HIGH RESOLUTION SPATIO-TEMPORAL ANALYSIS BY AN ACTIVE ARRAY

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ABSTRACT

We present in this paper a high resolution method for the joint estimation of Directions Of Arrival (DOA) and Travel Time (TT). This algorithm applies to active antenna for wich the transmitted signal of some sources is known. We show how to take into account the a priori information about signal in MUSIC.
The method is presented and a non asymptotic statistical performance analysis using perturbation expansions is applied. The major result is a formula for mean-squared error of the DOA and TT estimations. Simulation results verify the analytically predicted performance.

1. INTRODUCTION

The problem of localization of radiant sources in a propagation medium arises in several applications such as sonar, radar and seismology. In order to solve this problem, several methods have been proposed. Each of them depends on the a priori information available about the received signal: kind of sources, noise, geometry of the array, etc...
Since several years, high resolution methods have been developed in order to separate spatially close sources. They are based on an accurate modelization of the received signals upon the array (plane waves, uncorrelated sources,...). All those methods have been conceived for passive arrays. The signal is supposed to be random and stationary, the simplest case to be studied being the narrow band case.
In order to take into account an a priori information about signal (i.e. its spectral length), narrow band methods have been extended to wide-band signals. But, for spatial analysis as well as for temporal analysis of the signal, those wide-band methods used to consider the signal to be random, and they do not use informations about spectral characteristics (modulus and phase of the spectra) or temporal characteristics (waveform) of the signal.
More recently, methods have been proposed for cyclostationary signals (in narrow and wide band analysis). They integrate an a priori information about the nature of the signal and the performance in localization is, thus, appreciably improved.

In many applications such as active sonar or ocean acoustic tomography, the transmitted signal is known. This information have been introduced in the analysis of the signal received on a single sensor for the high resolution estimation of travel time [1], but it is few used in spatial analysis methods [2].
The method that is proposed here aims to use the a priori information about the signal and to introduce it into high resolution spatial or spatio-temporal analysis methods.

2. SPATIO-TEMPORAL ANALYSIS

2.1. Model and Notations

We consider an acoustic field composed of P sources incident on an array of M sensors.
The signal received on the m^{th} sensor is modelized as :

$$x_m(t) = \sum_{p=1}^{P} a_p.e_p(t - \tau_{m,p}) + b_m(t) \qquad (1)$$

with :

- $x_m(t)$: temporal domain received signal at the m^{th} sensor.
- $e_p(t)$: signal transmitted by the p^{th} source.
- a_p : amplitude of the p^{th} source.
- $\tau_{m,p}$: delay between the p^{th} source and the m^{th} sensor.
- $b_m(t)$: additive noise received at the m^{th} sensor.

In frequency domain, (1) is written as :

$$x_m(\nu) = \sum_{p=1}^{P} a_p e_p(\nu).e^{-2i\pi\nu\tau_{m,p}} + b_m(\nu) \qquad (2)$$

$x_m(\nu)$, $e_p(\nu)$ and $b_m(\nu)$ being, respectively, the Fourier Transforms of $x_m(t)$, $e_p(t)$ and $b_m(t)$.
The travel time $\tau_{m,p}$ can be expressed as follows :

$$\tau_{m,p} = T_p + t_m(\theta_p)$$

T_p represents the delay between the p^{th} source and the reference sensor, $t_m(\theta_p)$ is the delay between the reference sensor and the m^{th} sensor. $t_m(\theta_p)$ is function of θ_p which is the direction of arrival of the wave upon the array.

In (2), the term $e_p(\nu)$ is deterministic. It is composed of a modulus and a phase expressing the a priori information about the signal.
The a_p's are considered random and decorrelated. In ocean acoustic tomography, for example, each source corresponds to a path, $e(t)$ is the signal transmitted from the transmitter and a_p corresponds to the complex amplitude of each path.

2.2. Principle of the Algorithm

By separating the deterministic terms from the random terms, expression (2) can be written in the following vector form :

$$\mathbf{x_g} = \mathbf{H}.\mathbf{c} + \mathbf{b_g} \quad (3)$$

with :

- $\mathbf{x_g} = [\mathbf{x}^+(\nu_1), \mathbf{x}^+(\nu_2), \ldots, \mathbf{x}^+(\nu_F)]^+$
- $\mathbf{x}(\nu_i) = [x_1(\nu_i), x_2(\nu_i), \ldots, x_M(\nu_i)]^+$
- F: number of frequency bins of the signal.
- $\mathbf{b_g} = [\mathbf{b}^+(\nu_1), \mathbf{b}^+(\nu_2), \ldots, \mathbf{b}^+(\nu_F)]^+$
- $\mathbf{b}(\nu_i) = [b_1(\nu_i), b_2(\nu_i), \ldots, b_M(\nu_i)]^+$
- $\mathbf{c} = [a_1, a_2, \ldots, a_P]^+$
- $\mathbf{H} = [\mathbf{h_1}, \mathbf{h_2}, \ldots, \mathbf{h_P}]$
- $\mathbf{h_p} = [e_p(\nu_1).e^{-2i\pi\nu_1\tau_{1,p}}, \ldots, e_p(\nu_F).e^{-2i\pi\nu_F\tau_{M,p}}]^+$
- $^+$ meaning transposed.

