# STEERED RESPONSE CONTROL OF THE GENERALIZED SIDELOBE CANCELLER

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## ABSTRACT

This paper presents a new set of derivative constraints for the generalized sidelobe canceller (GSC) that can be used to reduce sensitivity to steering error. These constraints are designed to flatten the spatial null of the GSC blocking matrix so that for a small steering error, the desired signal is still blocked and the GSC does not experience signal cancellation. With this approach, the steered response of the GSC can be forced to locally approximate any realizable fixed-weight beampattern. A related set of constraints can be used with the eigenvector constraint calibrated GSC to control the steered response for use in the presence of array errors.

## 1. INTRODUCTION

The generalized sidelobe canceller (GSC) is a form of linearly constrained minimum variance (LCMV) beamforming [1]. LCMV beamformers can provide higher output signal-to-noise ratio (SNR) than fixed-weight beamformers when noise statistics are either time-varying or unknown. Unfortunately, LCMV beamformers (including the GSC) tend to be sensitive to signal model errors, especially when the input SNR is high [2]. Sensitivity to steering error results in a very narrow mainlobe steered response for the GSC compared to fixed-weight beamformers.

One way to improve robustness to direction error is to use derivative constraints to flatten the power response of the GSC in the look direction. Buckley and Griffiths [3] extend Er and Cantoni's work in [4] to an adaptive framework, and show how to implement the constraints using the GSC form of the LCMV beamformer. They also show that these constraints result in a beamformer with the undesirable property that the beamformer performance depends on the array spatial reference point (or phase reference). In [5], Tseng notes that the phase reference dependency is caused by an unnecessary constraint on the beamformer phase response. Tseng eliminates the phase reference problem by constraining only the power response and not the phase response. Thng, Cantoni, and Leung present similar constraints in [6]. In [7], Tseng and Griffiths extend the work in [5] for derivative constraints above 2nd order. In order to implement these power response derivative constraints for nth order derivative constraints ($n \geq 2$), nonlinear minimization is required to determine the constraint values of the linear constraint equations.

This paper considers the magnitude response of the spatial blocking filter in the GSC rather than the overall magnitude response of the GSC. Reduced sensitivity in the look direction for the GSC can be obtained by flattening the null of the spatial blocking filter. Thus, for a small steering error, the desired signal is still blocked from entering the noise cancelling path, and the GSC output is approximately the same as for no steering error.

*These constraints result in a GSC steered response that can approximate any realizable fixed-weight beamformer response near the look direction while still suppressing sidelobes.* Thus, the array designer can first select a fixed-weight beamformer that has the desired robustness to steering error, then apply enough spatial blocking filter derivative constraints so that the GSC steered response adequately approximates the fixed-weight beampattern. The eigenvector constraint (EVC) calibrated GSC [8] can be modified to incorporate constraints that are similar to spatial blocking filter derivative constraints, and which allow steered response control in the presence of array errors.

In a related approach, Claesson and Nordholm [9] suggest widening the null of the spatial blocking filter by using an equiripple approximation of a flat stopband rather than by using derivative constraints. We note that some of our theoretical results should hold (in a qualitative sense) for this particular choice of blocking matrix.

## 2. LOOK DIRECTION DERIVATIVE CONSTRAINTS FOR THE GSC

This section derives look direction derivative constraints designed specifically for the GSC. The output at time $k$ of an $M$-channel, $L$-tap GSC with $K$ constraints is given by

$$y(k) = (\mathbf{w}_f - \mathbf{B}\mathbf{w}_a(k))^{\mathrm{T}}\mathbf{x}_{sn}(k) \qquad (1)$$

where the $ML \times 1$ vector $\mathbf{w}_f$ forms a fixed-weight beamformer, $\mathbf{B}$ is an $ML \times ML - K$ matrix that blocks the desired signal, and $\mathbf{w}_a(k)$ is an $ML - K \times 1$ noise cancelling filter that adapts to minimize total output power. The input to the GSC is a stacked snapshot vector, consisting of the $M \times 1$ array output vector at $L$ consecutive times: $\mathbf{x}_{sn}(k) = [\mathbf{x}^{\mathrm{T}}(k), \ldots, \mathbf{x}^{\mathrm{T}}(k-L+1)]^{\mathrm{T}}$

