SIGNAL IDENTIFICATION BASED ON ORTHOGONAL TRANSFORM

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ABSTRACT

A new approach to the identification of a constant amplitude signal with frequency/phase modulation is investigated. We model the incoming signal phase as a linear combination of a set of orthogonal vectors and use the significant coefficients as features for identification. Because of the data compression ability of orthogonal transform, a few coefficients are sufficient for signal representation, thereby reducing the processing time and system complexity. The choices of transforms and feature size are discussed. The performance of the new identifier is studied through simulations.

I. INTRODUCTION

This paper addresses the problem of identifying a constant amplitude signal. Radio frequency signals contain intentional frequency/phase modulation such as linear FM, quadratic FM [1]. They may also have unintentional frequency/phase modulation which solely depends on the transmitter characteristics. It is possible to exploit these modulations to identify a signal.

Correlation of the incoming instantaneous phase (IP) or instantaneous frequency (IF) [2] sequence with those in the references is a straightforward way for identification. It is, however, computationally intensive and time consuming when the data size is large. An alternative is to classify from some distinct features. [3] models the IP sequence by a polynomial and the features are the polynomial coefficients. Computation of the coefficients requires the inverse of an ill-conditioned matrix, which may present numerical problems [3]. Polynomial phase transform [4] is a suboptimal approach to find the coefficients by computing them sequentially starting from the highest order one.

A polynomial model is suitable for a slow-varying phase sequence. It needs a very high order when the variation is rapid. Selection of the model order is not an easy task without any a priori knowledge. We instead model the phase sequence as a linear combination of orthogonal vectors and use the orthogonal transform coefficients as features for identification. The computation is simple with fast algorithm and has no numerical problems. Due to the data compression ability of orthogonal transform, classification only needs the first few largest coefficients, thereby reducing the complexity and permitting rapid identification.

The paper is organized as follows. Section II presents the new orthogonal vector phase modelling method, which also includes the selection of transform and feature size. The orthogonal transform identifier is given in Section III. Section IV compares the performance of the proposed identifier with different transforms and the optimum identifier.

II. MODELLING OF THE PHASE

The received signal in complex form is

$$z(k)=B\,e^{j\phi(k)}+\varepsilon(k) \tag{1}$$

where B is the amplitude, $\phi(k)$ is the IP and $\varepsilon(k)$ is a complex white Gaussian noise of power σ_ε^2. At high signal to noise ratio (SNR), the phase extracted from a quadrature demodulator can be modelled as [5]

$$\psi(k)=\phi(k)+\eta(k) \quad , \quad k=1,2,\ldots,N \tag{2}$$

where $\eta(k)$ is a white Gaussian noise of power $\sigma_n^2=\sigma_\varepsilon^2/2B^2$. The new method models the phase as a linear combination of orthogonal vectors so that (2) becomes

$$\psi=\boldsymbol{Q}_N\,\boldsymbol{c}_N+\eta \tag{3}$$

where $\psi=[\,\psi(1),\psi(2),\ldots,\psi(N)\,]^T$, $\eta=[\,\eta(1),\eta(2),\ldots,\eta(N)\,]^T$, $\boldsymbol{Q}_N$ is a transform with orthogonal column vectors $\boldsymbol{q}_i$ and $\boldsymbol{c}_N$ is the transform coefficient vector. Such representation is always possible since orthogonal transformation is a one-to-one mapping. Premultiplying both sides by $\boldsymbol{Q}_N^T$ forms

$$\hat{\boldsymbol{c}}_N=\boldsymbol{Q}_N^T\,\psi=\boldsymbol{c}_N+\boldsymbol{n}=\boldsymbol{c}_N+\boldsymbol{Q}_N^T\,\eta \tag{4}$$

which can be computed by fast algorithms. $\boldsymbol{n}$ is the coefficient noise which is Gaussian with power σ_n^2. We assume the elements of $\hat{\boldsymbol{c}}_N$ are in descending order (absolute value) for simplicity. Due to data compression ability of an orthogonal transform, a subset of the coefficients in $\hat{\boldsymbol{c}}_N$ is enough to represent the phase. Clearly, a better transform requires a smaller set of coefficients. In the extreme case, we have only one coefficient if the true phase is one of the orthogonal vectors in Q_N.

