SIMULATED ANNEALING APPROACH FOR THE DESIGN OF UNEQUALLY SPACED ARRAYS

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ABSTRACT

In this paper, a synthesis method aimed at designing an array antenna is proposed. Simulated Annealing (SA), which is a probabilistic methodology to solve combinatorial optimization problems, has been utilized to optimize the position and the weighting coefficients of array elements in order to improve the antenna performances. Sensor position and related weighting coefficients are considered as parameters to be tuned in order to constrain the directivity function (i.e., the beam power pattern) of an antenna to satisfy specific requirements. Conventional beamforming is utilized to compute the beam power pattern having desired properties, such as narrow width of the main lobe, side lobe amplitudes under a certain threshold, etc., taking also into account the need of reducing a small number of sensor and of little spatial aperture. Several results are presented showing a notable improvement of antenna performances utilizing the SA approach with respect to those considered in literature.

I. INTRODUCTION

The number of array elements strongly affects the cost of the array, the complexity of the control and processing devices, and the speed of the process. Therefore, it is very useful to succeed in decreasing the number of elements, while keeping the same spatial aperture of the array or the desired beam pattern shape. To reduce the number of elements and to prevent grating lobes, one may increase the inter-element spacing, breaking, at the same time, the periodicity of the elements positions. This operation leads to unequally spaced arrays (also called aperiodic arrays), where the average space between the elements, d_{av}, is some times as large as $\lambda/2$ [1]. The distribution and the amplitude of side lobes depend on the chosen positions of the elements along the spatial aperture and on the weight coefficients assigned to such elements. As a consequence, it is important to select both the best position and the best weight coefficient (shading) for each element of an array.

Some papers [2]-[5] address the problem of reducing the level of side lobes in unequally spaced arrays for which d_{av} is some times as large as $\lambda/2$. To this end, the number of elements and the spatial aperture of an array have been fixed *a priori*. Such works face only the case of arrays symmetrical with respect to their centers, and, in particular, the results obtained with such techniques refer to an array made up of 25 elements and with an overall aperture of 50λ ($d_{av} = 2.08\lambda$). Other papers [8], [9] address the problem of reducing the number of elements and the spatial aperture of the arrays maintaining the desired beam pattern shape. Also in this case, the papers face only the case of symmetrical arrays. A further constraint [5], [9] is imposed by the necessity for limiting to low values the Current Taper Ratio (*CTR*) which is the ratio between the maximum and the minimum weight coefficients. This constraint makes it possible to limit the consequences of possible unforeseen events degrading the performance of the elements with largest weight coefficients.

Simulated annealing (SA) is a probabilistic method of optimization. Initially, it aimed to simulate the behaviour of the molecules of a pure substance during the slow cooling that results in the formation of a perfect crystal (minimum-energy state) [6], [7]. The use of this technique to solve other types of problems is based on the analogy between the state of each molecule and the state of each variable that affects an energy function to be minimized. The principle of the algorithm is simple: at each iteration, a small random perturbation is induced in the current state configuration. If the new configuration causes the value of the energy function to decrease, then it is accepted. Instead, if the new configuration causes the value of the energy function to increase, it is accepted with a probability dependent on the system temperature, in accordance with the Boltzmann distribution. The higher the temperature, the higher the probability that the state configuration causing the energy function to increase may be accepted. As iterations go on, the temperature T is gradually lowered, until the configuration freezes in a certain final state. The solution, i.e. the best configuration of sensor positions and weights, is obtained when the energy function has reached the minimum value or when it does not vary more than a certain threshold along successive iterations. The SA approach allows one to overcome the difficulties due to a large number of local minima. The paper is organized as follows. Section 2 describes the SA approach, detailing the

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energy functions used in the experiments. In Section 3, the experimental trials are presented and the related results are compared with the examples considered in literature. Conclusions are drawn in Section 4.