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In order to minimize signal cancellation caused by steering error, the objective is to flatten the spatial null for each column of the blocking matrix $\mathbf{B}$. Let $\mathbf{b}$ represent a single column of the blocking matrix $\mathbf{B}$. Then the response $r(\omega,\theta)$ of the spatial filter $\mathbf{b}$ to a monochromatic planewave signal $\mathbf{a}(\omega,\theta)$ is given by $r(\omega,\theta) = \mathbf{b}^{\mathrm{H}}\mathbf{a}(\omega,\theta)$. The power response of $\mathbf{b}$ is then given by

$$H(\omega,\theta) = |r(\omega,\theta)|^2 = \mathbf{b}^{\mathrm{H}}\mathbf{a}\mathbf{a}^{\mathrm{H}}\mathbf{b} \tag{2}$$

The following theorem relates the amount of flattening of the spatial null to a set of orthogonality constraints:

**Theorem 1.** *Let $N \geq 0$ be an integer. Then*

$$\left.\frac{\partial^n H(\omega,\theta)}{\partial\theta^n}\right|_{\theta=\theta_0} = 0, \quad n = 0,\ldots,2N+1$$
$$\textit{iff} \quad \mathbf{b} \perp \mathbf{d}_0(\omega,\theta_0),\ldots,\mathbf{d}_N(\omega,\theta_0)$$

*where $\mathbf{d}_n(\omega,\theta_0)$ represents the nth partial derivative with respect to direction of the direction vector $\mathbf{a}(\omega,\theta)$, evaluated in the direction $\theta_0$.*

**Proof:** Proof follows from recursive application of the product rule for derivatives of products. □

Next, we want to consider the steered response of the GSC after applying these derivative constraints. Assume that the array receives a broadband signal from the direction $\theta_0$ in the presence of white sensor noise. The following theorem relates the orthogonality constraints to the behavior of the GSC steered response for the worst case environment (consisting of desired signal only plus additive white sensor noise). In this theorem, the GSC is assumed to have an orthonormal column blocking matrix.

**Theorem 2.** *Let the array output power spectral density matrix be given by $\mathbf{P}_{xx}(\omega) = \sigma_n^2\mathbf{I} + \mathbf{a}(\omega,\theta)\mathbf{a}^{\mathrm{H}}(\omega,\theta)$. Then $G_{s_N}(\omega,\theta)$ converges to $F_s(\omega,\theta)$ as $N \to ML$, where $G_{s_N}(\omega,\theta)$ is the steered response of the GSC with blocking matrix $\mathbf{B}$ satisfying the following $N+1$ orthogonality constraints:*

$$\mathbf{B}^{\mathrm{H}}\mathbf{D}_N(\omega,\theta_0) = \mathbf{0}$$

*where $\mathbf{D}_N(\omega,\theta_0) = [\mathbf{d}_0(\omega,\theta_0),\ldots,\mathbf{d}_N(\omega,\theta_0)]$.*

Note that as more orthogonality conditions are added, the blocking matrix $\mathbf{B}$ loses columns (less degrees of freedom). Obviously, $\mathbf{B}$ will eventually lose all columns, leading to a beamformer that is equivalent to a conventional beamformer. Thus, the convergence is applicable only while degrees of freedom are available.

**Proof:** Let the correlation matrix at a given frequency $\omega$ be given by $\mathbf{R}_{xx}(\theta) = \sigma_n^2\mathbf{I} + \mathbf{a}(\theta)\mathbf{a}^{\mathrm{H}}(\theta)$. Then the GSC and fixed-weight beamformer steered responses are given by

$$\begin{aligned} G_s(\theta) &= (\mathbf{w}_f - \mathbf{B}\mathbf{w}_a(\theta))^{\mathrm{H}}\mathbf{R}_{xx}(\theta)(\mathbf{w}_f - \mathbf{B}\mathbf{w}_a(\theta)) \\ F_s(\theta) &= \mathbf{w}_f^{\mathrm{H}}\mathbf{R}_{xx}(\theta)\mathbf{w}_f \end{aligned} \tag{3}$$

