ENERGY COMPACTION PERFORMANCE OF PARAUNITARY FIR FILTER BANKS FOR FINITE-LENGTH SIGNALS

Wolfgang Niehsen

Institut für Elektrische Nachrichtentechnik
Rheinisch-Westfälische Technische Hochschule (RWTH) Aachen
D-52056 Aachen, Germany
Email: niehsen@ient.rwth-aachen.de

ABSTRACT

The energy compaction performance of two-channel paraunitary finite impulse response (FIR) filter banks for finite-length signals is investigated. A detailed non-iterative design procedure for boundary filters which are optimal in a weighted mean square error (MSE) sense in the Fourier domain is presented. Simulation results are given for two-channel paraunitary FIR filter banks based on minimum-phase Daubechies filters and least-asymmetric Daubechies filters, respectively.

1. INTRODUCTION

The energy compaction performance is a rather general objective function for optimized design of signal decomposition algorithms [1]. It is well known that for two-channel paraunitary filter banks coding gain and energy compaction are equivalent [2].

Paraunitary FIR filter banks offer several desirable properties such as perfect reconstruction, energy conservation, and equal-length analysis and synthesis filters [3]. Furthermore, factorizations based on cascaded degree-one building blocks exist which are complete and minimal with respect to both, the number of delays and the the number of parameters [3].

Paraunitary FIR filter banks are of particular interest for signal-adapted filter bank trees with additive cost and distortion measures since the mean square quantization error equals the mean square reconstruction error. The single-tree algorithm which is essentially an adaptive wavelet packet algorithm and the double-tree algorithm which is essentially a spatially varying adaptive wavelet packet algorithm are prime examples with applications in the field of image coding [4].

In general, filter bank designs are based on the assumption of infinite-length signals. Therefore, finite-length signals, e.g., the rows and columns of an image or an image segment, require special treatment of the signal boundaries. Simple signal padding methods lead to distortion at the signal boundaries and introduce redundancy. Cyclic signal extension suffers from artificially induced high frequency components due to discontinuities at the signal boundaries. Symmetric signal extension and linear phase filtering is not compatible with two-channel paraunitary filter banks. It is possible to drop the linear phase requirement but paraunitarity is not easily preserved [5].

Boundary filters avoid these drawbacks, i.e., they allow for perfect reconstruction of finite-length signals without introducing artificially high frequency components or redundancy and they are compatible with two-channel paraunitary filter banks [6].

The scope of this paper is the investigation of the energy compaction performance of paraunitary FIR filter banks for finite-length signals. At first, a detailed non-iterative design procedure for boundary filters which are optimal in a weighted MSE sense in the Fourier domain will be developed. Afterwards, an appropriate energy compaction measure will be derived. Simulation results will be given for two-channel paraunitary FIR filter banks based on minimum-phase Daubechies filters and least-asymmetric Daubechies filters, respectively.

The following notation is used in the paper. Boldfaced quantities denote matrices or column vectors. Row vectors are denoted as transposed column vectors. The row and column indices of matrices and vectors are counted from zero. The quantities $\boldsymbol{A}', \boldsymbol{A}''$ and $\boldsymbol{A}^{\mathrm{T}}$ denote the real part, the imaginary part, and the transpose of $\boldsymbol{A}$, respectively. $\boldsymbol{O}_{N\times M}$ denotes the $N \times M$ zero matrix and $\boldsymbol{I}_N$ denotes the $N \times N$ identity matrix.

