COHERENCE EFFECTS OF THE INTERFERENCE ON THE PERFORMANCE OF OPTIMUM/ADAPTIVE BEAMFORMER

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ABSTRACT

We present in this correspondence the analyses of the interference coherence effects on the performance of antenna array processor, which maximizes the output signal-to-noise ratio (SNR) for a coherent wave. The analytical expression for the optimal weight vector, directional pattern and output SNR is derived as a function of the coherence coefficient, interference power and parameters of the adaptive processor. We show how different models of interference coherence affect the array performance. Finally, to illustrate the theory, several numerical results are given.

1. INTRODUCTION

A concern wnich arises, when considering the use of a very large aperture in order to achieve high array gain, is that the signal received at widely separated sensors may have reduced coherence due to complexities in the propagation of the sound waves from the source to spatially separated receivers. Causes of distorsions are very diversified: they are usually of two types. The first type relates to mechanical deformation of the array. The second type is inherent to the propagating medium. Actually, the medium is neither isotropic, nor homogeneous, nor deterministic. Different causes of perturbation of the propagation are for instance the existence of multipaths, local hėterogeneites and diffusion.

The effects of the signal coherence on the performance of optimum/adaptive beamformers have been studied by many authors [1-7]. Cox, using a simple exponential correlation model to describe the coherence of a plane-wave signal, examined the effect of signal coherence on the signal-to-noise ratio (SNR) gain of a line array. More recently, Morgan and Smith studied the effect of coherence on the detection performance of linear and quadratic array processors using an exponential-power-law model for the signal wave-front correlation in the uncorrelated noise field. However, the detection performance of an optimum/adaptive array may be substantially degraded also in the case of the interference coherence reduction.

The purpose of this paper is to examine the influence of reduced interference coherence on the performance of an adaptive processor, which maximizes the output SNR for coherent signal and interferences [6-9]. For simplicity, the single interference case is studied. The spatial coherence of the interference is assumed to fall off exponentially with separation of the elements. Analytical expressions for the optimal weight vector, directional pattern and output SNR are derived in terms of the interference coherence coefficient, interference power and parameters of the adaptive processor. It is shown that the performance of optimal/adaptive beamforming is substantially degraded for large arrays and typical coherence lengths.

2. PROBLEM FORMULATION

We consider a passive array having N sensors. We shall be limited by the assumptions that the array is linear, and the sensors are omnidirectional. The noise is assumed to be statistically independent from element to element. Besides, we shall assume that desired signal is fully coherent on the aperture of array. The N-dimensional complex vector W represents the N optimal weights of the array processor which maximizes the output SNR. A well-known expression for W is given by [7-9]

$$W_{opt}^{*} = b\mathbf{R}^{-1}S_0^{*}, \tag{1}$$

where b is a constant, the superscript * denotes complex conjugate, S_0 is an N-dimensional vector of the desired signal, and $\mathbf{R}$ is the array correlation matrix of the uncorrelated noise and interference alone, given by

$$\mathbf{R} = \sigma_0^2\mathbf{I} + \mathbf{Q}, \tag{2}$$

where $\mathbf{Q}$ is the array correlation matrix of the interferences only, $\mathbf{I}$ is an identity matrix and σ_0^2 is the power of the uncorrelated noise in each element. For a coherent wave, the correlation matrix of each single interference is written

$$\mathbf{Q} = \sigma_i^2 S_i S_i^H, \tag{3}$$

where σ_i^2 is the received single interference power per element, the superscript H denotes complex conjugate transpose and S is the Green's- function, vector describing the propagation of the interference to each element, normalized such that $S_i^H S_i = N$. This normalization is consistent with conventional plane-wave beamforming, whereby the components of steering vectors are unit-magnitude complex numbers. The rank of $\mathbf{Q}$ is equal to the number of independent interferences. A loss of interference coherence will degrade the ideal interference correlation matrix (3), so that the magnitude of the terms in the interference correlation matrix diminish away from the main diagonal, eventually going to zero for very large distances. In the general case, the interference correlation matrix $\mathbf{Q}$ has full rank even for a single interference. For a weak noise power the inverse correlation matrix $\mathbf{R}$ can be expressed as