Figure 1 compares the modelling ability of four commonly used transforms [6], Haar Transform (HT), Walsh-Hadamard Transform (WHT), Discrete Cosine Transform (DCT), Slant Transform (ST) and Discrete Fourier Transform (FFT), for a linear FM Chirp signal. The modelling mean squared error (MSE) is defined as

$$\mathrm{MSE} = \frac{1}{N} \| \phi - Q_M \hat{c}_M \|^2 \tag{5}$$

where $\| \cdot \|$ represents the second norm, $\phi = [\phi(1), \phi(2), \ldots, \phi(N)]^T$ is a vector of the original phase, Q_M is a matrix having the first M vectors of Q_N and $\hat{c}_M$ is a vector of the first M elements of $\hat{c}_N$. The ST is the best in this particular case and only four coefficients are sufficient for a representation with 0.1% error. DCT needs 8, WHT requires 13, HT takes 20 coefficients and FFT has the highest number of coefficients. Figure 2 is another example for the barker-13 Phase Code [7]. More coefficients are needed with 0.1% error in this case and the performance of the first four transforms are close.

The best transform for modelling depends on the phase types we are dealing with. Although DCT is the best in image processing under a first order Markov model, it is not always the most suitable candidate for signal identification. It will be the best for narrow-band phase modulation such as sinusoidal. The WHT has generally weaker data reduction capability, but requires less computations. Because of their discrete nature, WHT and HT appear to yield good representation for wide-band, step changing phases. ST, which bears similarity to both DCT and WHT in sequency but using slanted linear waveforms, is a compromise of the two and appears well-suited for applications involving a mix of wide-band and narrow-band phases.

One needs a criteria to select the number of retaining coefficients after a transform is chosen. A simple method is to keep the coefficients with magnitudes larger than a fixed fraction of the largest coefficient. Another may be setting the size to be a fraction of the total number of data points. The choice of the fractions may be crucial for the performance. Another difficulty is they do not take the SNR into account. We shall propose a method for the feature size.

In the presence of noise, the MSE in (5) becomes

$$\mathrm{MSE} = \frac{1}{N} E[\| Q_N c_N - Q_M c_M - Q_M n_M \|^2]$$

$$= \mathrm{MSE}_s + \mathrm{MSE}_n = \frac{1}{N} \sum_{i=M+1}^{N} c_i^2 + \frac{M}{N} \sigma_n^2 \tag{6}$$

The first term, MSE_s, is the signal modelling MSE and decreases with M. The second, MSE_n, is the noise MSE and increases with M. There is an optimum value for which the total MSE is minimum. A typical plot of the MSE for WHT modelling $\psi(k) = 3.2\cos(0.89k) + \eta(k)$ is in Figure 3, where the SNR is 4dB. The optimum M value in this case is around 25. We shall use this optimum M as the feature size.

Given a modelling phase, SNR and an orthogonal transform, numerical procedure can find the optimum M value. A simpler technique is to use a mathematical model for MSE_s and minimizes (6). Based on our study, MSE_s is well approximated by an exponential model,

$$\mathrm{MSE}_s \approx A\, e^{-\alpha M} \tag{7}$$

where A and α are determined by fitting (7) to MSE_s. Substituting (7) into (6) and performing minimization yields

$$M_{opt} = \left\lfloor \frac{1}{\alpha} \ln\left(\frac{\alpha A N}{\sigma_n^2} \right) \right\rfloor \tag{8}$$

where $\lfloor (\cdot) \rfloor$ denotes the integral part of $(\cdot)$. The accuracy improves by fitting (7) with a smaller data set around the computed optimum M, and then applies (8) again [8]. Note that since A corresponds to phase power, the optimum M is directly proportional to the phase SNR. At high phase SNR, even the smaller coefficients contain useful information and should therefore be retained for identification. On the other hand, the insignificant coefficients are highly corrupted by noise and have little effect on identification when the phase SNR is low. Hence, a smaller set of features can be used instead.

III. THE ORTHOGONAL TRANSFORM SIGNAL IDENTIFIER

Several assumptions about the identifier are in order: (A1) the noise in the signal environment is white, (A2) the demodulation oscillator exactly matches the signal carrier, so that there is no carrier frequency offset in the unwrapped phase, (A3) the input signal is accurately time-aligned and there is no sampling time mismatch between the input signal and those in the library, and (A4) noiseless signals are available for library generation. This is not an unrealistic assumption as several records of these signals could be appropriately processed to produce a high SNR specimen.

Figure 4 is a block diagram of the orthogonal signal identifier. The Pre-processor demodulates the unidentified signal by quadrature demodulation and produces the unwrapped phase. The identifier transforms the phase, and extracts a set of features from the transform domain representation. The extracted feature set is then compared with those of the L signals in the library for identification. The input is classified as the signal which has the smallest matching error with the input.

To generate the library, the L previously recorded signal phases are first tailored to a common size N. Each of them are transformed and then sorted. The DC component, which represents the phase offset, is discarded. Along with the coefficients, each entry contains a table of M_{opt} for a set of SNR's. Alternatively, several values of $\{A, \alpha\}$ for different ranges of SNR can be stored instead to reduce memory. M_{opt} is computed on-line from (8).

