PERFORMANCE ANALYSIS OF A DETECTOR FOR NONSTATIONARY RANDOM SIGNALS

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ABSTRACT

The detection of nonstationary random signals is an important sonar problem which also has potential applications in diverse areas such as biomedical signal processing and spread spectrum communications. The primary problem with applying a powerful test like the generalized likelihood ratio test (GLRT) is the computational effort required to search for the maximum likelihood model parameters for the observed signal. The computation required is multiplied many times over when a signal parameter is nonstationary. A computationally efficient detector of nonstationary Gaussian random signals based on the GLRT was presented at ICASSP94 [1]. A slightly enhanced version of the detector is described below, along with new simulation results demonstrating that the detector performs nearly optimally and is quite robust to signal model inaccuracy.

1. DETECTOR DESCRIPTION

The detection algorithm given in [1, 2] will be referred to as the General Viterbi implementation of the GLRT, due to its use of the Viterbi algorithm for efficient parameter searching. The General Viterbi method is most appropriate for the detection of signals which can be modeled accurately with a small number of AR coefficients and have a single nonstationary parameter, such as frequency. The method uses an AR(1) model and therefore performs best when the signal has a single spectral peak. It will be shown that the method is relatively robust to deviations from this assumed model. Figure 1 shows an example of a nonstationary AR(1) signal superimposed on a colored noise background.

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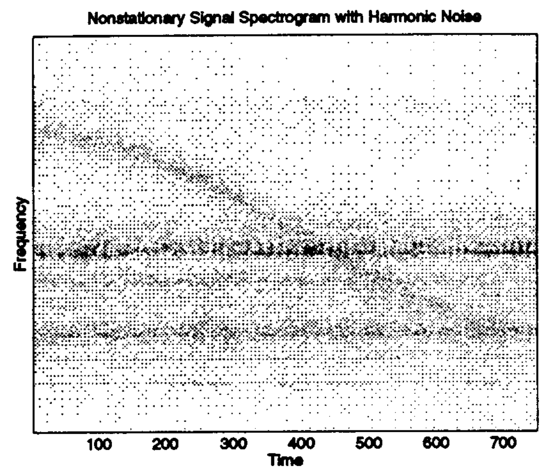

Figure 1: The spectrogram is the magnitude of 256 point FFTs with a rectangular window. The data blocks are non-overlapping. The "harmonic noise" is actually measured sonar data.

The following mathematical model gives a framework for describing the General Viterbi detection algorithm. Let the received signal be the zero mean complex Gaussian random vector $\underline{x}$. Then let $\underline{x}(p)$ be defined as a nonoverlapping rectangular windowed segment or "block" of $\underline{x}$ with

$$\underline{x}(p) = [x(pM), x(pM+1), \ldots, x(pM+(M-1))]^T \quad (1)$$

where M is the number of samples per block, and P is the number of blocks in the observation. Define $\mathbf{K}_x(p,q) = E[\underline{x}(p)\underline{x}'(q)]$ to be the cross-covariance matrix of any two blocks of $\underline{x}$. There are two possible hypotheses about $\underline{x}$, H_0 and H_1:

$$\begin{aligned} H_0: &\quad \mathbf{K}_x(p,p) = \mathbf{K}_n(p,p) && \text{for all } p \\ H_1: &\quad \mathbf{K}_x(p,p) = \mathbf{K}_s(p,p) + \mathbf{K}_n(p,p) && \text{for all } p. \end{aligned} \quad (2)$$

Only the diagonal blocks of the full observation covariance matrix are used. This simplification will be explained shortly. If s is assumed to be a nonstationary AR(1) process, $\mathbf{K}_1$ can be written entirely in terms of

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the signal parameters, G, r, θ, (signal gain, pole radius, and pole angle, respectively) and the noise covariance $\mathbf{K}_n$.

If an estimate of noise covariance is known, there are no parameters to maximize for H_0, and the GLRT for the entire observation may be computed. However, it is preferable to process each block of $\underline{x}$ independently, and the block version, or GBLRT detector, can be computed as

$$\max_{G,r,\underline{\theta}} \sum_{p=0}^{P-1} l_1(\underline{x}(p)|G,r,\theta(p)) - l_0(\underline{x}(p)) \underset{H_0}{\overset{H_1}{\gtrless}} 0 \quad (3)$$

where l_i is the log likelihood function under H_i. This alternative detector makes the implied assumption that the off-diagonal covariance submatrices are zero, or $\mathbf{K}_x(p,q) = 0$ when $p \neq q$, therefore the GBLRT only approximates the GLRT.

