ANALYSIS OF A DELAYLESS SUBBAND ADAPTIVE FILTER STRUCTURE

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ABSTRACT

In this paper, we present an analysis of the delayless subbabnd adaptive filter structure previously proposed by the authors. We derive a simple expression for the excess MSE of the proposed structure, and show that it requires up to 3.7 times less computational complexity than the corresponding fullband LMS structure. Also, we establish a connection between subband and block adaptive filtering, where the latter can be interpreted as a special case of the former. Some computer simulations are presented in order to verify the performance of the proposed structure and the theorectical results.

1. INTRODUCTION

It is known that when the conventional subband adaptive filter structure is implemented in an open loop scheme, a perfect reconstruction filter bank along with adaptive cross-filters among subbands are necessary to identify the unknown system correctly [1]. This increases the computational burden and the algorithm suffers from slow convergence in comparison with the fullband scheme. In a closed loop scheme, the MSE convergence is degraded due to the delay introduced in the adaptation algorithm. This problem was first addressed in [2], and it was proposed a delayless architecture. This paper presents an analysis of the new delayless structure proposed in [3], regarding the excess MSE, the computational complexity involved, and a relation between subband and a block adaptive filter proposed in [4].

2. THE DELAYLESS STRUCTURE

Fig. 1 illustrates the M-channel delayless structure in a closed loop configuration. The analysis filters correspond to a DFT filter bank, where the prototype filter has to be a *Nyquist(M)* filter. In this case, its polyphase components approximate fractional delays, an interpretation that allows a subband/wideband mapping of the adaptive filters [3]. It has been shown that this structure presents faster convergence speed than the corresponding fullband scheme, and perfect modeling is achieved if we use $L = N/M + 1$ length adaptive filters $\mathbf{w}_i(n)$, $i = 0,...,M-1$, where N is the length of the unknown system. The normalized LMS updating equations for the closed loop scheme are given by

$$\mathbf{w}_i(n+1) = \mathbf{w}_i(n) + \frac{2\mu}{\sigma_i^2(n)}\mathbf{u}_i^*(n)e_i'(n) \qquad (1)$$

$$\sigma_i^2(n) = \beta\sigma_i^2(n-1) + (1-\beta)\left|u_i(n)\right|^2, \qquad (2)$$

where $e(n)$ is the fullband error signal split in subbands to generate $e_i'(n)$, and $\mathbf{u}_i(n)$ is the input vector to the i-th adaptive filter. The recursion (2), with $0 < \beta < 1$, estimates the power of $u_i(n)$ assuming that the channel signals are stationary. The range of values for the convergence factor is typically $0 < \mu < 1/L$.

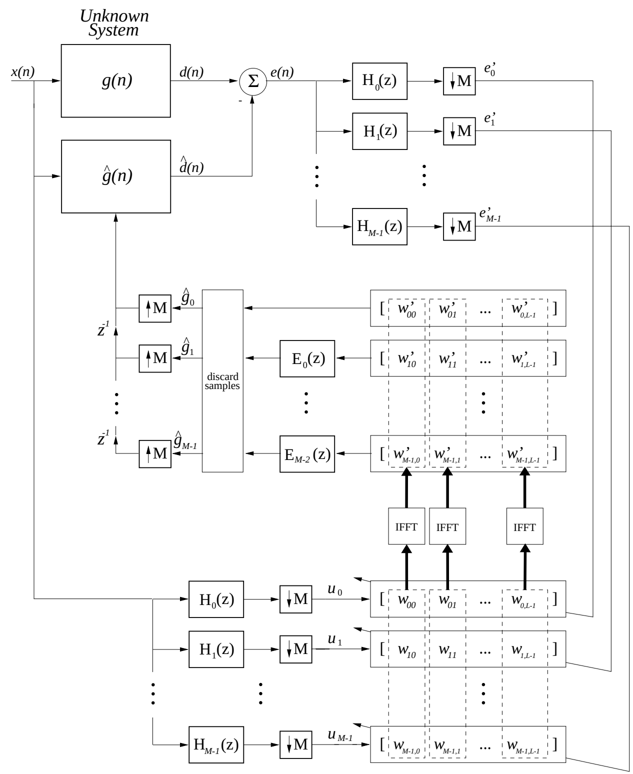

Figure 1: Delayless structure (closed loop).

