

EFFICIENT SUPER RESOLUTION TIME DELAY ESTIMATION TECHNIQUES

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ABSTRACT

In this paper, an efficient Weighted Fourier transform and RELAXation based algorithm (referred to as WRELAX) is first proposed for the well-known time delay estimation problem. WRELAX involves only a sequence of weighted Fourier transforms. Its resolution is much higher than that of the conventional matched filter approach. One disadvantage associated with WRELAX is that it converges slowly when the signals are spaced very closely. To overcome this problem, the well-known high resolution MODE (Method of Direction Estimation) algorithm, which was originally proposed for angle estimation in array processing, is modified and used with WRELAX for super resolution time delay estimation. The latter method is referred to as MODE-WRELAX. MODE-WRELAX provides better accuracy than MODE and higher resolution than WRELAX. Moreover, it applies to both complex- and real-valued signals (including those with highly oscillatory correlation functions). Numerical results show that the MODE-WRELAX estimates can approach the corresponding the Cramér-Rao bounds.

1. INTRODUCTION

Time delay estimation is a well-known problem that occurs frequently in sonar, radar, geophysical/seismic exploration, radio positioning and navigation. In this paper, we consider estimating time delays and amplitudes (real- or complex-valued) from the superposition of multiple signals with known shapes. The matched filter approach is the simplest method for this problem. However, it cannot resolve two signals with a time spacing less than the reciprocal of the signal bandwidth. Based on the observation that the frequency domain data model of this time delay estimation problem is similar to those used for the sinusoidal parameter and angle estimation problems except that the complex exponentials are weighted by the known signal spectrum, many existing sinusoidal frequency and angle estimation algorithms, such as MUSIC, linear prediction, and maximum likelihood, are applied to this problem [1, 2, 3]. However, they are best suited for complex-valued signals with special shapes (such as flat band-limited spectrum). A computationally efficient approach based on the expectation maximization (EM) algorithm is proposed in [4]. However, the EM method is very sensitive to initial conditions and no systematic initialization method is given in [4]. The separation of multipaths from real-valued bandpass underwater acoustic signals with highly oscillatory correlation functions is a very challenging issue and is addressed in [5, 6, 7]. The algorithms proposed in [5, 6, 7] are all based on the min-

imization of a nonlinear least squares (NLS) criterion and all of them resort to computationally demanding multidimensional search over the parameter space. In this paper, we present efficient super resolution time delay estimation algorithms that apply to complex-valued signals as well as real-valued signals (including those with highly oscillatory correlation functions).

2. MODEL DESCRIPTION

The time delay estimation data model considered in this paper has the following general form:

$$y(nT_s) = \sum_{l=1}^L \alpha_l s(nT_s - \tau_l) + e(nT_s), \quad 0 \leq n \leq N-1, \quad (1)$$

where T_s denotes the sampling interval, $s(nT_s)$, represents an *arbitrary* known transmitted signal, $y(nT_s)$ denotes the received signal, which is composed of L replicas of $s(nT_s)$ with different amplitudes $\{\alpha_l\}_{l=1}^L$ and delays $\{\tau_l\}_{l=1}^L$, and $e(nT_s)$ is the additive noise, which is modeled as a zero-mean white Gaussian random process. Without loss of generality, we assume that $s(nT_s)$, $y(nT_s)$, $e(nT_s)$, and $\{\alpha_l\}_{l=1}^L$ are either all complex-valued or all real-valued.

Our problem of interest herein is to estimate $\{\alpha_l, \tau_l\}_{l=1}^L$ from $\{y(nT_s)\}_{n=0}^{N-1}$ with known $\{s(nT_s)\}_{n=0}^{N-1}$ when the signals are very closely spaced.

