NON-LINEAR AMPLIFIER EFFECTS IN TRANSMIT BEAMFORMING ARRAYS

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ABSTRACT

The effects of memoryless (no feedback) amplifiers with amplitude non-linearities on the output of transmit beamforming arrays is studied. These studies are limited to "small signal" cases where the output signal is not undergoing significant compression by the amplifier; and where the amplifier non-linearities can be approximated well by a cubic polynomial [1]. The performance of three "optimal" beamforming techniques [2-4] are compared and contrasted when both linear and non-linear amplifiers are used in a two dimensional array. The two dimensional array considered is designed to simultaneously transmit two narrow band signals, at different frequencies, through a common aperture. Attention is focused on the non-linearity induced interaction of the stimulus signals, and the effect this has on the array's outputs as a function of beamforming technique.

1. INTRODUCTION

This paper studies some of the effects of final stage amplification on beam patterns from transmission arrays. In particular, we concentrate on the effects that amplitude non-linearities, inherent in the final stage amplifiers, have on these patterns. The problem considered here is one in which two narrow band signals of different frequency are to be transmitted through the same array aperture. That is, the array aperture is not split, but is shared in a non time multiplexed fashion by both signals. This might be done in small arrays to achieve maximum aperture for each signal without sacrificing dwell time. A simplified diagram of the signal paths for a single array element is shown in figure 1.

Since, in general, the desired output beam pattern will be different for each frequency, there will be a separate set of weights (amplitude and phase) for each stimulus signal. In this study, the multipliers, summers and array elements themselves are assumed ideal; and the computations are done to 32 bit floating point precision. This is done in order to focus attention primarily on the effects of the amplifier non-linearities. Effects due to quantization, anisotropic elements, element positional errors, and element cross loading were reported in earlier work for non-shared aperture arrays [5]. Similar results for shared aperture arrays are not reported here.

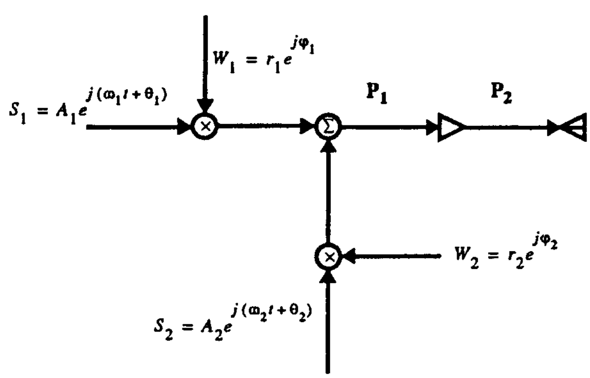

Figure 1

As stated, we will limit ourselves to the "small signal" case and to cubic approximations to the amplifier amplitude non-linearities. No other amplifier non-linearities are considered (e.g. the cross modulation, group delay, and AM to PM varieties). Starting from these assumptions, we develop an amplifier model and calculate the effects it has on the beam patterns produced by three well known "optimal" beamforming techniques. The beam formation techniques considered are outlined in the references [2-4].

It was found that the amplifier non-linearities produced undesirable effects for each of the beamforming techniques tested. However, some of the techniques proved more sensitive to the influences of the non-linearities than others. This sensitivity manifests itself in the amplitude of the unwanted harmonic and inter-modulation frequencies which result from non-linearities in the amplifier, and in the suddenness of occurrence of these effects as the beams scan in space. The amplitudes of these unwanted frequencies can be significant, even for "small" stimulus signals. Typically, it was the techniques which did not necessarily employ a smooth phase progression (ie. a small number of phase wraps) across the array aperture [3,4] which were the most sensitive to these effects as the beams scanned.

The array studied here is a 2 by 17 element rectangular array with uniform element spacing in both dimensions. Each of the elements is modeled as having a cosine shaped element pattern. Although we only consider a transmit array in this paper, similar effects should be seen when two signals close in frequency and signal strength arrive at a receive array aperture in which non-linear amplifiers precede beam formation (summation).