2. BOUNDARY FILTERS

Let $\boldsymbol{h}_0 = [h(N-1), h(N-2), ..., h(0)]^{\mathrm{T}} \in \mathbb{R}^N$ denote the reversed lowpass analysis filter impulse response vector of a causal two-channel FIR paraunitary filter bank. Then, the corresponding reversed highpass analysis filter impulse response vector is given by $\boldsymbol{h}_1 = [h(0), -h(1), ..., -h(N-1)]^{\mathrm{T}}$ [7]. Note $h(N-1) \neq 0$ and N even. $\boldsymbol{h}_0$ and $\boldsymbol{h}_1$ should have unit Euclidean-norm, i.e., $\boldsymbol{h}_0^{\mathrm{T}}\boldsymbol{h}_0 = \boldsymbol{h}_1^{\mathrm{T}}\boldsymbol{h}_1 = 1$. For the construction of boundary filters it is convenient to define the $(N-2) \times (N-2)$ triangular block matrices

$$\boldsymbol{A}_0 = \begin{bmatrix} \boldsymbol{H}_0 & \boldsymbol{H}_1 & \cdots & \boldsymbol{H}_{M-2} \\ & \boldsymbol{H}_0 & \cdots & \boldsymbol{H}_{M-3} \\ & & & \cdots \\ & & & \boldsymbol{H}_0 \end{bmatrix} \tag{1}$$

$$\boldsymbol{A}_1 = \begin{bmatrix} \boldsymbol{H}_{M-1} & & & \\ \boldsymbol{H}_{M-2} & \boldsymbol{H}_{M-1} & & \\ \cdots & & & \\ \boldsymbol{H}_1 & \boldsymbol{H}_2 & \cdots & \boldsymbol{H}_{M-1} \end{bmatrix} \tag{2}$$

with $M = N/2$ and

$$\boldsymbol{H}_m = \begin{bmatrix} h(N-1-2m) & h(N-2-2m) \\ h(2m) & -h(2m+1) \end{bmatrix}. \tag{3}$$

From [6] it is known that there exist exactly $M-1$ left and right boundary filters each of maximal length $N-2$. Therefore, construction of a set of left boundary filters requires determination of a real $(M-1)\times(N-2)$ matrix $\boldsymbol{R}_0$ such that the column vectors of $[\boldsymbol{R}_0^{\mathrm{T}} \ \boldsymbol{A}_0^{\mathrm{T}}]$ are mutually orthogonal. Similarly, construction of a set of right boundary filters requires determination of a real $(M-1) \times (N-2)$ matrix $\boldsymbol{R}_1$ such that the column vectors of $[\boldsymbol{A}_1^{\mathrm{T}} \ \boldsymbol{R}_1^{\mathrm{T}}]$ are mutually orthogonal. Canonical sets of boundary filters can be derived using the following facts.

Proposition 2.1. Let $\boldsymbol{a}_0^{\mathrm{T}}, ..., \boldsymbol{a}_{N-3}^{\mathrm{T}}$ denote the $N-2$ row vectors of $\boldsymbol{A}_1$. Then the $M-1$ row vectors $\boldsymbol{a}_1^{\mathrm{T}}, \boldsymbol{a}_3^{\mathrm{T}}, ..., \boldsymbol{a}_{N-3}^{\mathrm{T}}$ are linearly independent and orthogonal to the row space of $\boldsymbol{A}_0$.

Proof: Since $\boldsymbol{A}_0\boldsymbol{A}_1^{\mathrm{T}} = \boldsymbol{0}$ holds [6], $\boldsymbol{a}_1^{\mathrm{T}}, \boldsymbol{a}_3^{\mathrm{T}}, ..., \boldsymbol{a}_{N-3}^{\mathrm{T}}$ are orthogonal to the row space of $\boldsymbol{A}_0$. Because of $\det(\boldsymbol{u}_{0,N-2}, \boldsymbol{a}_1, \boldsymbol{u}_{2,N-2}, \boldsymbol{a}_3, ..., \boldsymbol{u}_{N-4,N-2}, \boldsymbol{a}_{N-3}) = (-h(N-1))^{M-1} \neq 0$ where $\boldsymbol{u}_{n,N-2}$ denotes the nth unity-vector of length $N-2$, $\boldsymbol{a}_1^{\mathrm{T}}, \boldsymbol{a}_3^{\mathrm{T}}, ..., \boldsymbol{a}_{N-3}^{\mathrm{T}}$ are linearly independent. □

Proposition 2.2. Let $\boldsymbol{b}_0^{\mathrm{T}}, ..., \boldsymbol{b}_{N-3}^{\mathrm{T}}$ denote the $N-2$ row vectors of $\boldsymbol{A}_0$. Then the $M-1$ row vectors $\boldsymbol{b}_0^{\mathrm{T}}, \boldsymbol{b}_2^{\mathrm{T}}, ..., \boldsymbol{b}_{N-4}^{\mathrm{T}}$ are linearly independent and orthogonal to the row space of $\boldsymbol{A}_1$.