Although we could solve the estimation problem in the time domain [4, 5, 7, 8], we prefer to do it in the frequency domain. This is because for the time domain processing methods, we could be restricted to using the discrete values of $\{\tau_l\}_{l=1}^L$ if we only know the sampled version of $s(t)$. For this case, if a more accurate delay estimate is required, then one has to resort to interpolation [5]. This inconvenience can be avoided by transforming the problem into the frequency domain, where $\{\tau_l\}_{l=1}^L$ can take on a continuum of values. Let $Y(k)$, $S(k)$, and $E(k)$, $k = -N/2, -N/2+1, \dots, N/2-1$, denote the discrete Fourier transforms (DFT's) of $y(nT_s)$, $s(nT_s)$, and $e(nT_s)$, respectively. Provided that aliasing is negligible, then $Y(k)$ can be written as:

$$Y(k) = S(k) \sum_{l=1}^L \alpha_l e^{j\omega_l k} + E(k), \quad (2)$$

where

$$\omega_l = -\frac{2\pi\tau_l}{NT_s}. \quad (3)$$

3. THE WRELAX ALGORITHM

3.1. WRELAX for Complex-Valued Signals

Let $\mathbf{Y} \in \mathcal{C}^{N \times 1}$, $\mathbf{E} \in \mathcal{C}^{N \times 1}$, and $\alpha \in \mathcal{C}^{L \times 1}$ denote the vectors formed by $\{Y(k)\}_{k=-N/2}^{N/2-1}$, $\{E(k)\}_{k=-N/2}^{N/2-1}$, $\{\alpha_l\}_{l=1}^L$, respectively, and let $\mathbf{S} \in \mathcal{C}^{N \times N}$ denote a diagonal matrix with $\{S(k)\}_{k=-N/2}^{N/2-1}$ as its diagonal elements. Then the data model (2) can be written in the following vector form:

$$\mathbf{Y} = \mathbf{S}\mathbf{A}\alpha + \mathbf{E}, \quad (4)$$

where

$$\mathbf{A} = [\mathbf{a}(\omega_1) \quad \mathbf{a}(\omega_2) \quad \cdots \quad \mathbf{a}(\omega_L)]^T, \quad (5)$$

with

$$\mathbf{a}(\omega_l) = [e^{j\omega_l(-\frac{N}{2})} \quad e^{j\omega_l(-\frac{N}{2}+1)} \quad \cdots \quad e^{j\omega_l(\frac{N}{2}-1)}]^T, \quad (6)$$

and $(\cdot)^T$ denotes the transpose.

WRELAX is a relaxation-based minimizer of the following nonlinear least-squares (NLS) criterion:

$$C_1(\{\alpha_l, \omega_l\}_{l=1}^L) = \|\mathbf{Y} - \sum_{l=1}^L \alpha_l \mathbf{S}\mathbf{a}(\omega_l)\|^2. \quad (7)$$

When $e(nT_s)$ is a zero-mean white Gaussian random process, $E(k)$ is also white since DFT is a unitary transformation. For this white noise case, the NLS approach is the same as the maximum likelihood (ML) method.

Before we present the WRELAX algorithm, let us consider the following preparations. Let

$$\mathbf{Y}_l = \mathbf{Y} - \sum_{i=1, i \neq l}^L \hat{\alpha}_i [\mathbf{S}\mathbf{a}(\hat{\omega}_i)], \quad (8)$$

where $\{\hat{\alpha}_i, \hat{\omega}_i\}_{i=1, i \neq l}$ are assumed to be given. Then (7) becomes

$$C_2(\alpha_l, \omega_l) = \|\mathbf{Y} - \alpha_l \mathbf{S}\mathbf{a}(\omega_l)\|^2. \quad (9)$$

Minimizing $C_2(\alpha_l, \omega_l)$ with respect to ω_l and the complex-valued α_l yields

$$\hat{\omega}_l = \arg \max_{\omega_l} |\mathbf{a}^H(\omega_l)(\mathbf{S}^* \mathbf{Y}_l)|^2, \quad (10)$$

and

$$\hat{\alpha}_l = \frac{\mathbf{a}^H(\omega_l)(\mathbf{S}^* \mathbf{Y}_l)}{\|\mathbf{S}\|_F^2} \Bigg|_{\omega_l = \hat{\omega}_l}, \quad (11)$$

where $(\cdot)^H$, $(\cdot)^*$, and $\|\cdot\|_F$ denote the conjugate transpose, complex conjugate, and the *Frobenius* norm.