The detector given in (3) is simple to compute except for the multiparameter maximization. A few reasonable simplifications, however, can make this maximization tractable. These simplifications are examined in the next section. The model parameters need not be continuous, but can be limited to a finite number of discrete values defined in advance. This approach can be applied to the model frequency $\theta(p)$, the model bandwidth r, and the gain G. Discretizing the parameters substantially reduces the size of the maximization problem.

The search for the maximum likelihood path through the discrete set of frequency values is equivalent to a tree-searching problem, and the Viterbi algorithm can be used to find the optimum solution [3]. The requirements of the Viterbi algorithm can be reduced if the search is restricted under the assumption that the frequency is slowly varying. The computational requirements can be further reduced by use of an efficient approximation to the likelihood function as suggested in [4]. This approximation is referred to as spectral diagonalization. The quadratic detector based on the spectral diagonalization has been shown to perform nearly as well as the optimum detector [2].

Now consider restricting the signal to have a slowly-varying frequency parameter. The detector in (3) chooses $\theta(p)$ to maximize l_1, but in low SNR problems this can give a discontinuous signal frequency path even when the actual path is smooth. To minimize erroneous path discontinuities it is useful to place a restriction on the rate of frequency change. The slowly-varying assumption is taken to mean that the frequency parameter can only vary up or down by one discrete value, or bin, from one block to the next. This reduces the computational requirements of the Viterbi algorithm to $O(VP)$. Based on simulations, the one bin restriction significantly improves the estimator's performance, even when there is a fairly rapid rate of frequency change. The detector described in [2]

used the stack algorithm as an approximation to the Viterbi algorithm, but further investigation showed that the Viterbi algorithm performs substantially better and becomes more computationally efficient when using the single bin transition restriction.

In its final form, the detector statistic of the General Viterbi method can be expressed as

$$\Lambda_{GBLRTsd} = \max_{G,r,\underline{\theta}} \sum_{p=0}^{P-1} \tilde{\underline{x}}'(p)[\mathcal{D}(\widetilde{\mathbf{K}}_0(p,p))^{-1} - \mathcal{D}(\widetilde{\mathbf{K}}_1(p,p;G,r,\theta(p)))^{-1}]\tilde{\underline{x}}(p) \quad (4)$$

where $\mathcal{D}()$ is defined as the operator which sets the off-diagonal elements to zero and where

$$\tilde{\underline{x}}(p) = \mathbf{W}\underline{x}(p) \quad (5)$$

and

$$\widetilde{\mathbf{K}}_i(p,p) = \mathbf{W}\mathbf{K}_i(p,p)\mathbf{W}' \quad (6)$$

where $\mathbf{W}$ is the FFT matrix and $\tilde{}$ indicates a spectral domain quantity. The name General Viterbi is inspired by the fact that the Viterbi algorithm is used to maximize the sequence of $\theta(p)$ values, and G and r, although fixed over the observation, are assumed unknown. The acronym GBLRTsd is used to denote the Generalized Block Likelihood Ratio Test (with) spectral diagonalization. Diagonalization in the frequency domain, or spectral diagonalization, is an effective approximation because the FFT tends to diagonalize the covariance matrix.

2. ANALYSIS

The distributions of $\Lambda_{GBLRTsd}$ under hypotheses H_0 and H_1 are very difficult to obtain due to the maximization step. However, if the parameter values are assumed known in advance, the maximization is eliminated and the test becomes an approximation to the LRT instead of the GLRT. The resulting detector statistic, including the approximations but not the unknown parameters, is given by

$$\Lambda_{BLRTsd} = \sum_{p=0}^{P-1} \tilde{\underline{x}}'(p)[\mathcal{D}(\widetilde{\mathbf{K}}_0(p,p))^{-1} - \mathcal{D}(\widetilde{\mathbf{K}}_1(p,p))^{-1}]\tilde{\underline{x}}(p). \quad (7)$$

The distribution of Λ_{BLRTsd} is also difficult to obtain in closed form, but its second order statistics can be used to approximate it by the Γ distribution. The evaluation of the Γ approximation is given in [2]. The Γ approximation technique may also be applied to approximate the distribution of Λ_{BLRT}, the detector employing only the block processing simplification (without spectral diagonalization). The distribution of Λ_{BLRT} is given by

$$\Lambda_{BLRT} = \sum_{p=0}^{P-1} \underline{x}'(p)[\mathbf{K}_0^{-1}(p,p) - \mathbf{K}_1^{-1}(p,p)]\underline{x}(p) \quad (8)$$

Figure 2 shows a comparison of the three detectors based on the Γ approximation of their distributions. The figure makes it clear that the block processing and spectral diagonalization simplifications have a very small effect on the detector performance. Since this analytical method cannot be applied when the signal parameters are unknown, the generalized methods must be evaluated using simulations.