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$$R^{-1} = Q^{-1} + E\,,$$

where $E = -\sigma_0^2 Q^{-2}$. We shall assume for the small uncorrelated noise:

$$\|Q\| \gg \sigma_0^2\|I\| \qquad and \qquad \|Q^{-1}\| \gg \|E\|, \tag{4}$$

where $\|\|$ is the matrix spectral norm. (The spectral norm of a matrix A, which is induced by the Euclidian vector norm, is given by the square root of the largest eigenvalue of $A^H A$). Then the inverse correlation matrix R^{-1} can be expressed as

$$R^{-1} \approx Q^{-1}(I - \sigma_0^2 Q^{-1}). \tag{5}$$

From (1) the weight vector can be written as

$$W^*_{opt} \approx b\{I - \sigma_0^2 Q^{-1}\}Q^{-1}S_0^*. \tag{6}$$

Taking only the first term of Eq.(6), we can obtain the following approximation for the weight vector

$$W^*_{opt} \approx bQ^{-1}S_0^*. \tag{7}$$

Now the well-known expression for the output SNR is derived. Thus, in general case the output SNR is given by

$$SNR_{out} = \sigma_s^2(S_0^H R^{-1} S_0)^{-1}, \tag{8}$$

where σ_s^2 is the desired signal power. Using Eq. (5) we obtain for the output SNR

$$SNR_{out} = \sigma_s^2\{S_0^H Q^{-1}(I - \sigma_0^2 Q^{-1})S_0\}. \tag{9}$$

3. INTERFERENCE COHERENCE INVESTIGATION

For simplicity we shall consider the case of a single strong interference. In order to introduce a decrease in interference coherence, with an increase in element separation, we shall consider an exponential dependence of the form

$$q_{lk} = exp\{j(l-k)u_i\}p_0^{|l-k|}, \tag{10}$$

so that p_0 is the coherence between adjacent sensors, where $u_i = 2\pi d sin\theta_i/\lambda$, λ is the wavelenght, d is the interelement distance, θ_i is the angle of arrival of interference in the absence of reduction of spatially coherence. Typically p_0 might be expected to depend on the separation between elements measured in wavelenghts such as

$$p_0 = exp\{(-d/L)\}, \tag{11}$$

where L is a characteristic correlation length. It is important to understand that the interference model of Eq.(10) specifies the interference coherence between all pairs of elements in terms of the coherence between adjacent elements. Note that the form of the decorrelation coefficient between sensors appears to be independent of the interference direction. In most situations, the decorrelation between elements is strongly direction dependent. In these situations, the interference coherence coefficient p_0 is a function of the angle of arrival θ_i.

Using matrix notation, the interference correlation matrix Q under the above formulation is expressed as

$$Q = \sigma_i^2 F^* P F, \tag{12}$$

where σ_i^2 is the interference power per element and P is the correlation matrix whose components are defined as

$$p_{lk} = p_0^{|l-k|},$$

and

$$F = diag\{1, exp(ju_i), ..., exp(j(N-1)u_i)\}$$

is a diagonal matrix. Using Eq.(12) the inverse matrix Q^{-1} is given by

$$Q^{-1} = (1/\sigma_i^2)F^* P^{-1} F, \tag{13}$$

where the matrix P^{-1} can be written as follows [12,13]

$$P^{-1} = \frac{1}{1-p_0^2}\begin{pmatrix} 1 & -p_0 & 0 & \cdots & 0 \\ -p_0 & 1+p_0^2 & -p_0 & \cdots & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & \cdots & -p_0 & 1+p_0^2 & -p_0 \\ 0 & 0 & \cdots & -p_0 & 1 \end{pmatrix}$$

Then for the steady-state weight vector from (6) we obtain

$$W^*_{opt} = (b/\sigma_i^2)\{F^* P^{-1} F S_0^* - \frac{\sigma_0^2}{\sigma_i^2} F^* P^{-1} P^{-1} F S_0^*\}. \tag{14}$$