With the above simple preparations, we now present the WRELAX algorithm.

Step (1): Assume $L = 1$. Obtain $\{\hat{\omega}_l, \hat{\alpha}_l\}_{l=1}$ from \mathbf{Y} by using (10) and (11).

Step (2): Assume $L = 2$. Compute \mathbf{Y}_2 with (8) by using $\{\hat{\omega}_l, \hat{\alpha}_l\}_{l=1}$ obtained in Step (1). Obtain $\{\hat{\omega}_l, \hat{\alpha}_l\}_{l=2}$ from \mathbf{Y}_2 . Next, compute \mathbf{Y}_1 by using $\{\hat{\omega}_l, \hat{\alpha}_l\}_{l=2}$ and then redetermine $\{\hat{\omega}_l, \hat{\alpha}_l\}_{l=1}$ from \mathbf{Y}_1 .

Iterate the previous two substeps until “practical convergence” is achieved (to be discussed later on).

Remaining Steps: Continue similarly until L is equal to the desired or estimated number of signals.

The “practical convergence” in the iterations of the above WRELAX algorithm may be determined by checking the relative change of the cost function $C_1(\{\hat{\omega}_l, \hat{\alpha}_l\}_{l=1}^L)$ in (7) between two consecutive iterations.

Once $\{\hat{\omega}_l\}_{l=1}^L$ are determined, the delay estimates $\{\hat{\tau}_l\}_{l=1}^L$ of $\{\tau_l\}_{l=1}^L$ can be computed by using (3) with $\{\omega_l\}_{l=1}^L$ replaced by $\{\hat{\omega}_l\}_{l=1}^L$.

3.2. WRELAX for Real-Valued Signals

Consider the data model expressed by (2). When the signals $s(t)$, $y(t)$, and $e(t)$ are all real-valued, their Fourier transforms are conjugate symmetric, hence we can use only $\{Y(k)\}_{k=0}^{N/2-1}$ to estimate the time delays and amplitudes without any performance degradation. Define

$$\begin{aligned} \mathbf{W} &= \text{diag} \left\{ W(0), W(1), \dots, W\left(\frac{N}{2}-1\right) \right\} \\ &= \text{diag} \left\{ \frac{1}{\sqrt{2}}, 1, \dots, 1 \right\}. \end{aligned} \quad (12)$$

Assume that $\bar{\mathbf{Y}} \in \mathcal{C}^{N/2 \times 1}$, $\bar{\mathbf{E}} \in \mathcal{C}^{N/2 \times 1}$, and $\bar{\alpha} \in \mathcal{R}^{L \times 1}$ denote the vectors formed by $\{W(k)Y(k)\}_{k=0}^{N/2-1}$, $\{W(k)E(k)\}_{k=0}^{N/2-1}$, $\{\alpha_l\}_{l=1}^L$, respectively. Let $\bar{\mathbf{S}} \in \mathcal{C}^{N/2 \times N/2}$ denote a diagonal matrix with $\{W(k)S(k)\}_{k=0}^{N/2-1}$ as its diagonal elements, and let $\bar{\mathbf{A}}$ and $\bar{\mathbf{a}}(\omega_l)$ denote the lower halves of \mathbf{A} and \mathbf{a} , respectively. Then it follows that

$$\bar{\mathbf{Y}} = \bar{\mathbf{S}}\bar{\mathbf{A}}\bar{\alpha} + \bar{\mathbf{E}}. \quad (13)$$