3. EXCESS MSE

In this section, we estimate the excess MSE for the closed loop scheme. The expression for the excess MSE for the open loop scheme will not be presented, since it can be derived in a similar manner.

From Fig. 1, the subband/wideband transformation can be slightly modified, so that it can be interpreted as a synthesis of the subband adaptive filters through a filter bank, as illustrated in Fig. 2.

Now, let

$$\mathbf{P}(z) = \sum_{n=0}^{N_{FD}} \mathcal{P}(n)z^{-n} \qquad (3)$$

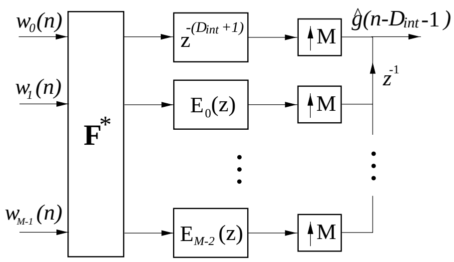

Figure 2: Filter bank representing the subband/fullband transformation.

be the polyphase matrix related to this filter bank and define **T** as the $N \times (N+M)$ matrix

$$\mathbf{T} \triangleq \left[\begin{array}{ccccccc} \mathcal{P}(D_{int}+1) & \cdots & \mathcal{P}(0) & \mathbf{0} & \mathbf{0} & \cdots & \cdots \\ \mathcal{P}(D_{int}+2) & \cdots & \mathcal{P}(1) & \mathcal{P}(0) & \mathbf{0} & \cdots & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \cdots \\ \mathcal{P}(D_{int}+L) & \cdots & \mathcal{P}(L-1) & & \cdots & \mathcal{P}(0) & \cdots \\ \mathbf{0} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{array}\right] \quad (4)$$

The wideband filter $\hat{\mathbf{g}}(n)$ can then be expressed as

$$\begin{aligned} \hat{\mathbf{g}}(n) &= \mathbf{TB}\mathcal{W}(n) \\ &= \mathbf{A}\mathcal{W}(n) \end{aligned} \quad (5)$$

where

$$\mathcal{W}(n) \triangleq \left[\begin{array}{cccc} \mathbf{w}_0^T(n) & \mathbf{w}_1^T(n) & \cdots & \mathbf{w}_{M-1}^T(n) \end{array}\right]^T, \quad (6)$$

$\mathbf{A} = \mathbf{TB}$ and **B** is a $(N+M) \times (N+M)$ matrix with one and zero entries that places the adaptive filter coefficients in an appropriate form to the matrix multiplication.

The discarding-of-samples step is eliminated by starting the convolution with the polyphase filters after $D_{int}+1$ samples, as can be observed from the first row of **T**.

The fullband excess MSE can be expressed as [5]

$$J_{\text{exc}}(n) = tr[\mathbf{RK}(n)] \quad (7)$$

where **R** is the correlation matrix of the input $\mathcal{X}(n)$ and $\mathbf{K}(n) = E[\Delta\hat{\mathbf{g}}(n)\Delta\hat{\mathbf{g}}^H(n)]$ is the covariance matrix of $\hat{\mathbf{g}}(n)$. Using the relation in Eq. (5), the excess MSE can be written as

$$\begin{aligned} J_{\text{exc}}(n) &= tr[\mathbf{R}E[\Delta\hat{\mathbf{g}}(n)\Delta\hat{\mathbf{g}}^H(n)]] \\ &= tr[\mathbf{RA}E[\Delta\mathcal{W}(n)\Delta\mathcal{W}^H(n)]\mathbf{A}^H] \\ &= tr[\mathbf{A}^H\mathbf{RA}E[\Delta\mathcal{W}(n)\Delta\mathcal{W}^H(n)]] \\ &= tr[E[\mathbf{A}^H\mathcal{X}(n)\mathcal{X}^H(n)\mathbf{A}]E[\Delta\mathcal{W}(n)\Delta\mathcal{W}^H(n)]] \end{aligned}$$