2. THE AMPLIFIER MODEL

Under the assumption of ideal multipliers and summers, the signal present at point P_1 in figure 1 is given by:

$$P_1 = r_1 A_1 e^{j(\omega_1 t + \theta_1 + \varphi_1)} + r_2 A_2 e^{j(\omega_2 t + \theta_2 + \varphi_2)} \quad (1)$$

If the amplifier were linear then the signal present at point P_2 would be:

$$P_2 = \alpha\left(r_1 A_1 e^{j(\omega_1 t + \theta_1 + \varphi_1)} + r_2 A_2 e^{j(\omega_2 t + \theta_2 + \varphi_2)}\right) + \beta \quad (2)$$

where α is the slope (gain) and β is the (constant) ordinate inter-

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cept; taken here as zero (no power in no power out). For real stimulus signals (and $\beta = 0$), equation (2) could be written as:

$$P_2 = \alpha\,(r_1 A_1 \cos(\omega_1 t + \Phi_1) + r_2 A_2 \cos(\omega_2 t + \Phi_2)) \qquad (3)$$

where $\Phi_i = \theta_i + \varphi_i, i = 1, 2$. For an ideal antenna element, equation (3) would be the transmitted signal. We assume that the stimulus signals at frequencies ω_1 and ω_2 are un-modulated in amplitude and phase, and constant across the array; and also that each amplifier has the same α.

Let V_i be the signal input to a memoryless amplifier with amplitude non-linearities. From equation (3) above, this input signal may be expressed as:

$$V_i = r_1 A_1 \cos(\omega_1 t + \Phi_1) + r_2 A_2 \cos(\omega_2 t + \Phi_2) \qquad (4)$$

Let V_o be the signal output from the amplifier. Assuming a cubic approximation to the amplitude non-linearities we have:

$$V_o = k_1 V_i + k_2 V_i^2 + k_3 V_i^3 \qquad (5)$$

where the $k_i, i = 1, 2, 3$ are the coefficients which describe the cubic polynomial representing the amplifier transfer function. These may be readily calculated via. intercept point analysis for a particular amplifier [1]. Note that for a linear amplifier, $k_1 = \alpha$ from equation (3) and $k_2 = k_3 = 0$. Expanding equation (5) in terms of equation (4) results in equation (6).

$$\begin{aligned}
V_o = {} & k_2\left(\frac{r_1^2 A_1^2 + r_2^2 A_2^2}{2}\right) + \\
& \left(k_1 (r_1 A_1) + k_3\left(\frac{3 r_1^3 A_1^3 + 6 r_1 A_1 r_2^2 A_2^2}{4}\right)\right)\cos(\omega_1 t + \Phi_1) + \\
& \left(k_1 (r_2 A_2) + k_3\left(\frac{3 r_2^3 A_2^3 + 6 r_2 A_2 r_1^2 A_1^2}{4}\right)\right)\cos(\omega_2 t + \Phi_2) + \\
& k_2\left(\frac{r_1^2 A_1^2}{2}\right)\cos(2(\omega_1 t + \Phi_1)) + k_2\left(\frac{r_2^2 A_2^2}{2}\right)\cos(2(\omega_2 t + \Phi_2)) + \\
& k_2 (r_1 A_1 r_2 A_2)\cos((\omega_1 - \omega_2)t + (\Phi_1 - \Phi_2)) + \\
& k_2 (r_1 A_1 r_2 A_2)\cos((\omega_1 + \omega_2)t + (\Phi_1 + \Phi_2)) + \\
& k_3\left(\frac{r_1^3 A_1^3}{4}\right)\cos(3(\omega_1 t + \Phi_1)) + k_3\left(\frac{r_2^3 A_2^3}{2}\right)\cos(3(\omega_2 t + \Phi_2)) + \\
& k_3\left(\frac{3 r_1 A_1 r_2^2 A_2^2}{4}\right)\cos((2\omega_2 - \omega_1)t + (2\Phi_2 - \Phi_1)) + \\
& k_3\left(\frac{3 r_1 A_1 r_2^2 A_2^2}{4}\right)\cos((2\omega_2 + \omega_1)t + (2\Phi_2 + \Phi_1)) + \\
& k_3\left(\frac{3 r_2 A_2 r_1^2 A_1^2}{4}\right)\cos((2\omega_1 - \omega_2)t + (2\Phi_1 - \Phi_2)) + \\
& k_3\left(\frac{3 r_2 A_2 r_1^2 A_1^2}{4}\right)\cos((2\omega_1 + \omega_2)t + (2\Phi_1 + \Phi_2))
\end{aligned} \qquad (6)$$

By examination of equation (6) the reader can determine the role of the coefficients, array tapers, steering phases, and stimulus amplitudes in corrupting the output signal. In particular, note the effects of the coefficients. High gain amplifiers generally have large coefficients, especially in the quadratic and cubic terms. High values in these terms can significantly raise the power of the unwanted harmonics and intermodulation products, even for small input signals.