Due to the conjugate symmetry of $Y(k)$, $S(k)$, and $E(k)$, it can be easily proven that minimizing $C_1(\{\alpha_l, \omega_l\}_{l=1}^L)$ is equivalent to minimizing

$$C_3(\{\alpha_l, \omega_l\}_{l=1}^L) = \|\bar{\mathbf{Y}} - \sum_{l=1}^L \alpha_l \bar{\mathbf{S}}\bar{\mathbf{a}}(\omega_l)\|^2. \quad (14)$$

For the case of white Gaussian noise, it can be proved the above NLS approach is the same as the ML method. Let

$$\bar{\mathbf{Y}}_l = \bar{\mathbf{Y}} - \sum_{i=1, i \neq l}^L \hat{\alpha}_i [\bar{\mathbf{S}}\bar{\mathbf{a}}(\hat{\omega}_i)], \quad (15)$$

where $\{\hat{\alpha}_i, \hat{\omega}_i\}_{i=1, i \neq l}$ are assumed to be given. Then (14) becomes

$$C_4(\alpha_l, \omega_l) = \|\bar{\mathbf{Y}} - \alpha_l \bar{\mathbf{S}}\bar{\mathbf{a}}(\omega_l)\|^2. \quad (16)$$

Minimizing $C_4(\alpha_l, \omega_l)$ with respect to ω_l and real-valued α_l yields

$$\hat{\omega}_l = \arg \max_{\omega_l} \text{Re}^2 [\bar{\mathbf{a}}^H(\omega_l)(\bar{\mathbf{S}}^* \bar{\mathbf{Y}}_l)], \quad (17)$$

and

$$\hat{\alpha}_l = \frac{\text{Re} [\bar{\mathbf{a}}^H(\omega_l)(\bar{\mathbf{S}}^* \bar{\mathbf{Y}}_l)]}{\|\bar{\mathbf{S}}\|_F^2} \Bigg|_{\omega_l = \hat{\omega}_l}. \quad (18)$$

The iteration steps of WRELAX for real-valued signals is the same as that for complex-valued signals except that the cost functions (10) and (11) are replaced by (17) and (18), respectively.

4. THE MODE-WRELAX ALGORITHM

WRELAX is an efficient algorithm that is both computationally and conceptually simple. For signals not spaced very closely, WRELAX usually converges after a few iterations. However, when the signals get more and more closer to each other, WRELAX will converge very slowly. In this section, we modify the popular angle estimation MODE algorithm and use it with WRELAX to increase the convergence speed of WRELAX.

4.1. MODE-WRELAX for Complex-Valued Signals

MODE is an asymptotically statistically efficient estimator of $\{\omega_l\}_{l=1}^L$ for complex-valued signals [9]. The MODE estimates $\{\hat{\omega}_l\}_{l=1}^L$ of $\{\omega_l\}_{l=1}^L$ are obtained by minimizing the following cost function

$$C_5(\{\omega_l\}_{l=1}^L) = \mathbf{Y}^H \mathbf{P}_{\hat{\mathbf{A}}}^\perp \mathbf{Y}, \quad (19)$$

where $\mathbf{P}_{\hat{\mathbf{A}}}^\perp = \mathbf{I} - \tilde{\mathbf{A}}(\tilde{\mathbf{A}}^H \tilde{\mathbf{A}})^{-1} \tilde{\mathbf{A}}^H$ with \mathbf{I} denoting the identity matrix and $\tilde{\mathbf{A}} = \mathbf{S}\mathbf{A}$. To avoid the search over the parameter space, $C_5(\{\omega_l\}_{l=1}^L)$ can also be reparametrized in terms of another parameter vector $\mathbf{b} = [b_0 \ b_1 \ \dots \ b_L]^T$, where $\{b_l\}_{l=0}^L$ are the coefficients of the following polynomial:

$$b(z) \triangleq \sum_{l=0}^L b_l z^{L-l} \triangleq b_0 \prod_{l=1}^L (z - e^{j\omega_l}); \quad b_0 \neq 0. \quad (20)$$