$\mathbf{x_g}$ is a vector of dimension $M.F$ obtained by concatenation of observation vectors at each frequency. $\mathbf{b_g}$ is a vector of dimension $M.F$ obtained by concatenation of noise vectors at each frequency. $\mathbf{c}$ is a vector of dimension P. $\mathbf{H}$ is a matrix of dimension $(M.F, P)$. $\mathbf{H}$ puts together the terms $e^{-2i\pi\tau_{m,p}}$ caracterizing the transfer functions between the sources and the sensors, and the term $e_p(\nu_i)$ characterizing the signal transmitted by the p^{th} source.

Let us consider $\mathbf{X}$, the covariance matrix of the received data :

$$\mathbf{X} = E(\mathbf{x_g}.\mathbf{x_g^*})$$

* meaning transpose-conjugated.
If the sources and the noise are uncorrelated, we have :

$$\mathbf{X} = \mathbf{H}.\mathbf{C}.\mathbf{H}^* + \mathbf{B} = \mathbf{Y} + \mathbf{B} \quad (4)$$

with :

- $\mathbf{C} = E(\mathbf{c}.\mathbf{c}^*)$: Covariance matrix of the sources.
- $\mathbf{Y} = \mathbf{H}.\mathbf{C}.\mathbf{H}^*$: Covariance matrix of the noiseless observation.
- $\mathbf{B} = E(\mathbf{b_g}.\mathbf{b_g^*})$: Covariance matrix of the noise.

The principle of the algorithm can be deduced easily from the classical MUSIC algorithm. If the sources are uncorrelated, $\mathbf{C}$ is diagonal and the rank of $\mathbf{Y}$ is P. If the noise is spatially white and temporally white (further hypothesis in comparison with the narrow band case), $\mathbf{B}$ is proportional to Identity. We can proceed to a subspace decomposition and define a signal subspace spanned by the first P eigen vectors of $\mathbf{X}$, and its complementary, the orthogonal subspace spanned by the $M.F - P$ last eigen vectors of $\mathbf{X}$. Thus, the algorithm consists in maximizing the following function :

$$d(\theta, T) = \frac{1}{\sum_{i=P+1}^{M.F} ||\mathbf{a}^*(\theta, T).\mathbf{v}_i||^2} \quad (5)$$

with :

- $\mathbf{a}(\theta, T)$: wide-band steering vector.
- $\mathbf{v_i}$: i^{th} eigen vector of $\mathbf{X}$.

The steering vector $\mathbf{a}(\theta, T)$ is written as follows :

$$\mathbf{a}(\theta, T) = \begin{bmatrix} e(\nu_1).e^{-2i\pi\nu_1 T}.\mathbf{d}^+(\nu_1, \theta) \\ \cdots \\ e(\nu_F).e^{-2i\pi\nu_F T}.\mathbf{d}^+(\nu_F, \theta) \end{bmatrix} \quad (6)$$

with :

$$\mathbf{d}(\nu_i, \theta) = [1, e^{-2i\pi\nu_i t_{1,2}(\theta)}, \ldots, e^{-2i\pi\nu_i t_{1,M-1}(\theta)}]^+$$

$\mathbf{d}(\nu_i, \theta)$ is the classical steering vector used in narrow band analysis. It contains the informations concerning the phase shifts between sensors at a given frequency and for a source of parameter θ.
$\mathbf{a}(\theta, T)$ is the concatenation of the vectors $\mathbf{d}(\nu_i, \theta)$ weighted by the frequency characteristics of the signal.

2.3. Simulation Results

In order to illustrate the characteristics of the algorithm, we have done some numerical simulations. A typical result is shown in Figure 1. The transmitted signal is a BPSK signal whose time-bandwidth product equals 15. The central frequency equals $180Hz$ and the main lobe is $120Hz$ and $240Hz$. The data are generated in time. The sampling frequency is equal to $1920Hz$ and the celerity of the medium is $1500m/s$. Three sensors are used and the spatial sampling corresponds to one half wavelength of the lowest frequency. The SNR is equal to $10dB$. Two echoes are received on the array whose DOAs and TTs are, respectively, $\theta_1 = 0\ deg$, $\theta_2 = 7\ deg$, $T_1 = 0\ sample$ and $T_2 = 1\ sample$. Figure 1 represents the logarithm of the function (5). Both sources are clearly identified with their correct DOA and TT. Such a result cannot be obtained by a classical analysis (Beamforming-Matched filtering) due to the time resolution of the signal (around 16 samples) and the array geometry (only 3 sensors).