Proof: Since $\boldsymbol{A}_1\boldsymbol{A}_0^{\mathrm{T}} = \boldsymbol{0}$ holds [6], $\boldsymbol{b}_0^{\mathrm{T}}, \boldsymbol{b}_2^{\mathrm{T}}, ..., \boldsymbol{b}_{N-4}^{\mathrm{T}}$ are orthogonal to the row space of $\boldsymbol{A}_1$. Because of $\det(\boldsymbol{b}_0, \boldsymbol{u}_{1,N-2}, \boldsymbol{b}_2, ..., \boldsymbol{b}_{N-4}, \boldsymbol{u}_{N-3,N-2}) = h(N-1)^{M-1} \neq 0$, $\boldsymbol{b}_0^{\mathrm{T}}, \boldsymbol{b}_2^{\mathrm{T}}, ..., \boldsymbol{b}_{N-4}^{\mathrm{T}}$ are linearly independent. □

The boundary filter matrices $\boldsymbol{R}_0$ and $\boldsymbol{R}_1$ are now readily obtained by Gram-Schmidt orthogonalization and subsequent orthonormalization of the above sets of linearly independent vectors $\boldsymbol{a}_1^{\mathrm{T}}, \boldsymbol{a}_3^{\mathrm{T}}, ..., \boldsymbol{a}_{N-3}^{\mathrm{T}}$ and $\boldsymbol{b}_0^{\mathrm{T}}, \boldsymbol{b}_2^{\mathrm{T}}, ..., \boldsymbol{b}_{N-4}^{\mathrm{T}}$, respectively. With $\boldsymbol{r}_1^{\mathrm{T}}, \boldsymbol{r}_3^{\mathrm{T}}, ..., \boldsymbol{r}_{N-3}^{\mathrm{T}}$ and $\boldsymbol{r}_0^{\mathrm{T}}, \boldsymbol{r}_2^{\mathrm{T}}, ..., \boldsymbol{r}_{N-4}^{\mathrm{T}}$ the obtained sets of orthonormal vectors, the boundary filter matrices are $\boldsymbol{R}_0^{\mathrm{T}} = [\boldsymbol{r}_1, \boldsymbol{r}_3, ..., \boldsymbol{r}_{N-3}]$ and $\boldsymbol{R}_1^{\mathrm{T}} = [\boldsymbol{r}_0, \boldsymbol{r}_2, ..., \boldsymbol{r}_{N-4}]$, respectively.

For optimization of the boundary filter matrices, a fact already given in [6] will be restated.

Proposition 2.3. If $\boldsymbol{R}_0$ is a left boundary filter matrix then

$$\boldsymbol{B}_0 = \boldsymbol{Q}_0^{\mathrm{T}}\boldsymbol{P}_0 = \boldsymbol{Q}_0^{\mathrm{T}} \begin{bmatrix} \boldsymbol{I}_{p_0} & \boldsymbol{0}_{p_0\times(N-2)} \\ \boldsymbol{0}_{(M-1)\times p_0} & \boldsymbol{R}_0 \end{bmatrix} \tag{4}$$

is also a left boundary matrix where $\boldsymbol{Q}_0$ denotes an $(M-1+p_0) \times (M-1+p_0)$ orthogonal matrix. Similarly, if $\boldsymbol{R}_1$ is a right boundary filter matrix then

$$\boldsymbol{B}_1 = \boldsymbol{Q}_1^{\mathrm{T}}\boldsymbol{P}_1 = \boldsymbol{Q}_1^{\mathrm{T}} \begin{bmatrix} \boldsymbol{R}_1 & \boldsymbol{0}_{(M-1)\times p_1} \\ \boldsymbol{0}_{p_1\times(N-2)} & \boldsymbol{I}_{p_1} \end{bmatrix} \tag{5}$$

is also a right boundary filter matrix where $\boldsymbol{Q}_1$ denotes an $(M-1+p_1) \times (M-1+p_1)$ orthogonal matrix.