Let

$$\mathbf{B} = \begin{bmatrix} b_0 & & & 0 \\ & \ddots & & \\ & & \ddots & \\ b_L & & & b_0 \\ & & \ddots & \\ 0 & & & b_L \end{bmatrix} \in \mathcal{C}^{N \times (N-L)}. \quad (21)$$

Assume that the diagonal elements of \mathbf{S} are nonzero (see **Remark 1** for more discussions). Let

$$\tilde{\mathbf{B}} = \mathbf{S}^{-H} \mathbf{B}. \quad (22)$$

It can be readily verified that $\mathbf{B}^H \mathbf{A} = \mathbf{0}$ and hence $\tilde{\mathbf{B}}^H \tilde{\mathbf{A}} = \mathbf{0}$. Then $\mathbf{P}_{\tilde{\mathbf{A}}}^\perp = \tilde{\mathbf{B}}(\tilde{\mathbf{B}}^H \tilde{\mathbf{B}})^{-1} \tilde{\mathbf{B}}^H$ and minimizing $C_5(\{\omega_l\}_{l=1}^L)$ in (7) is equivalent to minimizing

$$C_6(\{b_l\}_{l=0}^L) = \mathbf{Y}^H \tilde{\mathbf{B}}(\tilde{\mathbf{B}}^H \tilde{\mathbf{B}})^{-1} \tilde{\mathbf{B}}^H \mathbf{Y}. \quad (23)$$

Note that $\tilde{\mathbf{B}}^H \tilde{\mathbf{B}}$ in (9) can be replaced by a consistent estimate without affecting the asymptotically statistical efficiency of the minimizer of (9). Hence $\hat{\mathbf{b}}$ can be obtained computationally efficiently as follows:

$$\hat{\mathbf{b}} = \arg \min_{\mathbf{b}} \left[\mathbf{Y}^H \mathbf{S}^{-H} \mathbf{B} (\hat{\mathbf{B}}_0^H \mathbf{S}^{-1} \mathbf{S}^{-H} \hat{\mathbf{B}}_0)^{-1} \mathbf{B}^H \mathbf{S}^{-1} \mathbf{Y} \right], \quad (24)$$

where $\hat{\mathbf{B}}_0$ is the initial estimate of \mathbf{B} obtained by replacing \mathbf{b} with $\hat{\mathbf{b}}^{(0)}$ in (21). The initial value $\hat{\mathbf{b}}^{(0)}$ is obtained by setting $\tilde{\mathbf{B}}^H \tilde{\mathbf{B}}$ in (9) to \mathbf{I} :

$$\hat{\mathbf{b}}^{(0)} = \arg \min_{\mathbf{b}} \left[\mathbf{Y}^H \mathbf{S}^{-H} \mathbf{B} \mathbf{B}^H \mathbf{S}^{-1} \mathbf{Y} \right]. \quad (25)$$

To avoid the trivial solution $\mathbf{b} = \mathbf{0}$, we should impose $\|\mathbf{b}\| = 1$ (where $\|\cdot\|$ denotes the Euclidean norm) in (24) and (25) or some other similar constraints. The estimates $\{\hat{\omega}_l\}_{l=1}^L$ of $\{\omega_l\}_{l=1}^L$ are the phases of the roots of the polynomial $\sum_{l=0}^L \hat{b}_l z^{L-l}$. Once $\{\hat{\omega}_l\}_{l=1}^L$ are obtained, the amplitudes α are estimated by applying the linear least-squares approach to

$$\mathbf{Y} \approx \hat{\mathbf{S}} \hat{\mathbf{A}} \alpha, \quad (26)$$

where $\hat{\mathbf{A}}$ is formed by replacing $\{\omega_l\}_{l=1}^L$ with $\{\hat{\omega}_l\}_{l=1}^L$ in (5). **Remark 1:** MODE cannot be implemented efficiently to avoid the search over the parameter space when $S(k) = 0$ for some k . The most commonly used complex analytic signal $s(t)$ is low-pass. For this case, we can select a contiguous segment of \mathbf{Y} satisfying $|S(k)| > 0$, $K_1 \leq k \leq K_2$, and preferably with $|S(k)|$ above a certain threshold to avoid numerical problems. We can then apply MODE to the segment $\{Y(k)\}_{k=K_1}^{K_2}$ to estimate $\{\omega_l\}_{l=1}^L$.