Let the index set of the first largest M_{opt} coefficients of entry l be I_l. Comparison of input to the lth entry involves reconstructing the input phase from $\hat{c}_i$, $i \in I_l$ and computing the squared Euclidean distance (D) between the reconstructed phase and the library entry. Let the transform coefficient of the lth entry be $c_{l,k}$. Due to the orthogonal property of the transform, the distance D_l can be expressed as

$$\begin{aligned} D_l &= \| \sum_{k=1}^{N} c_{l,k} q_k - \sum_{k \in I_l} \hat{c}_k q_k \|^2 \\ &= \sum_{k \in I_l} (c_{l,k} - \hat{c}_k)^2 + \sum_{k \notin I_l} c_{l,k}^2 \\ &= \sum_{k \in I_l} (\hat{c}_k - 2c_{l,k})\hat{c}_k + \sum_{k=1}^{N} c_{l,k}^2 \qquad (9) \end{aligned}$$

Since the last term is the energy of the lth phase which can be computed before hand, the distance calculation requires only $M_{opt}+1$ additions and M_{opt} multiplications.

IV. SIMULATIONS

The library contained ten reference phase sequences. The first five were sinusoids of same amplitude but slightly different frequencies. They were given by $1.3\cos(2\pi f_i t)$, where $f_1 = 79kHz$, $f_2 = 95kHz$, $f_3 = 103kHz$, $f_4 = 191kHz$ and $f_5 = 254kHz$. The remaining were quadratic phases with small differences in chirp rates ρ_i, where $\rho_6 = 3.2GHz/\sec$, $\rho_7 = 3.19GHz/\sec$, $\rho_8 = 638.293986101MHz/\sec$, $\rho_9 = 638.293986102MHz/\sec$ and $\rho_{10} = 638.293986104MHz/\sec$. The sampling frequency was 64MHz. The confusion matrix at 10dB SNR with WHT is shown in Table I. The sinusoidal phases were correctly classified. Although signals (8)-(10) have very little difference in chirp rates, only 10% error was observed. Figure 5 compares the identifier performance when different transforms, HT, WHT, DCT and ST, were used. Their performance were close to the optimal identifier in which all coefficients were used. HT had the largest complexity while ST had the least, and DCT and WHT are in between. In particular, ST took less than 30% of the coefficients to achieve the near optimum result for SNR below 15dB.

To summarize, a signal identifier that uses orthogonal transform coefficients as features for identification was proposed and studied. Feature extraction requires modest computation and has no numerical problems. The performance is close to the optimum identifier, with a complexity less than half of that of the optimum identifier.

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reference phases

1	2	3	4	5	6	7	8	9	10
100	0	0	0	0	0	0	0	0	0
0	100	0	0	0	0	0	0	0	0
0	0	100	0	0	0	0	0	0	0
0	0	0	100	0	0	0	0	0	0
0	0	0	0	100	0	0	0	0	0
0	0	0	0	0	100	0	0	0	0
0	0	0	0	0	0	100	0	0	0
0	0	0	0	0	0	0	91	9	0
0	0	0	0	0	0	0	9	82	9
0	0	0	0	0	0	0	0	9	91