Taking the first terms only, we can write

$$W^*_{opt} \approx (b/\sigma_i^2)F^* P^{-1} C, \tag{15}$$

where C is a N-dimensional vector defined as

$$C = FS_0^* = [1, exp(j(u_i - u_s)), ..., exp(j(N-1)(u_i - u_s))]' \tag{16}$$

where $u_s = 2\pi d sin\theta_s/\lambda$, θ_s is the angle of arrival of the desired signal. Note that for the array processor which maximizes output SNR the constant b is given by [7]

$$b = \sigma_0^2. \tag{17}$$

Then from Eq.(15) we can obtain the following expressions for the components of the weight vector

$$\begin{aligned} w_1 &= \frac{1}{\nu_i(1-p_0^2)}\{1 - p_0 exp(j(u_i - u_s))\}, \\ w_k &= \frac{1}{\nu_i(1-p_0^2)}\{-p_0 exp(j(k-1)u_i) \\ &\quad +(1+p_0^2)exp(j(k-1)u_s) - p_0 exp(j(ku_s - (k-1)u_i))\}, \\ &\qquad k = 2, \cdots, (N-1) \qquad (18) \\ w_N &= \frac{1}{\nu_i(1-p_0^2)}\{1 - p_0 exp(-j(u_i - u_s))\}, \end{aligned}$$

where $\nu_i = \frac{\sigma_i^2}{\sigma_0^2}$. It will be further assumed, in order to simplify the calculation, that desired signal power is uniform over the array, so that $\theta_s = 0$. Then from (18) for w_l one can obtain

$$\begin{aligned} w_1 &= \frac{1}{\nu_i(1-p_0^2)}\{1 - p_0 exp(ju_i)\} \\ w_k &= \frac{1}{\nu_i(1-p_0^2)}\{1 + p_0^2 - 2p_0 cos(k-1)u_i\}, \\ &\qquad k = 2, \cdots, (N-1) \quad (19) \\ w_N &= \frac{1}{\nu_i(1-p_0^2)}\{1 - p_0 exp(-ju_i)\}. \end{aligned}$$

From (18),(19) we can derive the main characterictics of the adaptive processor. In particular, if $u_s = 0$ the expression for directional pattern of the adaptive array in the interference direction can be written as

$$\begin{aligned} | g(u_i) |^2 &= |W^H C|^2 = \\ &= |\sum_{k=1}^{N} w_k^* exp(j(k-1)u_i)|^2 = \\ &= (1/\nu_i(1-p_0^2))^2 |f(u_i) + \\ &+ (p_0^2 - 2p_0 cos(k-1)u_i) f_1(u_i) - \\ &- p_0(exp(-ju_i) + exp(jNu_i))|^2 \end{aligned} \quad (20)$$

where
$$f(u_i) = \sum_{k=1}^{N} exp(j(k-1)u_i,$$
$$f_1(u_i) = \sum_{k=2}^{N-1} exp(j(k-1)u_i$$

For isotropic interference ($p_0 \to 0$), the directional pattern is given by
$$| g(u_i) | \to (1/\nu_i)|f(u_i)|.$$

Now we derive the expression for the output SNR. Using Eq.(9) and (13) the output SNR in the first approximation can be written as

$$SNR_{out} = (\sigma_s^2/\sigma_i^2)\{S_0^H \mathbf{F}^* \mathbf{P}^{-1} \mathbf{F} S_0\}, \quad (21)$$

which describes the behavior of the output SNR as a function of the coherence coefficient, number of elements in the array, output SNR of the optimal processor. From (21) the expression for output SNR is given by

$$SNR_{out} = \frac{SNR_0}{N\nu_i(1-p_0^2)}\{N + (N-2)p_0^2 - 2p_0(N-1)cosu_i\}, \quad (22)$$

where $SNR_0 = N\nu_0 = N\frac{\sigma_s^2}{\sigma_0^2}$ is the output SNR of the optimal processor. From (22) the following observations can be made. 1) The output SNR is a monotonically decreasing function of the power of the interference, the coherence coefficient and the angle-of-arrival of the interference. 2) For the same coherence coefficient, the output SNR falls more rapidly by higher values of interference power. 3) For an array with a very large aperture ($N \gg 1$) from (22) the output SNR can be expressed as

$$SNR_{out} \approx SNR_0 \frac{1}{\nu_i(1-p_0^2)}(1 - 2p_0 cosu_i + p_0^2). \quad (23)$$

