TWO-STAGE KALMAN ESTIMATOR USING ADVANCED CIRCULAR PREDICTION FOR MANEUVERING TARGET TRACKING

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ABSTRACT

Maneuvering targets are difficult to track for the Kalman filter since the target model of tracking filter might not fit the real target trajectory and the statistical characteristics of the target maneuver are unknown in advance. In order to track such a heavy maneuvering target, the estimation of the target turn-direction is necessary. The two-stage estimator using advanced circular prediction which considers the target turn-direction is proposed for maneuvering target tracking. Simulation results are given for a comparison of the performances of our proposed scheme with that of conventional tracking filters.

1. INTRODUCTION

Track while scan (TWS) radar using phased array antenna is often used in air and sea surveillance. The Kalman filter or an α-β filter is used in single target tracking problem. The Kalman filter performs almost perfect tracking in the case that the target model is fit for the real target trajectory and the statistical characteristics of the target maneuver and measurement noise such as mean and variance are known[1]. In practice, it is difficult to know the statistical characteristics of the target in advance. Additionally, the Kalman filter requires growing computational requirements. On the other hand, an α-β filter can realize real-time tracking with an uniform data rate since it omits the calculation of error covariance and filter gain[2]. However, when the target maneuvers, the quality of the position and velocity estimates could be degraded significantly, and for a heavy maneuver, the target may be lost.

To track such a target, the two-stage Kalman filter could be used [3]. The two-stage Kalman filter consists of two parallel filters, the constant velocity and acceleration filter. When the maneuver detector declares a target maneuver, the acceleration filter is turned on to correct the estimates of the constant velocity filter. However, heavy maneuvering targets are difficult to track since the two-stage Kalman filter doesn't consider the target turn-direction and the optimization of thresholds in the maneuver detector is also difficult. In this paper, the two-stage estimator using advanced circular prediction for maneuvering target tracking is presented. It was shown that the combination of advanced circular prediction with the two-stage Kalman filter gives good maneuver-following capability and ease threshold setting problems on the target maneuver detection.

2. TWO-STAGE KALMAN ESTIMATOR USING ADVANCED CIRCULAR PREDICITON

In modeling of a target motion, the state equation is given by

$$\boldsymbol{X}_{k+1} = \boldsymbol{F}_k \boldsymbol{X}_k + \boldsymbol{G}_k \boldsymbol{b}_k + \boldsymbol{W}_k^X \qquad (1)$$

$$\boldsymbol{b}_{k+1} = \boldsymbol{b}_k + \boldsymbol{W}_k^b \qquad (2)$$

where $\boldsymbol{X}_k$ is the system state vector at sample k and $\boldsymbol{b}_k$ is the bias vector. This system may represent the dynamics of a maneuvering target, where the position and velocity are the system state and the bias represents the target acceleration. The $\boldsymbol{W}_k^X$ and $\boldsymbol{W}_k^b$ are white Gaussian sequences with zero means and variances. In noise environment, the measurement model at sample k is given by

$$\boldsymbol{Y}_k = \boldsymbol{H}_k \boldsymbol{X}_k + \boldsymbol{v}_k \qquad (3)$$

where $\boldsymbol{Y}_k$ is the measurement, $\boldsymbol{H}_k$ is the observation matrix and the measurement noise process $\boldsymbol{v}_k$ is also a white Gaussian sequence with zero means and variances. The measurement noise $\boldsymbol{v}_k$ is added on the polar component (r, θ, ϕ) since the measurement is obtained from the polar coordinates.

The two-stage Kalman estimator using advanced circular prediction consists of three parallel filters as shown Fig.1. The first filter, the "bias-free" filter, is the constant velocity filter based on the assumptions that the bias is nonexistent. The second filter, the bias filter, corresponds to the acceleration filter which produces an estimate of the bias filter. The third filter is the proposed advanced circular prediction filter. When the target maneuvers, the output of the first filter is corrected with output of the second and the third filter. These three filters are used in our proposed scheme.