Define $T_\theta$ as $T_\theta = \mathbf{a}^{\mathrm{H}}(\theta)\mathbf{B}\mathbf{w}_a(\theta)$. Then, using $\mathbf{B}^{\mathrm{H}}\mathbf{w}_f = \mathbf{0}$, one may obtain

$$\begin{aligned} |G_s(\theta) - F_s(\theta)| &\leq |T_\theta|^2 + 2\left|\mathbf{w}_f^{\mathrm{H}}\mathbf{a}(\theta)\right||T_\theta| \\ &+ \sigma_n^2\mathbf{w}_a^{\mathrm{H}}(\theta)\mathbf{B}^{\mathrm{H}}\mathbf{B}\mathbf{w}_a(\theta) \end{aligned} \tag{4}$$

Next, approximate $\mathbf{a}(\theta)$ with a Taylor series expansion about $\theta_0$ as follows:

$$\mathbf{a}(\theta) = \sum_{n=0}^{N}\mathbf{d}_n(\theta_0)\frac{(\theta-\theta_0)^n}{n!} + \Delta(\theta,N) \tag{5}$$

where $\Delta(\theta,N) \to \mathbf{0}$ either as $N \to \infty$ for a given $\theta$ or as $\theta \to \theta_0$ for a given $N$. Assuming that $\mathbf{B}$ satisfies the $N+1$ orthogonality conditions $\mathbf{B}^{\mathrm{H}}\mathbf{D}_N = \mathbf{0}$, $T_\theta$ is given by

$$\begin{aligned} T_\theta &= \left(\sum_{n=0}^{N}\mathbf{d}_n(\theta_0)\frac{(\theta-\theta_0)^n}{n!} + \Delta(\theta,N)\right)^{\mathrm{H}}\mathbf{B}\mathbf{w}_a(\theta) \\ &= \Delta^{\mathrm{H}}(\theta,N)\mathbf{B}\mathbf{w}_a(\theta) \end{aligned} \tag{6}$$

This implies that

$$\begin{aligned} |G_{s_N}(\theta) - F_s(\theta)| &\leq \left|\Delta^{\mathrm{H}}(\theta,N)\mathbf{B}\mathbf{w}_a(\theta)\right|^2 + \sigma_n^2\,||\mathbf{B}\mathbf{w}_a(\theta)||^2 \\ &+ 2\left|\mathbf{w}_f^{\mathrm{H}}\mathbf{a}(\theta)\Delta^{\mathrm{H}}(\theta,N)\mathbf{B}\mathbf{w}_a(\theta)\right| \end{aligned} \tag{7}$$

Next, assume that the adaptive weight vector is at the optimal value:

$$\begin{aligned} \mathbf{w}_a(\theta) &= \left(\mathbf{B}^{\mathrm{H}}\mathbf{R}_{xx}\mathbf{B}\right)^{-1}\mathbf{B}^{\mathrm{H}}\mathbf{R}_{xx}\mathbf{w}_f \\ &= (\mathbf{B}^{\mathrm{H}}(\sigma_n^2\mathbf{I} + \mathbf{a}(\theta)\mathbf{a}^{\mathrm{H}}(\theta))\mathbf{B})^{-1}\mathbf{B}^{\mathrm{H}} \\ &\quad \times(\sigma_n^2\mathbf{I} + \mathbf{a}(\theta)\mathbf{a}^{\mathrm{H}}(\theta))\mathbf{w}_f \\ &= \frac{1}{\sigma_n^2}\left(\mathbf{I} + \mathbf{B}^{\mathrm{H}}\Delta(\theta,N)\Delta^{\mathrm{H}}(\theta,N)\mathbf{B}\right)^{-1} \\ &\quad \times\left(\mathbf{B}^{\mathrm{H}}\Delta(\theta,N)\mathbf{a}^{\mathrm{H}}(\theta)\mathbf{w}_f\right) \end{aligned} \tag{8}$$