Note that the product $\mathbf{A}^H\mathcal{X}(n)$ represents the transpose of the synthesis filter bank operation applied to the input signal $\mathcal{X}(n)$, i.e., it represents an analysis operation applied to $x(n)$. Observing the polyphase structure of this synthesis filter bank, we see that its corresponding prototype filter is practically the same as the one of the analysis filter bank, as can be seen from their impulse responses shown in Fig. 3 (the only difference is that the last polyphase component now comes first). Hence, considering that the subband signals and the adaptive coefficients are uncorrelated, this expression reduces to

$$J_{\text{exc}}(n) = \sum_{i=0}^{M-1} tr[E[\mathbf{u}_i(n)\mathbf{u}_i^H(n)]\mathbf{K}_i(n)] = \sum_{i=0}^{M-1} J_{\text{exc}}(n)_i \quad (8)$$

where the overall excess MSE is given by the sum of the excess MSEs $J_{\text{exc}}(n)_i$ in the subbands. Considering that we have a prototype with good attenuation, this expression can be approximated as [5]

$$J_{\text{exc}}(n) = \sum_{i=0}^{M-1} \frac{\mu_i \sigma_{n_i}^2 tr[\mathbf{R}_i]}{1 - \mu_i tr[\mathbf{R}_i]} \quad (9)$$

where $\sigma_{n_i}^2 \approx \sigma_n^2/M$ and $\mathbf{R}_i = E[\mathbf{u}_i(n)\mathbf{u}_i^H(n)]$.

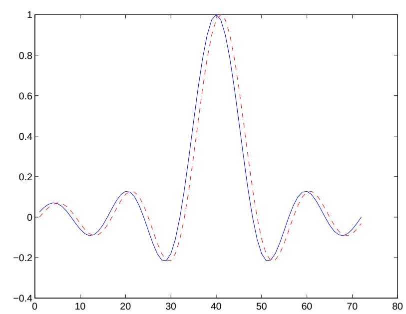

Figure 3: Prototype filters related to $\mathbf{E}(z)$ and $\mathbf{P}(z)$.

4. COMPUTATIONAL COMPLEXITY

Here, the computational complexity will be given in multiplies per input sample, considering that the product of complex values is implemented through 4 real multiplies.

The subband decomposition requires one convolution of a K-length prototype filter and one M-point FFT for each block of M input samples. Thus, the subband splitting requires

$$C_1 = 2K/M + 2\log_2 M \text{ mult./sample} \quad (10)$$

for the two analysis filter banks.

Because of the symmetry of the IFFT for real signals, we only have to process half of the M channel complex signals. Therefore, we have to update $M/2$ adaptive filters of length $L = N/M + 1$ for every M input samples. Thus, the subband processing requires

$$C_2 = 2N/M + 2 \text{ mult./sample} \quad (11)$$

For the open loop scheme we have to evaluate the adaptive filters outputs, which requires an additional C_2 real multiplies per input sample.

For the subband/wideband adaptive filters mapping, we have to compute L IFFTs for the weights transformation $\mathbf{w}_i(n) \rightarrow \mathbf{w}_i'(n)$ and perform $M-1$ convolutions with the polyphase filters. In practice, it is not necessary to evaluate the wideband filter for each M input samples, because its output cannot vary much faster then the length of its impulse response. The same idea was used in [2], and we only need to perform the subband/wideband mapping for