The algorithm of the two-stage Kalman estimator using advanced circular prediction is as follows. If the bias term is ignored ($\boldsymbol{b} = 0$), the constant velocity filter is the Kalman filter. The constant velocity filter is given by

$$\bar{\boldsymbol{X}}_{k|k} = \bar{\boldsymbol{X}}_{k|k-1} + \bar{\boldsymbol{K}}_k^X \left[\boldsymbol{Y}_k - \boldsymbol{H}_k \bar{\boldsymbol{X}}_{k|k-1}\right] \tag{4}$$

$$\bar{\boldsymbol{X}}_{k|k-1} = \boldsymbol{F}_{k-1} \bar{\boldsymbol{X}}_{k-1|k-1} \tag{5}$$

$$\bar{\boldsymbol{P}}_{k|k}^X = \left[\boldsymbol{I} - \bar{\boldsymbol{K}}_k^X \boldsymbol{H}_k\right] \bar{\boldsymbol{P}}_{k|k-1}^X \tag{6}$$

$$\bar{\boldsymbol{P}}_{k|k-1}^X = \boldsymbol{F}_{k-1} \bar{\boldsymbol{P}}_{k-1|k-1}^X \boldsymbol{F}_{k-1}^T + \bar{\boldsymbol{Q}}_{k-1}^X \tag{7}$$

$$\bar{\boldsymbol{K}}_k^X = \bar{\boldsymbol{P}}_{k|k-1}^X \boldsymbol{H}_k{}^T \left[\boldsymbol{H}_k \bar{\boldsymbol{P}}_{k|k-1}^X \boldsymbol{H}_k{}^T + \boldsymbol{R}_k\right]^{-1} \tag{8}$$

The $\bar{\boldsymbol{X}}_{(\cdot)}$ represents the estimate of state process when the bias is ignored, and $\bar{\boldsymbol{P}}_{(\cdot)}^X$ is the covariance of $\bar{\boldsymbol{X}}_{(\cdot)}$.

As in [3] the acceleration filter is used to estimate the bias vector from residual sequence of the bias-free filter as follows.

$$\boldsymbol{b}_{k|k-1} = \boldsymbol{b}_{k-1|k-1} \tag{9}$$

$$\boldsymbol{b}_{k|k} = \boldsymbol{b}_{k|k-1} + \boldsymbol{K}_k^b \left[\boldsymbol{d}_k - \boldsymbol{S}_k \boldsymbol{b}_{k|k-1}\right] \tag{10}$$

$$\boldsymbol{P}_{k|k-1}^b = \boldsymbol{P}_{k-1|k-1}^b + \boldsymbol{Q}_{k-1}^b \tag{11}$$

$$\boldsymbol{K}_k^b = \boldsymbol{P}_{k|k-1}^b \boldsymbol{S}_k^T \Big[\boldsymbol{S}_k \boldsymbol{P}_{k|k-1}^b \boldsymbol{S}_k^T + \boldsymbol{H}_k \boldsymbol{P}_{k|k-1}^{\bar{X}} \boldsymbol{H}_k^T + \boldsymbol{R}_k\Big]^{-1} \tag{12}$$

$$\boldsymbol{P}_{k|k}^b = \left[\boldsymbol{I} - \boldsymbol{K}_k^b \boldsymbol{S}_k\right] \boldsymbol{P}_{k|k-1}^b \tag{13}$$

$$\boldsymbol{S}_k = \boldsymbol{H}_k \boldsymbol{U}_k \tag{14}$$

$$\boldsymbol{U}_k = \boldsymbol{F}_{k-1} \boldsymbol{V}_{k-1} + \boldsymbol{G}_{k-1} \tag{15}$$

$$\boldsymbol{V}_k = \left[\boldsymbol{I} - \bar{\boldsymbol{K}}_k^X \boldsymbol{H}_k\right] \boldsymbol{U}_k \tag{16}$$

where $\boldsymbol{d}_k$ is the residuals of the bias-free filter.