One can see that output SNR depends only on the coefficient of coherence between adjacent sensors p_0. Note that expressions (22)-(23) are correct, if the conditions (4) are granted. It is easy to show that the conditions (4) can be written as follows

$$\nu_i \gg 1, \qquad \nu_i(1-p_0^2) \gg 1 \quad (24)$$

From (24) one can see that expression (22) is not correct if $p_0 = 1$ (the case of full coherence on the aperture) because the matrix **Q** degenerates (has the dyadic structure (3)). For this case the output SNR is given by well-known expression [14]

$$SNR_{out} = SNR_0(1 - (N\nu_i/(1 + N\nu_i))|f(u_i)|). \quad (25)$$

The dependence of normalized output SNR (SNR_{out}/SNR_0) as a function of the coherence coefficient p_0 is shown in Fig.1 for a ten-element line array for various values of the input interference power for $\theta_i = 30^o$ (curve (a) corresponds to $\nu_i = 10dB$, curve (b) - $\nu_i = 16dB$, curve (c) - $\nu_i = 20dB$). The solid lines are given dependence, which computes from (22), the dashed lines are given exact solutions, which are obtained by computer inversion of the correlation matrix **R**. From fig.1 one can see, that for strong interference (curve (c)) we have a good agreement between exact and approximate solutions for almost all values of the coherence coefficient p_0 .

4. OTHER MODELS OF COHERENCE

In this section we analyze how different models of interference can affect the linear array performance.

An exponential-power-law model [15] will be assumed for the signal coherence, whereby the correlation between the $i-th$ and $j-th$ element of an equally spaced linear array is written as

$$\rho(x) = \exp\{-(| d-k | /L)^r\}, \quad (26)$$

where the exponent-power r is a parameter that typically varies between 1 (exponential) and 2 (Gaussian) [16,17]. Thus values of $r < 2$ will be of most interest for the present application. The dependence of normalized output SNR as a function of a characteristic correlation length L expressed in element spacing units is shown in Fig.2 for ten-element linear array for various models of coherence function for $\theta_i = 30^o$, $\nu_i = 20dB$ (curve(a) corresponds to $r = 2$, curve (b) - $r = 1.5$, curve (c) - $r = 1$). From fig.2 one can see that increase of parameter r leads to increase of the output SNR especially for a characteristic correlation length L in order of size of the antenna aperture ($L/d \approx N$). For very small L the difference in the output SNR for various models is insignificant. Note, that the differences in the output SNR for various models and the characteristic correlation length L and aperture size N are small.

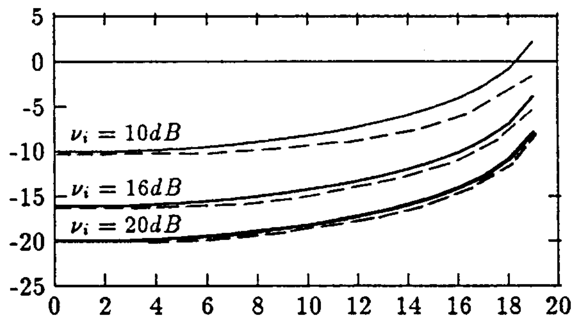

Figure 1: Dependence of (SNR_{out}/SNR_0) (dB), as a function of coefficient $p = 20 * p_0$, $N = 10$, $\theta_i = 30^o$.

5. CONCLUDING REMARKS

The use of the exponential model makes it possible to evaluate the number of adaptive processors maximizing the output SNR. Analytical expressions for the optimal weight vector, directional pattern and output SNR are derived in terms of the coherence coefficient of interference, the interference power, and the parameters of the adaptive processor. The different models of interference coherence are studed. It is shown that the detection performance of adaptive beamformer is substantially degraded for large arrays and typical coherence lengths. The different models of interference coherence are studed. Several numerical results are given.

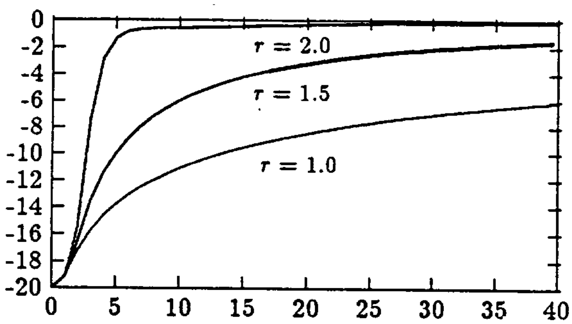

Figure 2: Dependence of (SNR_{out}/SNR_0)(dB), as a function of a characteristic correlation lenght L.

6. ACKNOWLEDGMENT

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