Proof: It is readily verified that $\boldsymbol{B}_0\boldsymbol{B}_0^{\mathrm{T}} = \boldsymbol{I}_{M-1+p_0}$, $\boldsymbol{B}_1\boldsymbol{B}_1^{\mathrm{T}} = \boldsymbol{I}_{M-1+p_1}$, $\boldsymbol{B}_0\,[\boldsymbol{0}_{(N-2)\times p_0} \ \ \boldsymbol{A}_0]^{\mathrm{T}} = \boldsymbol{0}$, and $\boldsymbol{B}_1\,[\boldsymbol{A}_1 \ \boldsymbol{0}_{(N-2)\times p_1}]^{\mathrm{T}} = \boldsymbol{0}$ hold. □

Note that there are $M-1+p_0$ left boundary filters each of maximal length $N-2+p_0$ and $M-1+p_1$ right boundary filters each of maximal length $N-2+p_1$. The objective function for optimization of the boundary filters will be the weighted MSE between the frequency responses of the boundary filters and the associated stationary filters. If the numbers of boundary filters $M-1+p_0$ and $M-1+p_1$ are restricted to be even, the associated stationary filter matrices are

$$\boldsymbol{S}_i = \begin{bmatrix} \boldsymbol{H}_0 & \boldsymbol{H}_1 & \cdots & \boldsymbol{H}_{M-1} \\ \cdots & & & \\ \boldsymbol{H}_0 & \cdots & & \boldsymbol{H}_{M-1} \end{bmatrix} \tag{6}$$

where $\boldsymbol{S}_i \in \mathbb{R}^{(M-1+p_i)\times(N-2)}$, $(i = 0, 1)$ holds. With the Fourier matrices $\boldsymbol{F}_{K\times L} = [W^{kl}]_{0\leq k\leq K-1,\,0\leq l\leq L-1}$, $\boldsymbol{F} = \boldsymbol{F}_{N\times L}$, $\boldsymbol{F}_i = \boldsymbol{F}_{(M-1+p_i)\times L}$, $W = \exp(-j2\pi/L)$, and the positive definite frequency weight matrix $\boldsymbol{W} = \mathrm{diag}(w_0, ..., w_{L-1})$, the boundary filter optimization problems can be stated as

$$\min_{\boldsymbol{Q}_i} \|\boldsymbol{S}_i\boldsymbol{F}\boldsymbol{W} - \boldsymbol{Q}_i^{\mathrm{T}}\boldsymbol{P}_i\boldsymbol{F}_i\boldsymbol{W}\|_F \tag{7}$$

subject to the constraint $\boldsymbol{Q}_i^{\mathrm{T}}\boldsymbol{Q}_i = \boldsymbol{I}_{M-1+p_i}$. It was already noted in [8] that (7) is a slight modification of the orthogonal Procrustes problem [9].