MI input samples. The convergence behavior for different values of I will be verified in the simulations. In this case, we will show that by increasing I we can maintain the convergence rate and also reduce the computational complexity of this part. Since we need to evaluate N/M samples at the output of the $M-1$ polyphase filters $E_i(z)$, the computational complexity of the convolution is $(M-1)(N/M)\min(K/M, N/M+1)$. Considering that $K/M \leq N/M+1$, i.e., the polyphase filters are shorter then the adaptive filters, this part requires the complexity

$$C_3 = \left((N/M+1)\log_2 M + \frac{NK(M-1)}{M^3}\right)\frac{1}{I}$$

The wideband convolution can be performed in the same way described in [2] by partitioning the wideband filter in p segments and using fast convolution techniques [6], [7]. In this case, the number of multiplies per input sample is given by

$$C_4 = N/p + 2(p+1)\log_2(2N/p) + 4(p-1). \qquad (12)$$

The number of segments p can also be optimized so that the complexity over the direct convolution is minimized. Therefore, the total computational complexity for the closed loop scheme is $C = C_1 + C_2 + C_3 + C_4$, whereas for the open loop version we count C_2 twice.

In order to compare with the example presented in [2], consider $N = 512$ taps, $M = 32$ subbands, and a $K = 128$ tap prototype filter. In this case, Eq. (12) is optimized with $p = 6$. For the structure of [2], when the wideband transformation was performed for each N input samples, the total computational load consists of 401 multiplies per input sample in the case of the closed loop configuration. This corresponds to a reduction by a factor of 2.5 in the computational complexity of the fullband LMS. The open loop scheme required 529 multiplies per input sample, which corresponds to a reduction of 1.9. Note that the filter bank used in [2] is oversampled by $M/2$, while here we use a maximally decimated structure. For the proposed closed loop structure, calculating the subband/wideband mapping for each N input samples corresponds to using $I = 16$, and it gives $C_1 = 18$, $C_2 = 34$, $C_3 = 9$ and $C_4 = 218$, with the total of 279 real multiplies per input sample. For the open loop version we have one additional C_2 resulting in a total of 313 real multiplies per input sample. We see that the complexity over the fullband LMS is reduced by a factor of 3.7 for the closed loop version and 3.5 for the open loop. Using $I = 1$ gives $C_3 = 147$, and the total complexity of 417 multiplies/sample is comparable to the one proposed in [2].

5. COMPUTER SIMULATIONS

The effect of increasing the factor I on the convergence behavior can be observed in Fig. 4, where we used a white noise input for a 64 tap unknown system, and an 8-channel filter bank with a 64 prototype filter. The convergence factor μ was chosen such that a maximum convergence rate was achieved, while still preserving stability. For I ranging from 1 to 5 the convergence rate does not change significantly. For $I = 8$, we are actually updating the wideband filter each N input samples. In conclusion, we can reduce the computational complexity by using higher values of I without reducing the convergence speed to the level of the fullband LMS.

In order to verify the theorectical estimate of the MSE, we simulated the proposed structure using an additive noise of variance

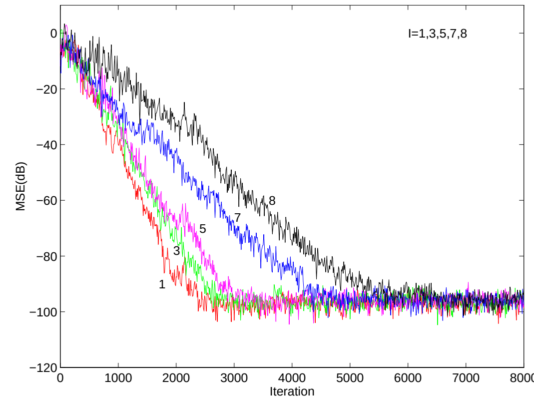

Figure 4: Effect of factor I in the MSE decay. I = 1,3,5,7,8.

-40 dB at the output with colored noise input signal. Table I shows the results of some experiments for different values of the number of subbands M and the prototype length K. We notice that these results are in good agreement with Eq. (9), provided that we have designed a good prototype filter or a filter bank with sufficient number of subbands.