For tracking of heavy maneuvering targets, the correction of the estimates by the acceleration filter isn't enough to get better maneuver-following capability. The advanced circular prediction filter regards the target maneuvering trajectory as a part of circle and produces circular prediction. The inputs to the advanced circular prediction filter is the three observed positions as

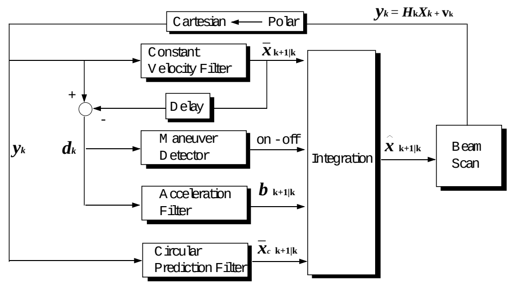

Figure 1: Block diagram of the proposed scheme.

shown in Fig.2. The algorithm of advanced circular prediction is as follows. The coordinates of a circle which is through the last three observed position is given by

$$\begin{aligned} \left(\frac{\boldsymbol{Y}_{k-3}+\boldsymbol{Y}_{k-2}}{2} - \boldsymbol{X}_{o\ k}\right)\left(\boldsymbol{Y}_{k-3} - \boldsymbol{Y}_{k-2}\right) &= \mathbf{o} \\ \left(\frac{\boldsymbol{Y}_{k-2}+\boldsymbol{Y}_{k-1}}{2} - \boldsymbol{X}_{o\ k}\right)\left(\boldsymbol{Y}_{k-2} - \boldsymbol{Y}_{k-1}\right) &= \mathbf{o} \end{aligned} \tag{17}$$

where $\boldsymbol{X}_{o\ k}$ denotes the center of circle. In two dimensional measurement space, the coordinates of the center and the radius of part of a circle are given by

$$\begin{aligned} x_{o\ k} &= \frac{\nabla_k \nabla_{k-1}(y_{k-1}^m - y_k^m) + \nabla_k x_{k-1}^m - \nabla_{k-1} x_k^m}{\nabla_k - \nabla_{k-1}} \\ y_{o\ k} &= -\frac{x_{o\ k} - x_k^m}{\nabla_k} + y_k^m \\ r_k &= \sqrt{\left(x_{o\ k} - x_k\right)^2 + \left(y_{o\ k} - y_k\right)^2} \end{aligned} \tag{18}$$

where $(x_{o\ k},\ y_{o\ k})$ is the coordinates of the center at sample k, $(x_k,\ y_k)$ is the coordinates of the observed position at sample k. ∇_k denotes the gradient from $\boldsymbol{Y}_{k-1}$ to $\boldsymbol{Y}_k$. $(x_k^m,\ y_k^m)$ represents the middle point between $\boldsymbol{Y}_{k-1}$ and $\boldsymbol{Y}_k$. And the angle ψ_k to calculate the circular prediction is given by

$$\begin{aligned} \psi_k &= \theta_k - \phi_k \quad , \text{ right turn} \\ \psi_k &= \theta_k + \phi_k \quad , \text{ left turn} \end{aligned} \tag{19}$$

From the angle ψ_k, the circular prediction is given by

$$\bar{\boldsymbol{X}}_{c\ k+1|k} = \boldsymbol{X}_{o\ k} + r_k \boldsymbol{\Phi}_k \tag{20}$$

where $\Phi_k = \left[\begin{array}{cc} \cos\psi_k & \sin\psi_k \end{array}\right]^{\mathrm{T}}$ and $\boldsymbol{X}_{o\ k}$ denotes the center of a circle.

The acceleration and the advanced circular prediction filter can be turned on or off as needed. The algorithm of the maneuver detection for the acceleration and the advanced circular prediction filter is as follows. A maneuver detector for the acceleration filter monitors a weighted sum of the predicted position residuals

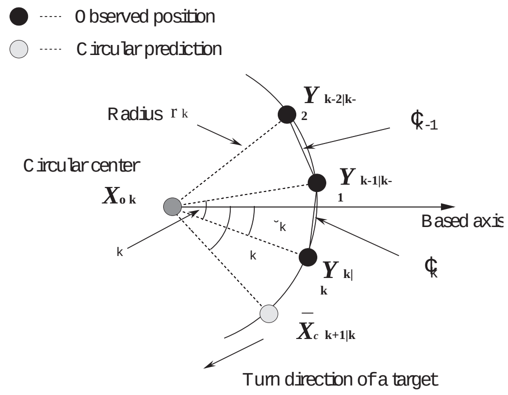

Figure 2: Principle of the circular prediction.