2. THE SIMULATED ANNEALING APPROACH

If the array is linear and made up of M punctiform and omnidirectional elements placed along the x axis, then the beam pattern, $p(u)$, can be expressed as (see Fig. 1):

$$p(u) = \left| \sum_{i=0}^{M-1} w_i \cdot e^{j\frac{2\pi}{\lambda} x_i \cdot u} \right| \quad (1)$$

where λ is the wavelength, w_i is the weight coefficient of the i-th element and $u = \sin\theta - \sin\theta_0$, θ and θ_0 being the angle of incidence of the wave and the steering angle, respectively, measured with respect to the y axis. When the beam pattern is plotted as a function of u, the main lobe is always in $u = 0$ and its width does not depend on θ_0. The expression in decibels for the beam pattern, normalized to 0 dB, is $20\cdot\log[p(u)/Q]$ where Q is the sum of all w_i's. The beam pattern is even with respect to u, i.e., $p(u) = p(-u)$. Therefore, it is sufficient to study the behaviour of the beam pattern in the range $0 \le u \le 2$. Moreover, when the distance of each element from the origin of the coordinates is a multiple of $\lambda/2$, then $p(1-\Delta u) = p(1+\Delta u)$, that is, the beam pattern is symmetrical with respect to $u = 1$. In this case, it is sufficient to study the beam pattern in the range $0 \le u \le 1$.

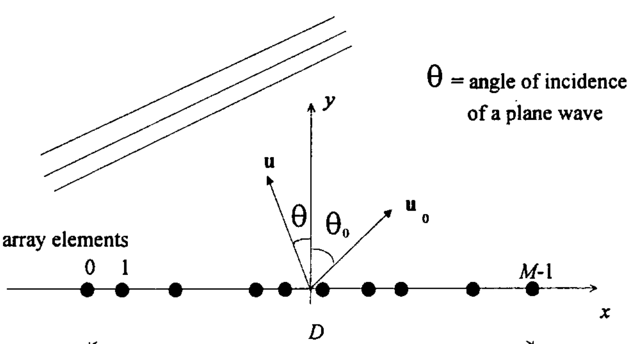


Fig. 1. Geometry and notations used for the linear array.

The cited methods [2]-[5], [8], [9], to obtain unequally spaced linear arrays with low density of elements, considered the case of symmetrical arrays in order to use a simplified expression for the beam pattern. Moreover, to maintain the symmetry, one must reduce the number of elements that can be positioned randomly. If one uses SA, the simplification involved in the symmetry is not necessary; therefore, the number of elements that can be positioned randomly is greater, and this notably increases the degrees of freedom. In addition, SA allows one to optimize positions and weight coefficients at the same time and in parallel.

The use of SA requires the definition of a suitable energy function, according to the purpose of the array design. When the purpose is to minimize the side lobe's peaks with spatial aperture and number of elements *a priori* fixed, the energy function $f(\mathbf{X}, \mathbf{W})$ must embed information about the peak value of the side lobes of the related beam pattern, on the basis of the vector of the element positions $\mathbf{X}=[x_1,..,x_M]$ and the vector of the weight coefficients $\mathbf{W}=[w_1,..,w_M]$. To this end, one can choose:

$$f(\mathbf{X}, \mathbf{W}) = \max_{u_{start} \le u \le u_{end}} \left\{ \left(p(u)/Q\right)^2 \right\} \quad (2)$$

where the beam pattern $p(u)$ is derived from the positions $\mathbf{X}$ and the weights $\mathbf{W}$; u_{end} may be equal to 1 or 2, depending on whether or not the positions are multiples of $\lambda/2$; u_{start} allows one to exclude the main lobe (which occurs at $u = 0$) from the computation of the peak of the side lobes.

Instead, when the purpose is to minimize the number of elements and the spatial aperture in order to obtain a beam pattern very close to a desired normalized beam pattern $p_d(u)$, the energy function $f(\mathbf{X}, \mathbf{W})$ must be able to provide a measure of the difference between the desired beam pattern and the current one obtained by using the actual positions $\mathbf{X}$ and the actual weights $\mathbf{W}$. To this end, one can choose:

$$f(\mathbf{X}, \mathbf{W}) = \int_{u_{start}}^{u_{end}} \left[\frac{p(u)}{Q} - p_d(u) \right]^2 du \quad (3)$$

The energy functions so formulated should be minimized in order to obtain the array configurations respecting the imposed conditions.

3. RESULTS AND COMPARISONS

In the papers dealing with an unequally spaced array made up of 25 elements arranged over a spatial aperture of 50λ [2]-[5], the height of side lobes is evaluated after fixing the initial point of the measurement, i.e., $u_{start} = 0.04$.