Often the most troublesome frequencies are the intermodulation products defined by $2\omega_2 - \omega_1$ and $2\omega_1 - \omega_2$. When ω_1 and ω_2 are close in frequency these intermodulation products can be within the pass band of the amplifier; and if k_3 is too large, these terms can have significant amplitudes. Note also that a large k_3 can corrupt the beams formed at the fundamental frequencies ω_1 and ω_2 as well.

3. RESULTS

Figures 2 and 3 illustrate the phase steered, Dolph-Chebyshev weighted (30 dB side lobes), beam patterns resulting from the two fundamental frequencies indicated in the figures (refer also to equation (6)). The steered angles are -10 degrees OBA (Off Broadside Angle) azimuth, 0 degrees OBA elevation for lambda = 2.02: and 20 degrees OBA azimuth, 0 degrees OBA elevation

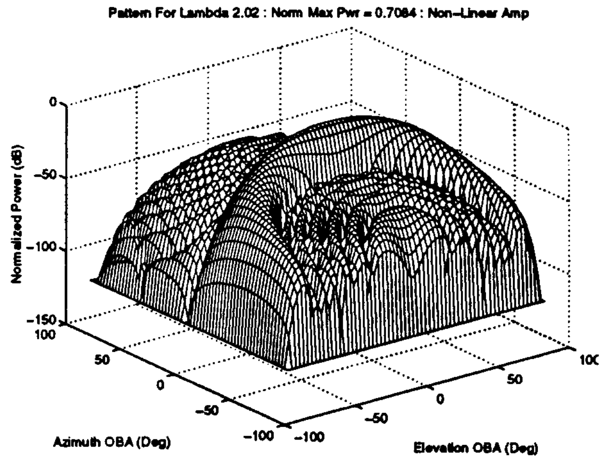

Figure 2: Dolph-Chebyshev - ω_1

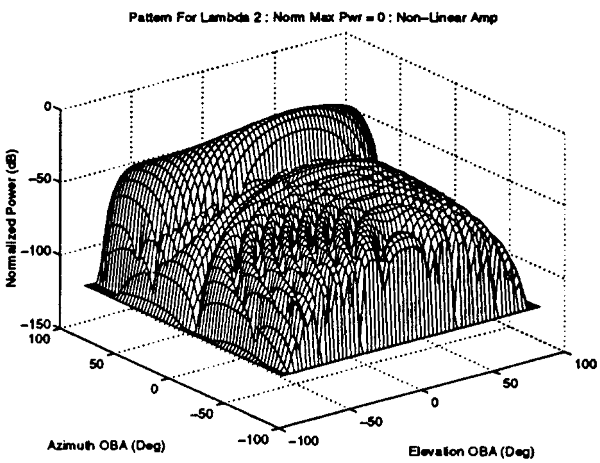

Figure 3: Dolph-Chebyshev - ω_2

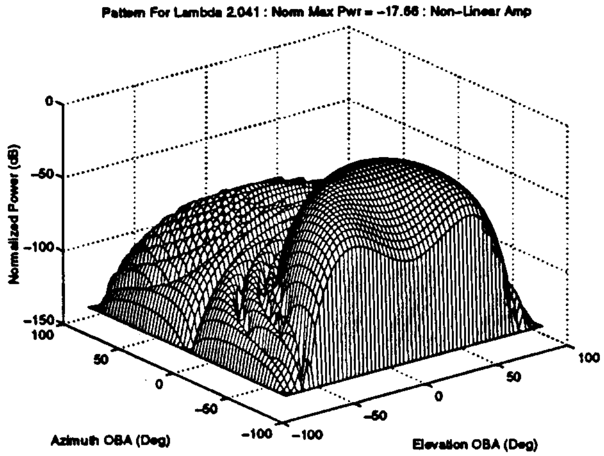