Tests	Theory	Experiment
$M = 8, K = 64$	-37.5 dB	-36.5 dB
$M = 8, K = 128$	-39.5 dB	-39.5 dB
$M = 16, K = 128$	-39.5 dB	-39.6 dB
$M = 16, K = 256$	-39.5 dB	-39.5 dB

Table 1: Comparison of theorectical and measured MSEs

6. RELATION WITH BLOCK ADAPTIVE FILTERING

Consider a linear time invariant system $g(n)$ and its Wiener solution resulted by the minimization of the error signal energy $E[|e(n)|^2]$. A corresponding multichannel Wiener solution can be derived by blocking the scalar input-output signal description, as ilustrated in Fig. 5. In this case, for jointly-WSS signals $\mathbf{x}(n)$

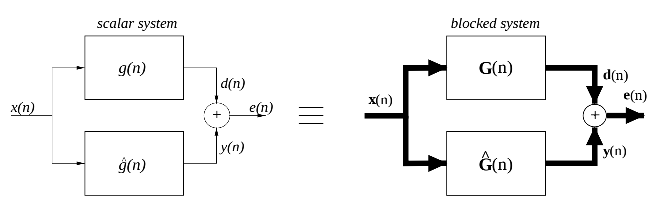

Figure 5: Equivalent input-output description.

and $\mathbf{d}(n)$, the optimal matrix filter obtained when we minimize the block error signal cost function $\xi_{bl} = E[\mathbf{e}^H(n)\mathbf{e}(n)]$ is given by [4]

$$\mathbf{G}_O(z) = \tilde{\mathbf{S}}_{\mathrm{xd}}(z)\mathbf{S}_{\mathrm{xx}}^{-1}(z), \qquad (13)$$

where $\mathbf{S}_{\mathrm{xx}}(z)$ and $\tilde{\mathbf{S}}_{\mathrm{xd}}(z)$ are the z-transforms of the autocorrelation $\mathbf{R}_{\mathrm{xx}}(k) = E[\mathbf{x}(n)\mathbf{x}^H(n-k)]$ and crosscorrelation $\mathbf{R}_{\mathrm{xd}}(k) = E[\mathbf{x}(n)\mathbf{d}^H(n-k)]$, respectively. The tilde operator denotes transpose, replacement of z by z^{-1}, and complex conjugate of the coefficients of $\mathbf{S}_{\mathbf{xd}}(z)$.

The multichannel Wiener solution can be reached iteratively by the block LMS algorithm given by

$$\mathbf{G}(n+1) = \mathbf{G}(n) + 2\mu\mathcal{X}(n)\mathbf{e}^H(n), \tag{14}$$

where $\mathbf{e}(n)$ is the block error signal. The block LMS was proposed by Sathe and Vaidyanathan for identification of bandlimited channels [4]. In this reference, it is shown that for this type of application, the input to the adaptive filter is in general cyclostationary. Thus, the use of a scalar adaptive filter for a cyclo-wide sense stationary input with period M ((CWSS)$_M$) will not be optimal in terms of a Wiener solution. Since the blocking version of a (CWSS)$_M$ input is a WSS vector, a matrix updating equation defined by Eq. (14) is best suited for that type of application. The problem concerning this scheme is that convolution and updating of a $(N+M)\times M$ matrix must be performed, turning its computational complexity so high that its usefulness is limited.

However, this computational burden can be reduced by diagonalizing the matrix $\mathbf{G}(z)$ as in [3]. Fig. 6(a), illustrates this procedure, where the transformed version of the blocked error $\mathbf{e}(n)$, say $\mathbf{e}'(n)$, is used to update the matrix coefficients. Since $\mathbf{E}(z) = \mathbf{Q}^{-1}(z)$, this scheme can be modified equivalently to Fig. 6(b), where we still preserve the fullband error information. In fact, this scheme represents the conventional closed loop subband structure, where the channel signals are reconstructed so that we can evaluate the error signal in fullband. We can go further, and instead of using a conventional subband structure, we can use a delayless closed loop scheme for this aplication. Note that the original matrix adaptive filter is a matrix polynomial of length $N/M+1$. This is in good agreement with the fact that the adaptive filters in each subband should be able to represent the unknown system (N/M length polyphase components) plus one sample split in fractional delays when we perform the diagonalization of $\mathbf{G}(z)$.