of the bias free filter within a time window of length N. Let

$$\boldsymbol{D}_k = \boldsymbol{H}_k \bar{\boldsymbol{P}}^X_{k|k-1} \boldsymbol{H}^T_k + \boldsymbol{R}_k \quad (21)$$

$$\delta_k = \boldsymbol{d}^T_k \boldsymbol{D}^{-1}_k \boldsymbol{d}_k \quad (22)$$

where $\boldsymbol{d}_k$ is the constant velocity filter residuals at sample k. For maneuver detection, a fading memory average of the innovations $\mu_k = \alpha\mu_{k-1} + \delta_k$ is computed. α takes a value from zero to one. The effective window length is given by $N = \frac{1}{1-\alpha}$. When μ_k exceeds a given threshold h_1, a maneuver is declared and the acceleration filter is turned on to correct the estimates of the constant velocity filter. When μ_k falls below a threshold h_2, the end of maneuver is declared and the acceleration filter is turned off.

On the other hand, when a target maneuvers heavily, the advanced circular prediction filter is turned on and its predicted position is used to correct the prediction of the constant velocity filter. The advanced circular prediction filter monitors the change of circular radius at sample k and decides the target heavy maneuver. The radius of a circle changes each scan, depending the degree of a target maneuver. When the radius falls below a given threshold r_1, a target heavy maneuver is declared and when it exceeds a threshold r_2, the end of heavy maneuver is declared.

During a maneuver and/or a heavy maneuver, the estimates of three filter are integrated. When only the maneuver detector for the acceleration filter declares a target maneuver, the corrected estimates are calculated as follows.

$$\hat{\boldsymbol{X}}_{k|k} = \bar{\boldsymbol{X}}_{k|k} + \boldsymbol{V}_k \boldsymbol{b}_{k|k} \quad (23)$$

$$\hat{\boldsymbol{X}}_{k|k-1} = \left[\bar{\boldsymbol{X}}_{k|k-1} + \boldsymbol{U}_k \boldsymbol{b}_{k|k-1}\right] \quad (24)$$

When only the advanced circular prediction detects a target heavy maneuver, $\bar{\boldsymbol{X}}_{c\ k|k-1}$ is used as the predicted position of $\hat{\boldsymbol{X}}_{k|k-1}$. Both filters detect a target maneuver, $\bar{\boldsymbol{X}}_{c\ k|k-1}$ takes priority over the acceleration filter outputs as the predicted position of $\hat{\boldsymbol{X}}_{k|k-1}$.

3. SIMULATION RESULTS

Computer simulation is done to show the effectiveness of our proposed scheme over conventional filters. We assume that measurement space is x-y plane for the sake of convenience. This condition is the same as r-θ dimensional space. Consider a target moving at a constant velocity 200m/s from t=0 to t=90s. Then the target maneuvers a 360° turn with turn gravity 2G. It completes the turn at t=165s and takes a straight course until t=250s. The sampling rate is T=5s and the standard deviation of measurement noise is 10m in range direction and 1.0 mil in azimuth direction and it is simulated by zero-mean independent white Gaussian random sequences. For all filters, the ratio of a maneuver to measurement noise 0.0003 is selected. The value of the effective window length of the maneuver detector 5 is used for the two-stage Kalman and our proposed scheme[3]. And the threshold of the maneuver detection for the acceleration filter h_1 3000 is selected. The end of the maneuver is detected when μ_k falls below a threshold h_2 of 2000. The advanced circular filter declares the begining of a heavy maneuver when the radius falls r_1=50000. The end of a heavy maneuver is detected when r_k exceeds a threshold r_2 of 30000.

A Monte-Carlo simulation of 50 runs was done. As the results of computer simulation, the change of radius and relationships between average RMS(Root Mean Square) position errors versus time are shown in Fig.3-Fig.6.