Figure 4: Dolph-Chebyshev - $2\omega_2 - \omega_1$

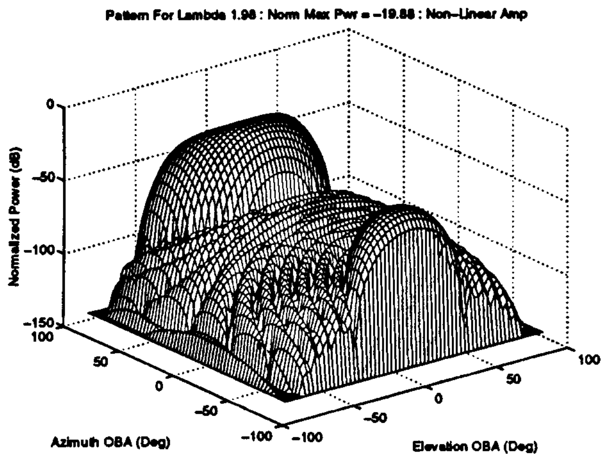

Figure 5: Dolph-Chebyshev - $2\omega_1 - \omega_2$

for lambda = 2. These patterns are not substantially different from the corresponding ideal patterns (not shown). Note that the wavelengths are given in terms of element spacing, and recall that the elements have a cosine element pattern. Figures 4 and 5 illustrate beam patterns of the intermodulation products defined by $2\omega_2 - \omega_1$ and $2\omega_1 - \omega_2$. Figure 6 shows a composite plot of slices in azimuth through the maximum of a given beam pattern for all

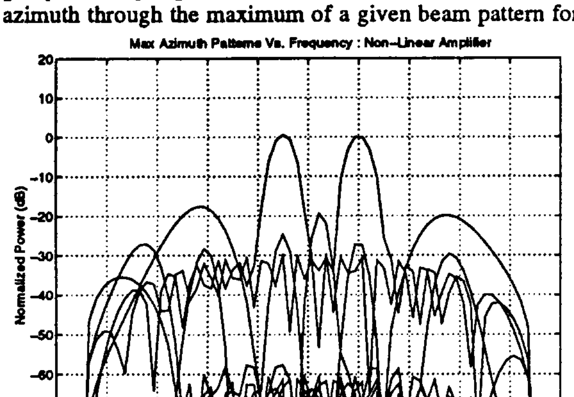

Figure 6: Dolph-Chebyshev

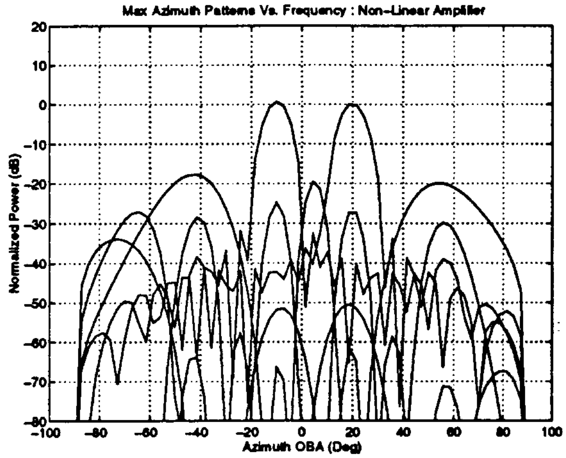

Figure 7: CWLMS

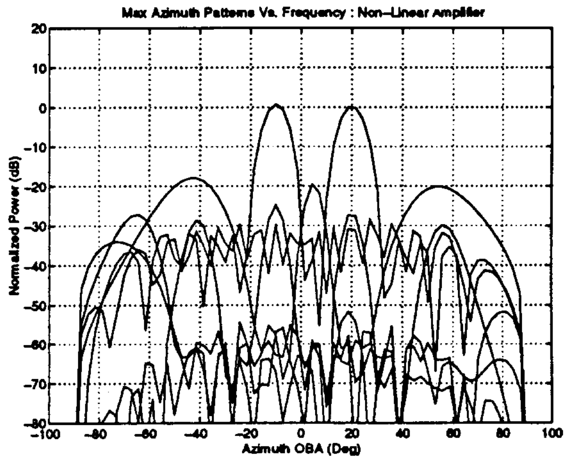

Figure 8: Maximum Gain

harmonics and intermodulation products generated by equation (6). Figures 7 and 8 show composite plots for the Constrained Weighted Least Mean Square (CWLMS) [3] and maximum gain [4] techniques respectively.