To assess the potentialities of SA procedure, two types of experiments have been performed: one aimed to optimize only the positions (using unit weight coefficients, $w_i = 1$), and the other one aimed to optimize both the positions and weights. In the former case, the obtained results are satisfactory, as the side lobes were limited to -12.07 dB. The improvement obtained by simulated annealing is twofold: the side lobes are more limited (1.9 dB lower) and the main lobe is narrower as compared with

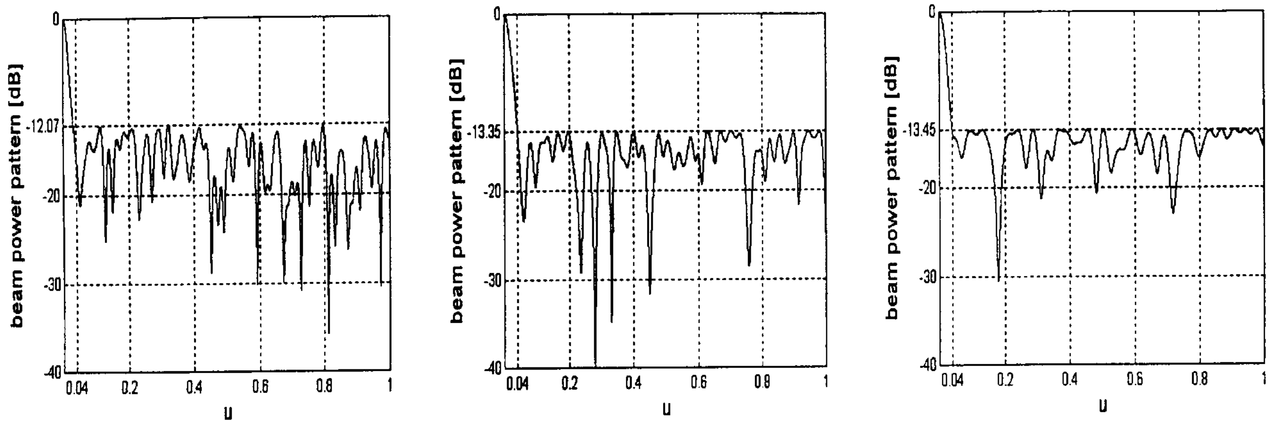


Fig. 2. The beam power patterns obtained when simulated annealing optimizes only positions (left panel) or positions and weights (center panel CTR = 3.2, right panel CTR = 9.8).

the best results present in the literature [4]. In the latter case, if SA optimizes both positions and weights, the side lobes were limited to -13.45 dB with CTR = 9.8 or to -13.35 dB with CTR = 3.2.

The best results presented in literature [5] show -12.20 dB of side lobes attenuation with CTR = 5.8. As a consequence, also in this case, the obtained results are very satisfactory (see Fig. 2). In both cases, the energy function in (2) was used.

A further experiment is aimed at minimizing the number of elements and the spatial aperture maintaining the desired beam pattern shape. In [9], a maximally sparse array was designed having certain characteristics in terms of maximum main lobe width (5^o degrees at -13.4 dB point) and side lobe peaks differently limited in function of the angle of steering θ (or, equivalently, of u).

$u = \sin\theta - \sin\theta_0$	maximum peak allowed [dB]
0.0	0
$0.042 \le u \le 0.31$	≤ -13.4
$0.31 < u \le 0.45$	≤ -26.9
$0.45 < u \le 0.80$	≤ -13.4
$0.8 < u \le 1.0$	≤ 0

Table A. Specifications of the beam pattern maximum values in function of the steering angle u.

The desired beam pattern was obtained by using the *simplex* method with two antenna configurations: with a non-equispaced and symmetric antenna formed by 26 sensors, having a whole aperture of 26 λ, or with an array λ/2-spaced formed by 32 sensors, having a whole aperture of 17 λ, and using the Dolph-Chebyshev weighting. SA method reaches the same beam pattern shape with an array of only 20 sensors on a 19.5 λ of spatial aperture, by minimizing the energy function in (3). Figure 3 shows the beam pattern with the desired properties and Fig. 4 shows the associated sensor position (a) and weighting (b) configurations. In this figure, the abscissas are referred to the element position expressed in wavelengths λ. Table A reports the desired characteristics of the beam pattern in function of the steering direction u.