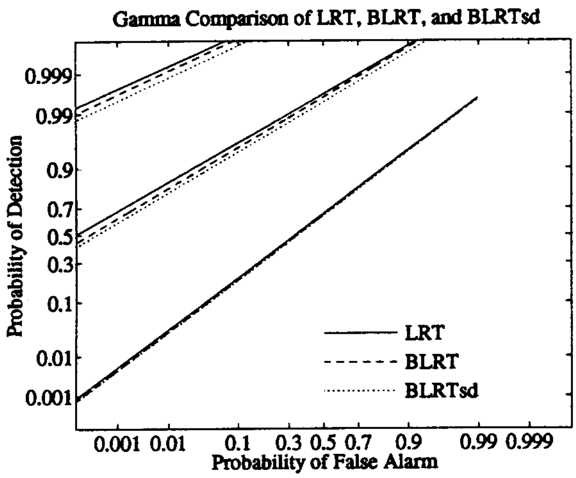

Figure 2: The Γ approximation for the LRT, the BLRT, and the BLRTsd detectors with SNR values of $-25, -15$, and -11dB.

3. DETECTOR PERFORMANCE

Figures 3 and 4 summarize the new results in performance analysis for the General Viterbi detector. Figure 3 shows the receiver operating characteristic of two detectors. The SUF (Stationary Unknown Frequency) detector is an implementation of the GBLRT for a first-order autoregressive AR(1) model. The SUF assumes that the signal has a stationary, unknown frequency, and the AR pole radius and signal power are known. Because the SUF detector assumes that two of the signal parameters are known in advance, it can be used as an upper-bound comparison standard for the General Viterbi detector which allows the frequency to be nonstationary. The General Viterbi detector is the new method, also based on the GBLRT, which assumes that all the signal parameters are unknown. The General Viterbi detector also includes several approximations to increase computational efficiency. It must be computationally efficient to estimate all the model parameters: the stationary signal power, the stationary pole radius, and the nonstationary pole angle or frequency. Figure 3 shows that the General Viterbi method performs nearly as well as the SUF even though more information is available to the SUF detector, and the General Viterbi detector must estimate the many new parameters introduced by the nonstationary frequency assumption.

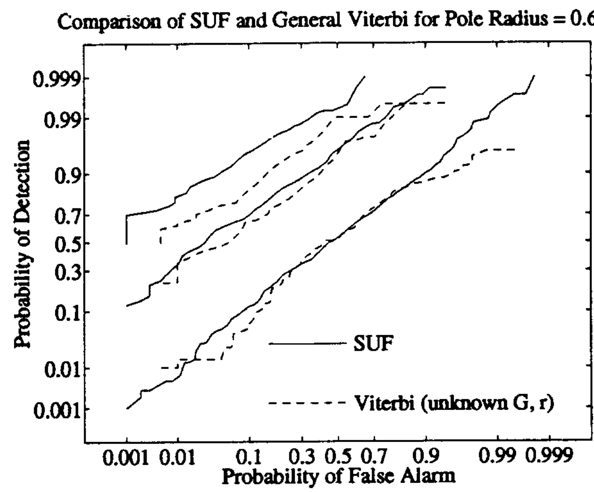

Figure 3: The curves shown are for SNR values of $-25, -13$, and -11dB. The SUF curve is for 1000 iterations per hypothesis and the Viterbi curve is for 200 iterations per hypothesis.

The General Viterbi algorithm assumes an AR(1) model for the signal, but its usefulness is not limited to perfect AR(1) signals. It has been applied to signals with AR(2) synthetic signals (a second order pole at one location), and to moving average (MA) signals shaped by a frequency-shifted 6th-order lowpass FIR filter. In each of these cases the signal meets the necessary requirements of being wideband and carrying most energy in a single spectral peak. Figure 4 shows a comparison of the General Viterbi method based on the AR(1) model with the SUF FIR detector. The SUF FIR detector is simply the GBLRT for the MA(6) signal where the only unknown parameter is the stationary frequency. The figure shows that the General Viterbi method performs nearly optimally even though it uses an AR(1) model instead of the correct MA(6) model. Therefore, the General Viterbi method is useful for a variety of signals which do not conform exactly to the AR(1) signal model assumed.