Fig.3 shows the change of the radius on the advanced circular prediction. The radius begins to decrease about 95s and increase about 170s. This result shows the maneuver detection based on the change of radius can declare the target maneuver faster than the maneuver detector for the acceleration filter. The advanced circular prediction uses the proposed fast switching to track heavy maneuvering targets. Additionally, the threshold setting problem of a maneuver detector for the acceleration filter is improved by the combination with the advanced circular prediction. Setting a high threshold for detecting a target maneuver is possible due to the fast reaction of the advanced circular prediction as shown in Fig.3. The acceleration filter for a maneuver and the advanced circular prediction filter for a heavy maneuver are used simultaneously.

Fig.4 shows average RMS of Euclid distance position errors versus time. As can be seen from the simulation result, the two-stage Kalman filter and our proposed scheme show good maneuver-following capability

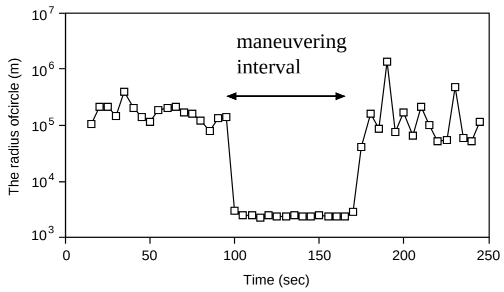

Figure 3: The change of the radius in advanced circular prediction

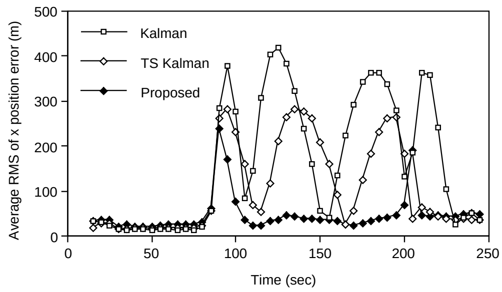

Figure 5: Average RMS x-axis position errors versus time

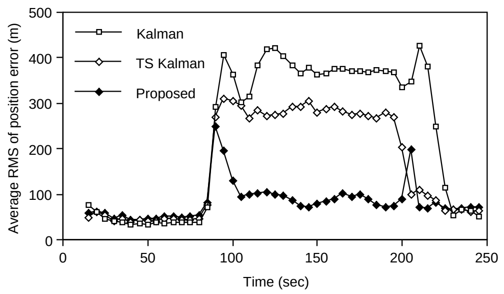

Figure 4: Average RMS position errors versus time

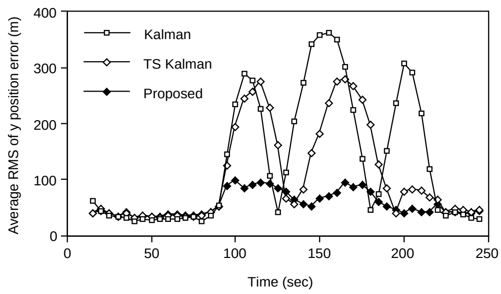

Figure 6: Average RMS y-axis position errors versus time

since they can detect the target maneuver. Our proposed scheme produces 248m error at the start of turn but the prediction error diminishes more rapid than that of the two-stage Kalman filter. This is because our proposed scheme takes the advanced circular prediction.

Fig.5 and Fig.6 shows average RMS of x and y position errors versus time. Average prediction errors in x and y axis of the Kalman and the two-stage Kalman filter increase and decrease repeatedly. This unstable prediciton error characteristic shows a weak point of linear prediction filter, that is, the difficulty for tracking a heavy maneuvering target. On the other hand, our proposed scheme shows stable and good performances, especially y position errors. A target moves along the y axis and then the target maneuvers a 360° turn. The constant velocity filter can't track a target especially in x position which is the target moving direction. Our proposed scheme is also affected by this characteristic of the constant velocity filter. However, our proposed scheme can get over the weak point of the constant velocity filter by the advanced circular prediction.

4. CONCLUSIONS

In order to get better maneuver-following capability for heavy maneuvering targets, we have proposed the two-stage Kalman estimater using advanced circular prediction which can detect a target heavy maneuver and ease the threshold setting problems for the acceleration filter. As the results of simulation, it was shown that our proposed scheme gives good maneuver-following capability and keep the small prediction error for nonmaneuvering target.

Further research is needed to develop this proposed scheme for multiple targets tracking.

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