Remark 2: The amplitude estimates given above can be very poor when the SNR is not sufficiently high. This is because some of the MODE estimates $\{\hat{\omega}_l\}_{l=1}^L$ can be so closely spaced that $\hat{\mathbf{A}}$ in (26) is seriously ill-conditioned. We use a simple spacing adjustment scheme to avoid this problem. After obtaining the MODE estimates $\{\hat{\omega}_l\}_{l=1}^L$ of $\{\omega_l\}_{l=1}^L$, we first sort them in the ascending order and then check the spacing between any two adjacent estimates. If the distance between any two estimates, say $\hat{\omega}_1$ and $\hat{\omega}_2$ ($\hat{\omega}_1 \leq \hat{\omega}_2$), is smaller than a predefined threshold, say $\Delta\omega_l$, we adjust the estimates by replacing $\hat{\omega}_1$ with $\hat{\omega}_1 - 0.5\Delta\omega_l$ and $\hat{\omega}_2$ with $\hat{\omega}_2 + 0.5\Delta\omega_l$. The amplitudes are then estimated using the adjusted estimates of $\{\omega_l\}_{l=1}^L$.

With the above preparations, we now present the steps of the MODE-WRELAX algorithm for complex-valued signals.

Step (1): Select a contiguous segment of data vector \mathbf{Y} (for MODE use only) so that $|S(k)| > 0$, $K_1 \leq k \leq K_2$. Apply MODE to the segment to obtain $\{\hat{\omega}_l\}_{l=1}^L$. Adjust $\{\hat{\omega}_l\}_{l=1}^L$ so that the minimum spacing of $\{\hat{\omega}_l\}_{l=1}^L$ is at least $\Delta\omega_l$. Obtain the estimates $\{\hat{\alpha}_l\}_{l=1}^L$ of $\{\alpha_l\}_{l=1}^L$ by using (26).

Step (2): Refine the estimates obtained in Step (1) by using the last step of WRELAX for complex-valued signals.

4.2. MODE-WRELAX for Real-Valued Signals

Real-valued signals are often bandpass signals that occur, for example, in underwater sonar and ultra wideband ground penetrating radar applications. Bandpass signals have highly oscillatory correlation functions, which makes the super resolution time delay estimation problem more difficult. The larger the center frequency of the pass band, the sharper the oscillation of the correlation function. However, by assuming the real-valued amplitudes to be complex-valued, we can obtain a much smoother cost function that is much easier to find its global minimum. For the case of one signal without noise, Figure 1 compares the true cost function (solid line) with the one obtained by assuming the real-valued amplitudes to be complex-valued (dashed line), where the signal is a chirp signal whose carrier frequency is twice of its bandwidth (see Section 5 for details). Based on this observation, we can apply the MODE-WRELAX algorithm derived above to $\tilde{\mathbf{Y}}$ first by assuming the real-valued amplitudes $\{\alpha_l\}_{l=1}^L$ to be complex-valued, and then use the last step of WRELAX for real-valued signals to refine the estimates.

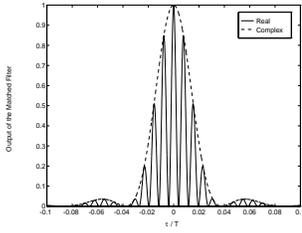


Figure 1. Comparison of the cost functions for complex-valued (dashed line) and real-valued signal amplitudes (solid line).