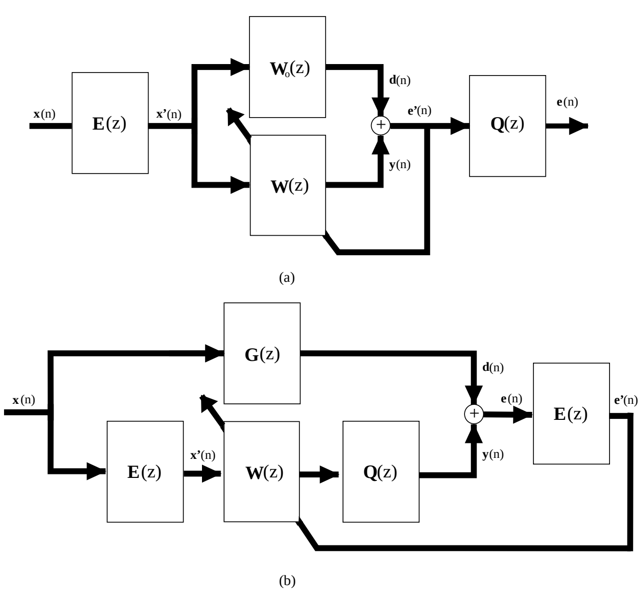

Figure 6: Conventional closed loop structure.

The closed loop algorithm is updated by using the transformed error vector $\mathbf{e}'(n)$, i.e., clearly, it minimizes the cost function $\xi_{cl} = E[\mathbf{e}'^H(n)\mathbf{e}'(n)]$. Let $\mathbf{S}_{\mathbf{ee}}(e^{j\omega})$ and $\mathbf{S}_{\mathbf{e}'\mathbf{e}'}(e^{j\omega})$ be the power density spectrum of the vector processes $\mathbf{e}(n)$ and $\mathbf{e}'(n)$. The above equation can be written as

$$\begin{aligned}\xi_{cl} &= \frac{1}{2\pi}\int_{-\pi}^{\pi} tr[\mathbf{S}_{\mathbf{e}'\mathbf{e}'}(e^{j\omega})]d\omega \\ &= \frac{1}{2\pi}\int_{-\pi}^{\pi} tr[\mathbf{E}(e^{j\omega})\mathbf{S}_{\mathbf{ee}}(e^{j\omega})\mathbf{E}^H(e^{j\omega})]d\omega \\ &= \frac{1}{2\pi}\int_{-\pi}^{\pi} tr[\mathbf{E}^H(e^{j\omega})\mathbf{E}(e^{j\omega})\mathbf{S}_{\mathbf{ee}}(e^{j\omega})]d\omega \end{aligned} \tag{15}$$

Since $\mathbf{E}^H(e^{j\omega})\mathbf{E}(e^{j\omega}) = \mathbf{I}$, the term inside the trace becomes

$$\xi_{cl} = \frac{1}{2\pi}\int_{-\pi}^{\pi}\sum_{i=0}^{M-1}\mathbf{S}_{e_i e_i}(e^{j\omega})d\omega = E[\mathbf{e}^H(n)\mathbf{e}(n)], \tag{16}$$

which is ξ_{bl}. In particular, for stationary signals we have

$$\xi_{cl} = E[|e(n)|^2]. \tag{17}$$

7. CONCLUSIONS

We have derived a simple expression for the excess MSE for the delayless subband adaptive filter structure proposed in [3], and verified that the new structure can reduce in 3.7 times the computational complexity of the fullband LMS algorithm. We have shown that the delayless strucuture can also be interpreted as a block adaptive filtering structure, which presents low computational burden when compared to the scheme proposed in [4].

8. REFERENCES

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