Table 1 Confusion matrix, WHT and SNR=10dB

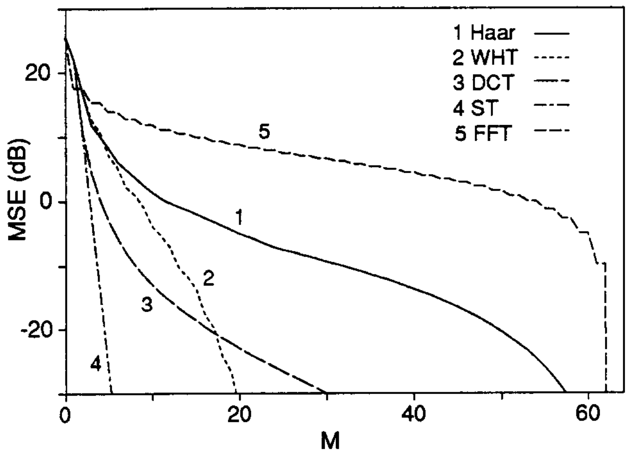

Fig. 1. Transform modelling MSE curves for linear FM chirp signal

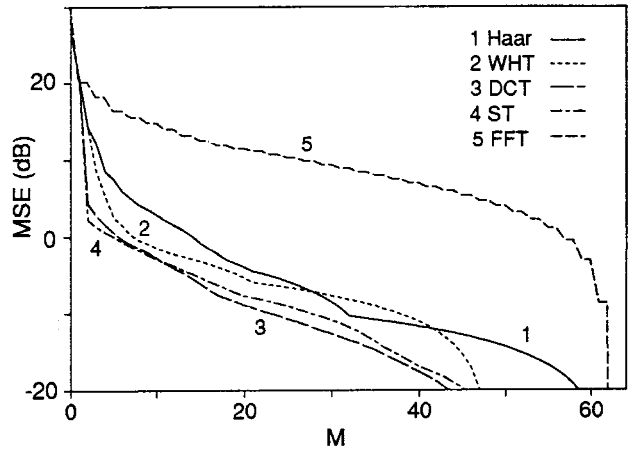

Fig. 2. Transform modelling MSE curves for a Barker-13 phase-coded signal

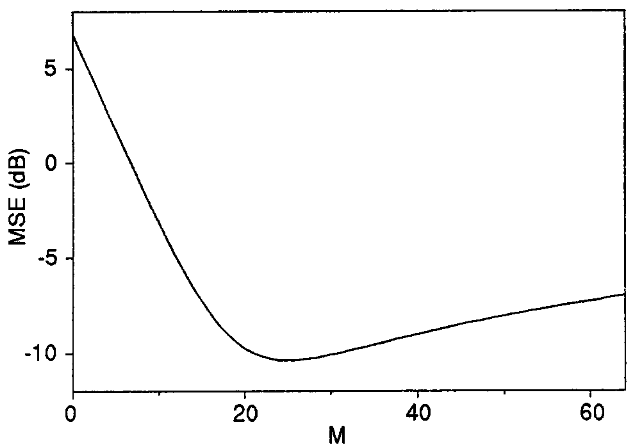

Fig. 3. MSE curve for WHT modelling $3.2\cos(0.89k)+\eta(k)$ at 4dB SNR

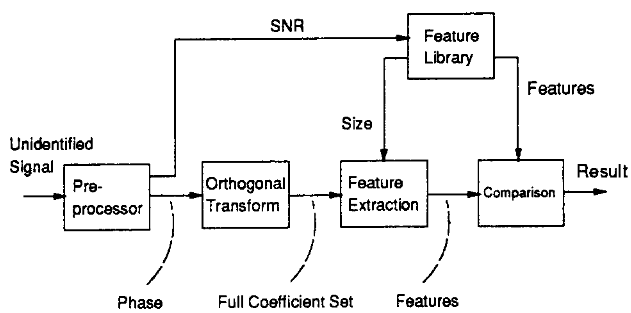

Fig. 4. The orthogonal transform signal identifier

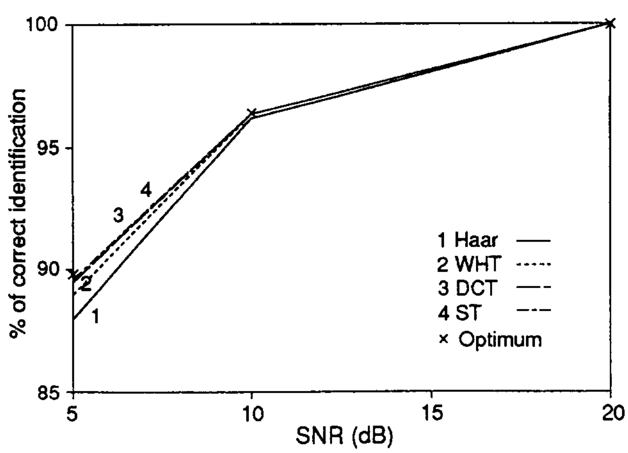

Fig. 5*a*. Performance comparison in accuracy

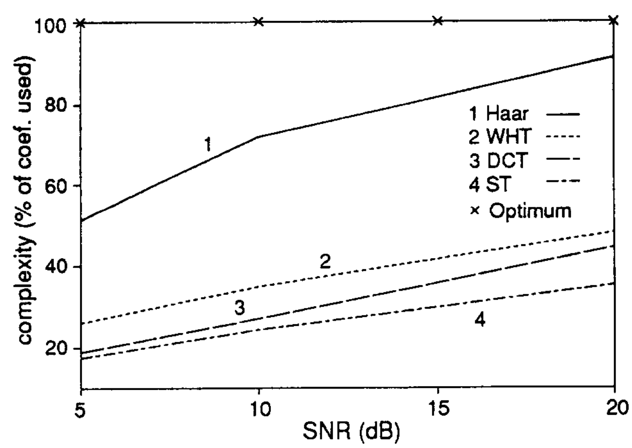

Fig. 5*b*. Performance comparison in complexity