Proposition 2.4. Given $\boldsymbol{T}_i = \boldsymbol{T}'_i + j\,\boldsymbol{T}''_i \in \mathbb{C}^{K\times L}$, an orthogonal matrix $\boldsymbol{Q} \in \mathbb{R}^{K\times K}$ which minimizes

$$\|\boldsymbol{T}_1 - \boldsymbol{Q}^{\mathrm{T}}\boldsymbol{T}_2\|_F \tag{8}$$

is $\boldsymbol{Q} = \boldsymbol{U}\boldsymbol{V}^{\mathrm{T}}$ where $\boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{V}^{\mathrm{T}}$ is the singular value decomposition (SVD) of

$$\boldsymbol{C}_1 = \boldsymbol{T}'_2{\boldsymbol{T}'_1}^{\mathrm{T}} + \boldsymbol{T}''_2{\boldsymbol{T}''_1}^{\mathrm{T}} . \tag{9}$$

Proof: With

$$\boldsymbol{C}_0 = {\boldsymbol{T}'_1}^{\mathrm{T}}\boldsymbol{T}'_1 + {\boldsymbol{T}'_2}^{\mathrm{T}}\boldsymbol{T}'_2 + {\boldsymbol{T}''_1}^{\mathrm{T}}\boldsymbol{T}''_1 + {\boldsymbol{T}''_2}^{\mathrm{T}}\boldsymbol{T}''_2 \tag{10}$$

the objective function becomes

$$\|\boldsymbol{T}_1 - \boldsymbol{Q}^{\mathrm{T}}\boldsymbol{T}_2\|_F^2 = \mathrm{tr}(\boldsymbol{C}_0 - 2\boldsymbol{Q}^{\mathrm{T}}\boldsymbol{C}_1) . \tag{11}$$

Therefore, minimization of $\|\boldsymbol{T}_1 - \boldsymbol{Q}^{\mathrm{T}}\boldsymbol{T}_2\|_F$ is equivalent to maximization of $\mathrm{tr}(\boldsymbol{Q}^{\mathrm{T}}\boldsymbol{C}_1)$. Because of $\mathrm{tr}(\boldsymbol{Q}^{\mathrm{T}}\boldsymbol{C}_1) = \mathrm{tr}(\boldsymbol{Z}\boldsymbol{\Sigma})$ where $\boldsymbol{Z} = \boldsymbol{V}^{\mathrm{T}}\boldsymbol{Q}^{\mathrm{T}}\boldsymbol{U}$ is an orthogonal $K \times K$ matrix, the upper bound

$$\mathrm{tr}(\boldsymbol{Q}^{\mathrm{T}}\boldsymbol{C}_1) = \sum_{i=0}^{K-1} z_{ii}\sigma_i \le \sum_{i=0}^{K-1} \sigma_i \tag{12}$$

can be readily derived. The upper bound is achieved by setting $\boldsymbol{Z} = \boldsymbol{I}_K$ what is equivalent to $\boldsymbol{Q} = \boldsymbol{U}\boldsymbol{V}^{\mathrm{T}}$. □

The above proposition provides a non-iterative algorithm for the computation of boundary filters which are optimal in the weighted MSE sense in the Fourier domain. The optimal boundary filter matrices $\boldsymbol{B}_i = \boldsymbol{Q}_i^{\mathrm{T}}\boldsymbol{P}_i$ are obtained by setting $\boldsymbol{T}_1 = \boldsymbol{S}_i\boldsymbol{F}\boldsymbol{W}$ and $\boldsymbol{T}_2 = \boldsymbol{P}_i\boldsymbol{F}_i\boldsymbol{W}$, respectively.

Composition of an $L_0 \times L_0$ orthogonal subband decomposition matrix $\boldsymbol{G}$ which contains the non-overlapping boundary filter matrices $\boldsymbol{B}_i$ along with the smallest possible set of stationary filters reveals the minimal required signal length L_0. With the decompositions $\boldsymbol{B}_0 = [\boldsymbol{B}_{00}\ \boldsymbol{B}_{01}]$ and $\boldsymbol{B}_1 = [\boldsymbol{B}_{11}\ \boldsymbol{B}_{10}]$, $\boldsymbol{B}_{i0} \in \mathbb{R}^{(M-1+p_i)\times p_i}$, $\boldsymbol{B}_{i1} \in \mathbb{R}^{(M-1+p_i)\times(N-2)}$, the smallest possible orthogonal matrix is

$$\boldsymbol{G} = \begin{bmatrix} \boldsymbol{B}_{00} & \boldsymbol{B}_{01} & & \\ & \boldsymbol{A}_0 & \boldsymbol{A}_1 & \\ & & \boldsymbol{B}_{11} & \boldsymbol{B}_{10} \end{bmatrix} . \tag{13}$$