There are several options for controlling the spurious emissions of the array (ie. those resulting from harmonic and intermodulation frequencies). Two will be considered here. The simplest approach to reducing the magnitude of the spurious beams is to reduce the level of the driving voltage at the input to the power amplifier. As the amplifier output power recedes from saturation (maximum output power), the harmonic and intermodulation outputs of the amplifier reduce rapidly. For every one dB reduction of the input power, the second order harmonic and intermodulation products reduce by 2 dB while the third order products reduce by 3 dB. Recall that the third order intermodulation frequencies $2\omega_2 - \omega_1$ and $2\omega_1 - \omega_2$ are the ones most likely to be within the pass band of the amplifier. Obviously, this approach also reduces the power radiated in the desired beams, but at a one to one input/output ratio.

One method to compensate for this loss in output power is to increase the array aperture via additional elements. Note, however, that to achieve an additional 3 dB in radiated power in this manner requires a 41 percent increase in the number of active

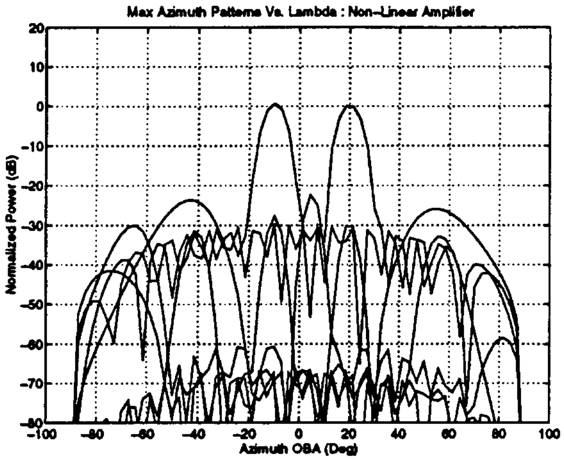

Figure 9: Dolph-Chebyshev

array elements. These additional elements add cost and weight to the array, and require more prime power.

An alternative approach to decreasing the input power is to increase the amplifier's second and third order intercept points without changing its gain or maximum output power. If this is done, the intermodulation products will be reduced without changing the radiated power at the desired frequencies. The second order products will be reduced one dB for every dB the second order intercept is raised. The third order products will go down 2 dB for every dB the third order intercept is raised. This effect is illustrated in figure 9 for the Dolph-Chebyshev case. Comparing figure 9 with figure 6 we see a 3 dB reduction in second order spurious beams and a 6 dB reduction in third order spurious beams due to a 3 dB increase in both intercept points. The same effect is observed in the spurious response patterns of both the CWLMS and maximum gain techniques.

4. SUMMARY AND DISCUSSION

The effect of amplitude non-linearities [1] present in the final stage amplifiers of a two dimensional transmit array on resulting beam patterns was presented. The focus of this study was the magnitude and distribution of spurious frequency beam patterns resulting from the simultaneous transmission of two narrow band signals, closely spaced in frequency, through a common array aperture. The results were presented as a function of beam forming technique [2-4]. The studies were limited to the "small signal" case in which the signals output from the amplifiers were not undergoing significant compression. Further, non-linearities such as cross modulation, group delay and AM to PM conversion were not considered.

The results indicate that while the non-linearities produced significant unwanted spurious frequency beam patterns in each of the techniques considered, the CWLMS and maximum gain techniques [3,4] proved more spatially sensitive to the amplifier non-linearities. This is thought to be due to the tendency of these techniques to design rapid phase fluctuations into their weight vectors when called upon to produce narrow beam widths (high gain) for a given side lobe level.

Two methods for reducing the magnitude of the beam patterns of the spurs were also outlined. Clearly, of the two proposed methods, the most effective means of reducing the spurious beam level is achieved by improving the power amplifier's intercept performance. Reducing the stimulus drive level is an option for applications where array size, weight and power consumption are not critical factors.

A third method for reducing spurious frequency beam patterns, in particular azimuth and elevation directions for specified frequencies, may be possible. In this method, point nulls for pre-determined azimuth and elevation directions and for spurious frequencies of interest are designed directly into the primary beam weight vectors produced by CWLMS. That is, the CWLMS algorithm [3] is modified to allow directional null placement at frequencies other than those of the desired transmission frequency.

Preliminary calculations indicate that this approach might be fruitful, but studies have not gone beyond this point at this time. In any event this nulling approach is practical only if one can identify the angles where spurious beams must be controlled; and this, in turn, would be highly application dependent.

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