From (8), $\|\mathbf{w}_a(\theta)\| \to 0$ either as $N \to \infty$ for given $\theta$ or as $\theta \to \theta_0$ for a given $N$. Combining this result with (7) proves the desired result. Since $\Delta(\theta,N)$ is in the first two terms of (7), part of the convergence is a consequence of the orthogonality conditions directly, and part of the convergence is due to the orthogonality conditions forcing $\|\mathbf{w}_a\| \to 0$. □

Theorem 2 implies that by using spatial blocking filter derivative constraints we can force the GSC steered response to approximate the fixed-weight beamformer steered response in the vicinity of the look direction. Although the convergence eventually ends (when the blocking matrix runs out of columns), simulations show that even for fairly small arrays (11 sensors), the mainlobe steered response of the fixed-weight beamformer can be closely approximated while still suppressing sidelobes almost as effectively as the GSC without derivative constraints.

## 3. COMBINING DERIVATIVE CONSTRAINTS WITH CALIBRATION

The spatial blocking filter derivative constraints presented here can be used directly with channel equalization (CE) calibration [10] since the array errors are corrected by filtering prior to the data reaching the GSC. An alternative method of calibration for the GSC is the EVC calibrated GSC [8]. A method of applying spatial blocking filter derivative constraints compatible with the EVC calibrated GSC is to include calibration data from directions near broadside along with the broadside calibration signal. Thus, the blocking matrix is forced to pass directions near broadside, effectively flattening the spatial null of $\mathbf{B}$. In practice, these point constraints are similar to using derivative constraints to flatten the spatial null. The basic procedure is a simple extension of the EVC calibration procedure presented in [8], and is outlined below.

Record $N >> ML$ stacked snapshots of a wideband calibration signal from the look direction $\theta_0$, and let the $N \times$

$ML$ data matrix $\mathbf{X}_0$ be given by $\mathbf{X}_0 = [\mathbf{x}_{sn}(1), \ldots, \mathbf{x}_{sn}(N)]^T$. For a perfect planewave without array errors, $\text{rank}\{\mathbf{X}_0\} = L$. Repeat this procedure for directions $\theta_1, \ldots, \theta_{K_1-1}$ in the vicinity of the look direction, with the array steered to $\theta_0$, and construct $K_1 - 1$ data matrices $\mathbf{X}_1, \ldots \mathbf{X}_{K_1-1}$. Let the $NK_1 \times ML$ matrix $\mathbf{X}$ be given by $\mathbf{X} = [\mathbf{X}_0^T, \mathbf{X}_1^T, \ldots, \mathbf{X}_{K_1-1}^T]^T$. Due to a combination of direction errors and array errors, $\mathbf{X}$ will be full rank. Next, solve for the $ML \times ML - K$ blocking matrix $\mathbf{B}_{cal}$ that maximally blocks the calibration data. In other words, $\mathbf{B}_{cal}$ should have a maximally flat spatial/spectral null in order to block perturbed direction vectors in the vicinity of broadside over a frequency set that effectively spans the desired signal subspace.

Let the singular value decomposition (SVD) of $\mathbf{X}$ be given by $\mathbf{X} = \mathbf{U\Sigma V}^T$, where $\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_{ML}$. Using the properties of the SVD [11], the rank $ML - K$ matrix $\mathbf{B}_{cal}$ that minimizes $\| \mathbf{XB}_{cal} \|_F^2$ and the rank $K$ passing matrix $\mathbf{P}_{cal}$ that maximizes $\| \mathbf{XP}_{cal} \|_F^2$ are given by

$$\begin{aligned} \mathbf{P}_{cal} &= [\mathbf{v}_1, \ldots, \mathbf{v}_K] \\ \mathbf{B}_{cal} &= [\mathbf{v}_{K+1}, \ldots, \mathbf{v}_{ML}] \end{aligned}$$

The fixed-weight beamformer that best approximates $\mathbf{w}_f$ is given by $\mathbf{w}_{f,cal} = \mathbf{P}_{cal}\mathbf{P}_{cal}^T\mathbf{w}_f$.