Hence, the minimal required signal length is $L_0 = 2(N-2) + p_0 + p_1$. For signal lengths $L > L_0$, the associated subband decomposition matrix can be readily obtained by straightforward extension of the block matrix $[\boldsymbol{A}_0\ \boldsymbol{A}_1]$. In the sequel, it is assumed that the signal length L and the numbers of left and right boundary filters are even. Then, the $L\times L$ subband decomposition matrix $\boldsymbol{G}$ contains exactly $L/2$ lowpass and $L/2$ highpass filter vectors which can be rearranged to obtain the $L/2 \times L$ lowpass filter matrix $\boldsymbol{G}_0$ and the $L/2 \times L$ highpass filter matrix $\boldsymbol{G}_1$, respectively. Note that $\boldsymbol{G}_0\boldsymbol{G}_0^{\mathrm{T}} = \boldsymbol{G}_1\boldsymbol{G}_1^{\mathrm{T}} = \boldsymbol{I}_{L/2}$ and $\boldsymbol{G}_0\boldsymbol{G}_1^{\mathrm{T}} = \boldsymbol{0}$ hold.

3. ENERGY COMPACTION

In this section, an energy compaction measure for two-channel paraunitary FIR filter banks for finite-length signals is derived. The input signal is assumed to be a zero-mean wide-sense stationary (WSS) random vector process with exponentially decaying covariance sequence, i.e., if $\boldsymbol{X} = [X_0, ..., X_{L-1}]^{\mathrm{T}}$ denotes the WSS random vector process of length L, $\mathrm{E}(\boldsymbol{X}) = \boldsymbol{0}$ holds and the covariance matrix of $\boldsymbol{X}$ is

$$\boldsymbol{C}_{\boldsymbol{X}} = \mathrm{E}(\boldsymbol{X}\boldsymbol{X}^{\mathrm{T}}) = [\rho^{|k-l|}]_{0\le k,l\le L-1} \tag{14}$$

where ρ denotes the correlation coefficient. The WSS property implies $|\rho| < 1$. Note that the individual components of $\boldsymbol{X}$ are normalized to have unit variance.

With the above defined matrices $\boldsymbol{G}_0$ and $\boldsymbol{G}_1$, $\boldsymbol{Y}_0 = \boldsymbol{G}_0\boldsymbol{X}$ and $\boldsymbol{Y}_1 = \boldsymbol{G}_1\boldsymbol{X}$ denote the output random vector processes of the two-channel paraunitary FIR filter bank. Because of $\mathrm{E}(\boldsymbol{Y}_i) = \boldsymbol{G}_i\mathrm{E}(\boldsymbol{X}) = \boldsymbol{0}$, the $\boldsymbol{Y}_i$ are zero-mean random vector processes. The covariance matrices are given by

$$\boldsymbol{C}_{\boldsymbol{Y}_i} = \mathrm{E}(\boldsymbol{Y}_i\boldsymbol{Y}_i^{\mathrm{T}}) = \boldsymbol{G}_i\boldsymbol{C}_{\boldsymbol{X}}\boldsymbol{G}_i^{\mathrm{T}} . \tag{15}$$

