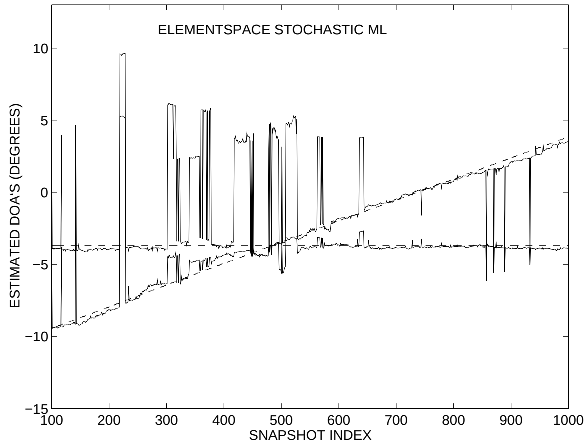

Figure 5: Results of real data processing.

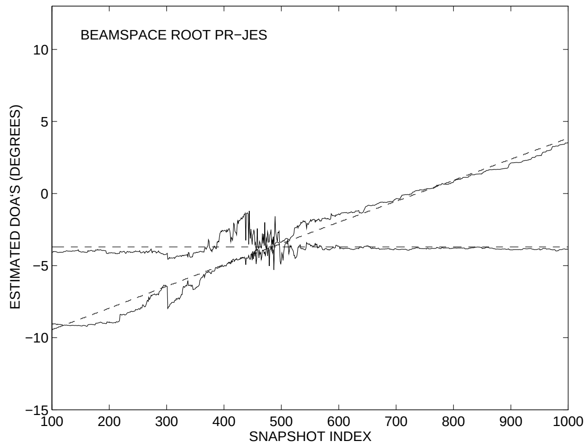

Figure 6: Results of real data processing.