While Theorem 1 is not used to achieve zero derivatives of the blocking filter spatial null, the basic result of Theorem 2 still holds. In other words, by using calibration data near broadside in the EVC calibrated GSC, the GSC steered response should still approximate the fixed-weight beamformer steered response in the vicinity of the look direction. Simulations demonstrate that this is the case.

## 4. SIMULATIONS

Figures 1 - 3 show how blocking filter derivative constraints can be used to control the steered response of the GSC. The array is linear with 11 sensors and 5 taps (sensor spacing is equal to half-wavelength spacing for a 4 kHz signal). The desired source is a 3 kHz signal located at array broadside, with additive white Gaussian sensor noise (input SNR 20 dB). The array is steered from left endfire to right endfire, and the resulting output power versus steer direction $\theta$ is plotted for the GSC with LMS adaptation (normalized step size of .1). The three fixed-weight beamformers considered are uniform shading, equiripple passband inside $90° \pm 21$ and stopband outside $90° \pm 26$, and fixed-weight beamformer resulting from power response derivative constraints of [5]. Note that as higher order constraints are applied, the steered response begins to approximate the fixed-weight beamformer response.

Figure 4 illustrates combining spatial blocking filter derivative constraints with CE calibration and with EVC calibration for a 16-channel, 10-tap array with simulated errors in Table 1. The desired signal is a synthetic /i/ originating from broadside, with formant frequencies at 297, 2313, and 3016 Hz. The fixed-weight beamformer (dotted line) is equiripple with passband inside $90° \pm 18$ and stopband outside $90° \pm 23$. The channel equalizer has 10 taps, followed by orthogonality constraints $\mathbf{B}^H\mathbf{D}_4 = 0$. Derivative constraints are combined with EVC calibration by using calibration data from 87 to 93 degrees with 40 constraints. The calibration signal is an /i/ sound recorded by a male speaker. Although significant signal cancellation occurs for the CE calibrated GSC, the derivative constraints still broaden the steered response.

## 5. SUMMARY

This paper has presented a new approach to derivative constraints for the GSC that leads to reduced sensitivity to direction error. These new derivative constraints can be used to control the GSC steered response in the vicinity of the look direction, while still suppressing the sidelobes. The amount of robustness to steering error can be controlled via the shape of the fixed-weight beamformer response, as well as by the order of spatial blocking filter derivative constraints. These derivative constraints can be combined indirectly with the EVC calibrated GSC, thus providing effective steered response control even in the presence of array errors.

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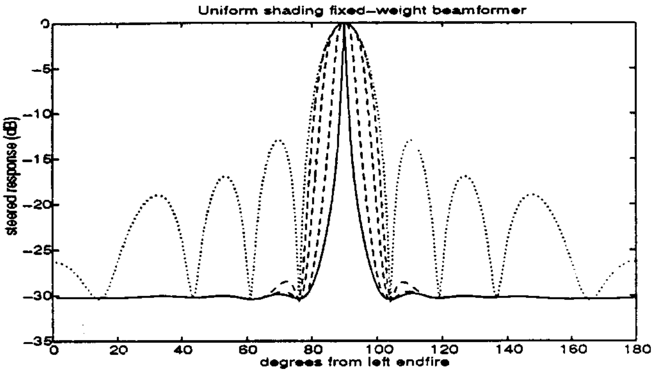


Figure 1: GSC steered response to narrowband signal with derivative constraints, uniform shading fixed-weight beamformer. Signal originates from array broadside (90°). The dashed lines show the GSC steered response for increasing orders of orthogonality derivative constraints (innermost dashed line is $\mathbf{B}^H\mathbf{D}_1 = \mathbf{0}$, outermost dashed line is $\mathbf{B}^H\mathbf{D}_3 = \mathbf{0}$). Response of GSC without derivative constraints (solid line) shown for comparison.