In general, the $\boldsymbol{C}_{\boldsymbol{Y}_i}$ are not persymmetric, i.e, $\boldsymbol{J}\boldsymbol{C}_{\boldsymbol{Y}_i}\boldsymbol{J} \neq \boldsymbol{C}_{\boldsymbol{Y}_i}$ where $\boldsymbol{J}$ denotes the $L/2\times L/2$ reversal matrix. Since symmetric Toeplitz matrices are persymmetric, the $\boldsymbol{Y}_i$ are not WSS. An energy compaction measure can be defined based on the cumulated variances of the output random vector processes $\boldsymbol{Y}_i$. With

$$J = J_0 + J_1 = \mathrm{tr}(\boldsymbol{C}_{\boldsymbol{Y}_0}) + \mathrm{tr}(\boldsymbol{C}_{\boldsymbol{Y}_1}) = L \tag{16}$$

an energy compaction measure for two-channel paraunitary FIR filter banks for finite-length signals is given by

$$\eta = \frac{J_0}{J} = \frac{1}{L}\,\mathrm{tr}(\boldsymbol{C}_{\boldsymbol{Y}_0}) . \tag{17}$$

Note that $0 \le \eta \le 1$ holds. For infinite-length signals, the energy compaction measure becomes

$$\eta_\infty = \lim_{L\to\infty} \eta = \frac{1}{2}\boldsymbol{h}_0^{\mathrm{T}}\boldsymbol{C}_{\boldsymbol{X}}\boldsymbol{h}_0 \tag{18}$$

where $\boldsymbol{h}_0$ denotes the reversed lowpass analysis filter impulse response vector of length N and where $\boldsymbol{C}_{\boldsymbol{X}}$ denotes the $N \times N$ covariance matrix of the WSS random vector process $\boldsymbol{X} = [X_0, ..., X_{N-1}]^{\mathrm{T}}$.

4. SIMULATION RESULTS

The simulation results are based on Daubechies filters [10] of length $N = 4$, $N = 6$, and $N = 8$, respectively. Both, minimum-phase filters denoted by $D_{m,M}$ and least-asymmetric filters denoted by $D_{s,M}$ were applied. Note that

$M = N/2$ is the number of zeros at $\omega = \pi$ of the lowpass filters frequency response. The boundary filters were computed according to the method presented in section 2. The numbers of left and right boundary filters were chosen to the smallest possible even number, i.e., for $N = 4$ and $N = 8$, $p_0 = p_1 = 1$ and for $N = 6$, $p_0 = p_1 = 0$, respectively. The frequency weight matrix was chosen to $\boldsymbol{W} = \boldsymbol{I}$. Table 1 summarizes the numbers of boundary filters $M - 1 + p_i$, the minimal required signal lengths L_0, and the energy compaction coefficients for infinite-length signals, η_∞. Figure 1 and Figure 2 show the simulation results for the correlation coefficients $\rho = 0.95$ and $\rho = 0.35$, respectively.

$D_{m,M}/D_{s,M}$	D_2	D_3	D_4
$M - 1 + p_i$	2	2	4
L_0	6	8	14
$\eta_\infty(\rho = 0.95)$	0.9808	0.9820	0.9825
$\eta_\infty(\rho = 0.35)$	0.6942	0.7010	0.7043

Table 1: Numbers of boundary filters $M - 1 + p_i$, minimal required signal lengths L_0, and energy compaction coefficients η_∞ for Daubechies filters $D_2 - D_4$.

In both cases, the minimum-phase filters $D_{m,2}$ and $D_{m,3}$ perform better than their least-asymmetric counterparts. For $\rho = 0.95$, $D_{m,3}$ is the best choice with respect to energy compaction performance. For $\rho = 0.35$, the best choice depends on the signal length. For short signals, i.e., $L \leq 32$, $D_{m,2}$ performs best whereas for medium signal lengths, i.e., $32 < L \leq 128$, $D_{m,3}$ should be used. $D_{s,4}$ is predestined for long signals, i.e., $L > 128$.