Figure 5 shows the performance of the General

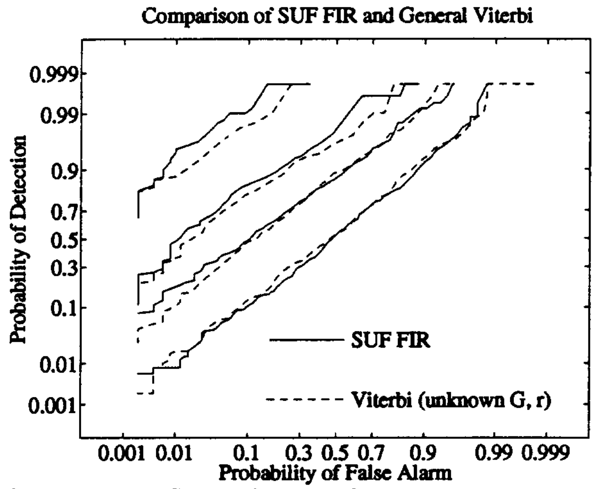

Figure 4: Four SNR values are shown, $-25, -15, -13$, and -11dB. Each curve was generated with 500 trials per hypothesis.

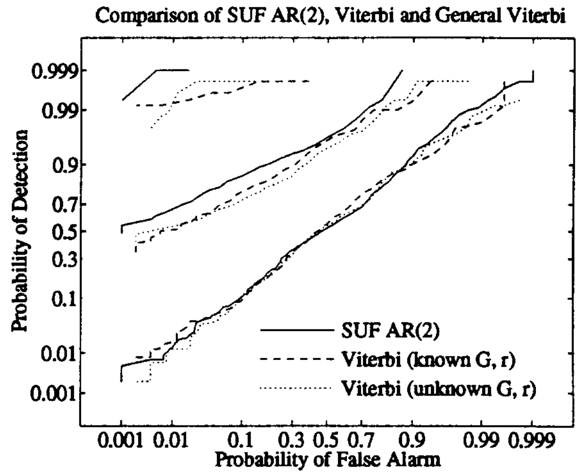

Figure 5: The SUF AR(2) is the near-optimum detector for a stationary AR(2) signal with unknown frequency. The Viterbi detector assuming known G and r for the AR(1) case is shown along with the stack detector making the same assumptions. The general Viterbi detector estimates the values of G and r which gives more freedom to compensate for the model inaccuracy. The SNR values shown are $-25, -15$, and -11dB. Each curve was simulated with 500 trials per hypothesis, except for the SUF AR(2), which required 1000 trials.

Viterbi method when the signal is an AR(2) signal instead of the presumed AR(1) signal. The AR(2) signal is formed using a pole of multiplicity two so that the spectrum will continue to have only a single peak, although the peak will be sharper than before. The General Viterbi method is compared with two other detectors. The first is the Viterbi method assuming known G and r values, and the second is the SUF detector optimized for an AR(2) signal. Once again, the SUF AR(2) detector assumes all signal parameters are known, including the shape of the spectrum, but the frequency is assumed unknown. Since the SUF assumes the stationary signal is known to be stationary, while the Viterbi methods assume the signal is not stationary in frequency, the SUF detector has a distinct advantage, and represents an upper bound for the General Viterbi method.

4. CONCLUSIONS

The performance of the General Viterbi detector has been shown to be similar to that of nearly optimal methods, and computationally practical as well. The excellent performance obtained vindicates the approximations made to simplify the calculation of the likelihood function. The computational efficiency of the algorithm is derived from the block processing approximation, the spectral diagonalization approximation, and the restricted version of the Viterbi maximization algorithm. Each of these reductions in computation have a small effect on the detector's performance, so that the resulting detector is comparable to the ideal from which it is derived.

In addition to showing that the computationally efficient detector has good performance, the structure on which it is based is easily converted to use with spectral models other than the AR(1) model assumed at the beginning. The same basic structure is employed in the implementation of the SUF FIR and SUF AR(2) detectors for signals with non-AR(1) spectra. Therefore, the General Viterbi detector may be useful as the basis for computationally efficient detectors for nonstationary signals with a variety of spectral models.

5. REFERENCES

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