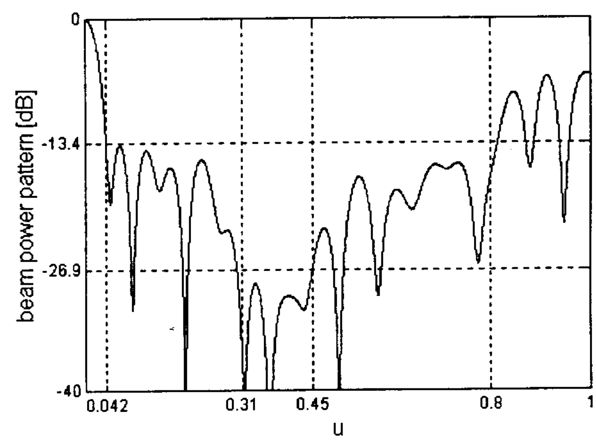
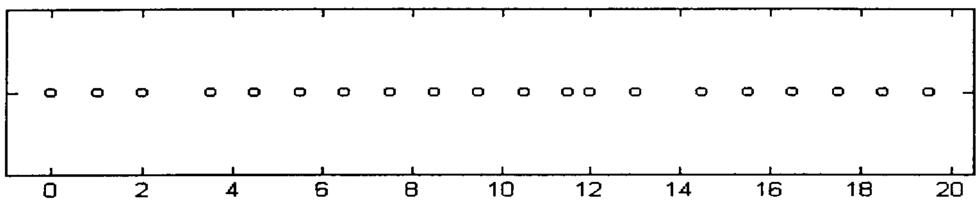


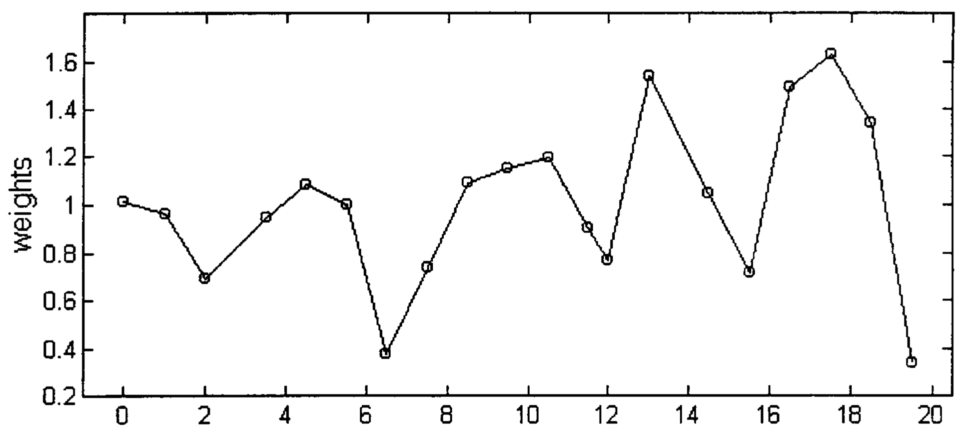
Fig. 3. Beam power pattern obtained with SA technique with an antenna of 20 sensors on 19.5λ of aperture.

4. CONCLUSIONS

In this paper, the Simulated Annealing approach to the synthesis of array antennas is proposed. This technique is resulted suitable to the synthesis of non-equispaced, non-symmetric arrays unlike several methods present in literature. Moreover, it gives rise to better results in many cases. This technique is based on the definition of a cost function (energy) to be minimized. This function is built by modelling the specific properties that a beam power pattern of an antenna must have in an analytic way in function of the sensor position and of the related weight coefficients (shading). The limited number of sensors spaced in a little spatial aperture is also taken into account to reduce the physical dimensions of an array. The results shown prove the usefulness and the efficiency of this technique that improves the best results obtained in literature in the different conditions considered.



(a)



(b)

Fig. 4. Sensor position of the 20 sensors (maximum spatial aperture 19.5λ) (a) and associated weighting coefficients (CTR = 4.8) of the antenna (b) whose associated beam power pattern is shown in Fig. 3.

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