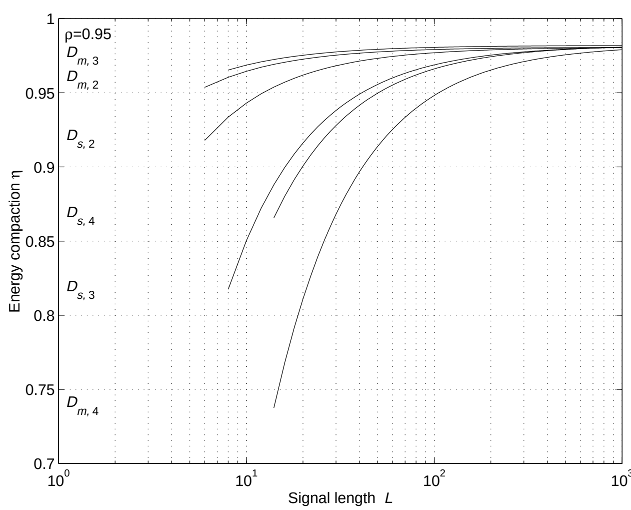

Figure 1: Energy compaction coefficients η for Daubechies filters $D_2 - D_4$ and correlation coefficient $\rho = 0.95$.

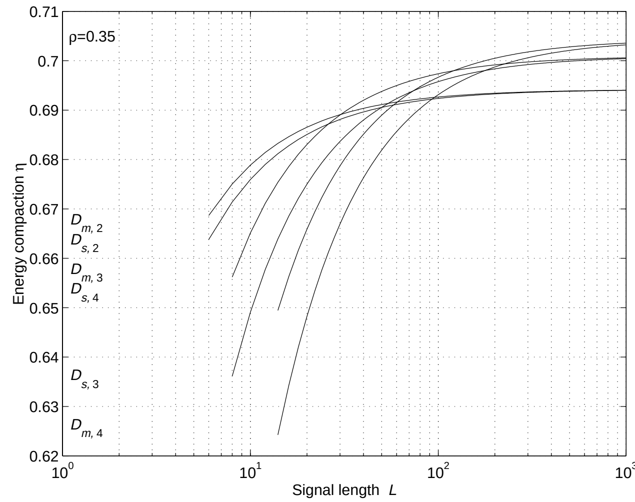

Figure 2: Energy compaction coefficients η for Daubechies filters $D_2 - D_4$ and correlation coefficient $\rho = 0.35$.

5. REFERENCES

[1] P. P. Vaidyanathan, "Theory of optimal orthonormal filter banks", *Proc. Int. Conf. ASSP*, pp. 1487-1490, May 1996.

[2] A. K. Soman and P. P. Vaidyanathan, "Coding gain in paraunitary analysis/synthesis systems", *IEEE Trans. Signal Proc.*, vol. 41, no. 5, pp. 1824-1835, May 1993.

[3] P. P. Vaidyanathan, *Multirate Systems and Filter Banks*, Englewood Cliffs, NJ: Prentice Hall, 1993.

[4] C. Herley, J. Kovačević, K. Ramchandran, and M. Vetterli, "Tilings of the time-frequency plane: Construction of arbitrary orthogonal bases and fast tiling algorithms", *IEEE Trans. Signal Proc.*, vol. 41, no. 12, pp. 3341-3359, Dec. 1993.

[5] R. L. de Queiroz, "Subband processing of finite length signals without border distortions", *Proc. Int. Conf. ASSP*, pp. 613-616, May 1992.

[6] C. Herley and M. Vetterli, "Orthogonal time-varying filter banks and wavelet packets", *IEEE Trans. Signal Proc.*, vol. 42, no. 10, pp. 2650-2663, Oct. 1994.

[7] M. Vetterli, "Wavelets and filter banks: Theory and design", *IEEE Trans. Signal Proc.*, vol. 40, no. 9, pp. 2207-2232, Sept. 1992.

[8] T. Kalker, "On optimal boundary and transition filters in time-varying filter banks", *Proc. Int. Conf. Image Proc.*, pp. 625-628, Sept. 1996.

[9] G. H. Golub and C. F. van Loan, *Matrix Computations*, Baltimore and London: The Johns Hopkins Univ. Press, 2nd ed., 1989.

[10] I. Daubechies, "Orthonormal bases of compactly supported wavelets", *Commun. Pure and Appl. Math.*, vol. XLI, pp. 909-996, 1988.