5. NUMERICAL EXAMPLE

Due to the limited space, in this section we use one example to show the performance of MODE-WRELAX. In this example, we use the following real-valued chirp signal,

$$s(t) = w(t)\cos\left[2\pi f_0 t + \beta\left(t - \frac{T_0}{2}\right)^2\right], 0 \leq t \leq T_0, \quad (27)$$

where f_0 denotes the carrier frequency, β represents the chirp rate, and $w(t)$ is a window function to avoid aliasing. The signal parameters are chosen as $\beta = \pi \times 10^5$, $N = 256$, the signal bandwidth $B_s = \beta T_0 / \pi$, the carrier frequency $f_0 = 2B_s$, and the sampling frequency $f_s = 8B_s$. T_0 is chosen in such a way that $T_0 = (N/2 - 1)T_s$. Assume $L = 2$, $\alpha_1 = \alpha_2 = 1.0$, $|\tau_2 - \tau_1| = 0.2\tau_e$, where τ_e is the resolution limit of the conventional matched filter method and is equal to $1/B_s$. The sampled noise $\{e(nT_s)\}$ is assumed to be a real-valued zero-mean white Gaussian random process with variance σ^2 . The SNR for each signal is defined as $10\log_{10}(\alpha_i^2/2\sigma^2)$. We have used $\epsilon = 0.001$ to test the convergence of WRELAX. The one-dimensional search of WRELAX is performed in two steps, a coarse search using FFT followed by a fine search using the Golden section search method. The MODE amplitude estimates are obtained without the spacing adjustment.

The mean-squared errors (MSEs) of MODE (“o”), and MODE-WRELAX (“*”) are compared with the CRBs (solid line) in Figure 2. Since the MODE amplitude estimates are obtained without spacing adjustment, they are so poor at low SNR that some of their MSEs are above the axis limit due to the inversion of ill-conditioned matrices corresponding to very closely spaced delay estimates. By assuming the real-valued amplitudes to be complex valued (the same assumption used by MODE), the MODE-WRELAX algorithm for complex-valued amplitudes (notated as MODE-WRELAX(C) (“x”)) performs better than MODE and can approach the CRBs corresponding to complex-valued signals (notated as CRB(C) (dashed lines)). Nevertheless, these wrong CRBs can be larger than the true CRBs, which corresponds to the real-valued amplitudes, by approximately 30 dB. MODE-WRELAX significantly outperforms MODE and WRELAX and can approach the true CRBs.

6. CONCLUSION

In this paper, we first present a computationally and conceptually simple algorithm (WRELAX) for time delay estimation. One major advantage of WRELAX is that it estimates the time delays and the amplitudes jointly and does not have the ill-conditioning problem suffered by many separate delay and amplitude estimation techniques. The combination of MODE with WRELAX significantly improves

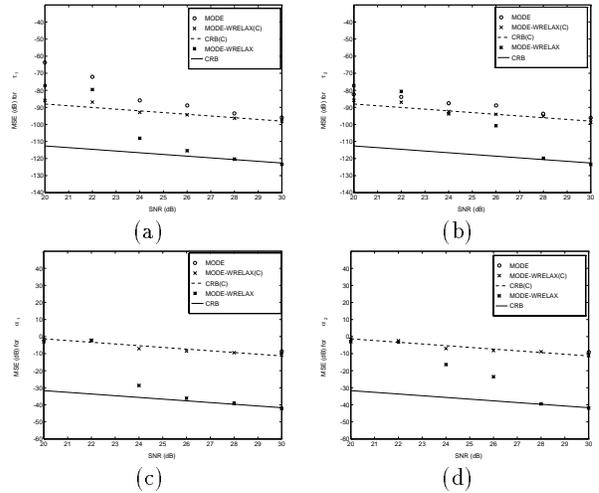


Figure 2. Comparison of the MSEs of MODE (“o”), MODE-WRELAX(C) (“x”), and MODE-WRELAX (“*”) with the true CRBs (solid lines) and the wrong CRBs (dashed lines) corresponding to real- and complex-valued signals, respectively, for (a) τ_1 , (b) τ_2 , (c) α_1 , and (d) α_2 .

the convergence speed of WRELAX and the estimation accuracy of MODE.

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