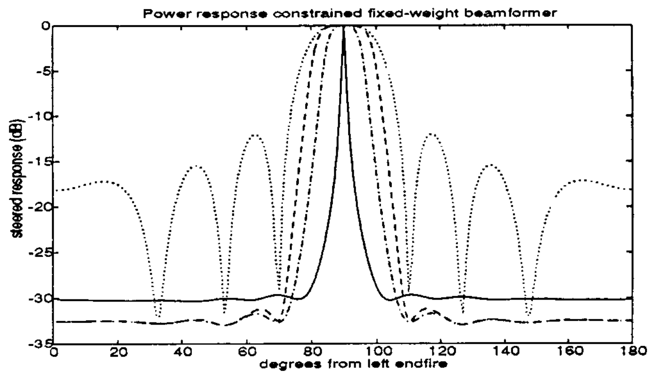


Figure 3: GSC steered response to narrowband signal with derivative constraints, fixed-weight beamformer $\mathbf{w}_f$ based on power response derivative constraints. Same simulation as above, except that $\mathbf{w}_f$ (dotted line) is derived from the power response 2nd order derivative constraints in [5]. Dash dot line shows the GSC with 2nd order derivative constraints (both $\mathbf{w}_f$ and blocking matrix based on [5]). Dashed line shows the GSC response with derivative constraints $\mathbf{B}^H\mathbf{D}_3 = \mathbf{0}$ ($\mathbf{w}_f$ same as for dash-dot line).

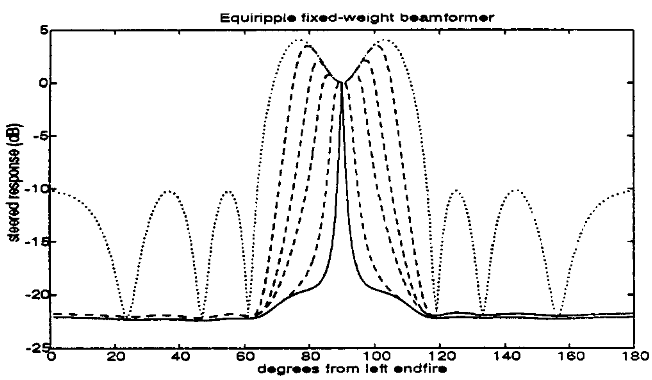


Figure 2: GSC steered response to narrowband signal with derivative constraints, equiripple shading. Same simulation as above, except that the fixed-weight beamformer has equiripple passband and stopband. The innermost dashed line is $\mathbf{B}^H\mathbf{D}_1 = \mathbf{0}$ and the outermost dashed line is $\mathbf{B}^H\mathbf{D}_4 = \mathbf{0}$.

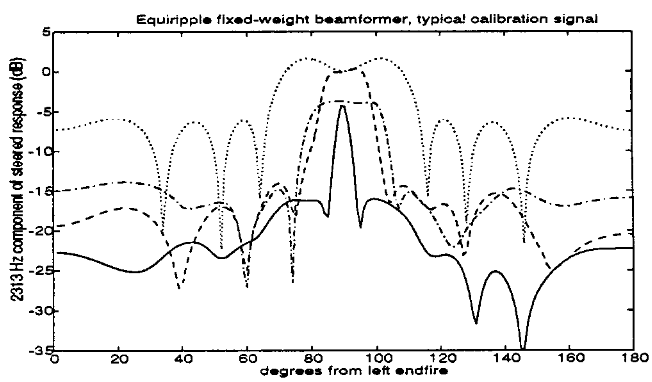


Figure 4: Steered response (2313 hz component) of broadband signal for 16-channel, 10-tap array with simulated errors, equiripple fixed-weight beamformer. EVC calibrated GSC (solid line) compared to CE calibrated GSC with derivative constraints $\mathbf{B}^H\mathbf{D}_4 = \mathbf{0}$ (dash dot line) and EVC calibrated GSC with spatial blocking filter null from 87 to 93 degrees (dashed line).

| Type of error | Channel number | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| Channel gain | 1.12 | 1.06 | 1.01 | 1.04 | .93 | 1.17 | 1.01 | 1.18 | 1.03 | 1.09 | .86 | .93 | 1.12 | .94 | 1.06 | .96 |
| Channel delay in samples | -.04 | -.40 | -.38 | .30 | -.01 | -.24 | -.23 | .26 | -.02 | .15 | .12 | .23 | .12 | -.40 | .11 | .34 |

Table 1: Array errors used for simulation.