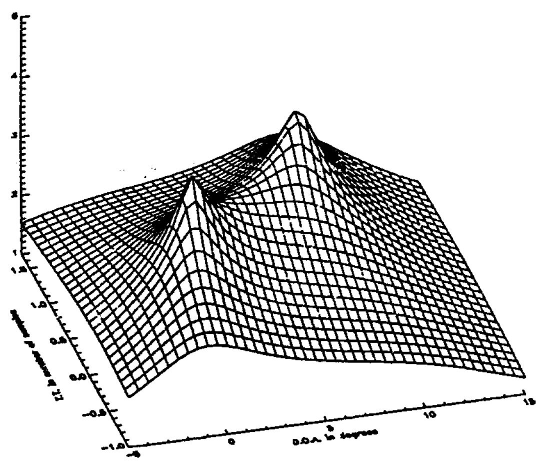

Figure 1: Example of DOA and TT Estimation

This algorithm have many interesting points and a complete description of its characteristics can be found in [3].

3. STATISTICAL PERFORMANCES

The success of subspace-based DOA estimation is based on its ability to perform a complete separation of signal and orthogonal subspaces. Observation noise is always present in practice which results in the perturbation of estimated subspaces when only finite measurements are available or when the noise has unknown covariance structure. This degrades the performance of DOA and TT estimations by causing an incomplete separation of the two subspaces.

3.1. Subspace Perturbation

Suppose that K snapshots are available. The K snapshots can be arranged as columns in a data matrix as follows :

$$\mathbf{X_g} = [\mathbf{x}_g(1), \mathbf{x}_g(2), \ldots, \mathbf{x}_g(K)]$$

which dimensions are $(M.F, K)$. The noisy data matrix can be written as :

$$\tilde{\mathbf{X}}_\mathbf{g} = \mathbf{X_g} + \Delta\mathbf{X_g}$$

The subspace decomposition can be either performed on the covariance matrix $\mathbf{X}$ by an eigenvalue decomposition or on the data matrix $\mathbf{X_g}$ by a singular value decomposition (SVD). The subspace decomposition using SVD on the direct data matrix $\mathbf{X_g}$ is as follows :

$$\mathbf{X_g} = \mathbf{U}.\mathbf{\Lambda}.\mathbf{V}^* = [\mathbf{U_S}\ \mathbf{U_O}] \begin{bmatrix} \mathbf{\Lambda_S} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{V_S}^* \\ \mathbf{V_O}^* \end{bmatrix}$$

where $\mathbf{U_S}$ are the singular vectors associated with the P non-zero singular values, while $\mathbf{U_O}$ are the singular vectors associated with the zero singular values.
The subspace decomposition of noisy data by SVD generates a noisy orthogonal subspace $\tilde{\mathbf{U}}_\mathbf{O}$ which columns are the estimated orthogonal subspace vectors associated with smallest singular values of the noisy data matrix $\tilde{\mathbf{X}}_\mathbf{g}$.

A noise matrix $\Delta\mathbf{X_g}$ induces perturbations in the estimated signal and orthogonal subspaces as follows [4] :

$$\Delta\mathbf{U_0} = \tilde{\mathbf{U}}_\mathbf{O} - \mathbf{U_O} = -\mathbf{U_S}\mathbf{\Lambda_S}^{-1}\mathbf{V_S}^*\Delta\mathbf{X_g}^*\mathbf{U_0} \quad (7)$$

$$\Delta\mathbf{U_S} = \tilde{\mathbf{U}}_\mathbf{S} - \mathbf{U_S} = \mathbf{U_0}\mathbf{U_0}^*\Delta\mathbf{X_g}\mathbf{V_S}\mathbf{\Lambda_S}^{-1} \quad (8)$$

where $\Delta\mathbf{U_O}$ is the perturbation in the estimated orthogonal subspace and $\Delta\mathbf{U_S}$ is the perturbation in the estimated signal subspace.

3.2. Perturbation of DOA's and TT's estimates

In a noisy environment, the estimated DOA's and TT's obtained by maximizing the function (5) are denoted as perturbations from the true directions of arrival and travel times as :

$$\tilde{\theta}_p = \theta_p + \Delta\vartheta_p$$

$$\tilde{T}_p = T_p + \Delta T_p$$

where $\Delta\theta_p$ and ΔT_p are the perturbations of the p^{th} directions of arrival and travel times.

The subspace perturbations (7) and (8) cause perturbations in the estimated DOA's and TT's. By approximating the derivative of the null-spectrum function by its first two terms in its Taylor series expansion about the true angles and times of arrival, we derive an analytical expression relating the perturbation in the estimated orthogonal subspace to perturbations in the directions and times of arrival. We get the following expression for the perturbation in the p^{th} DOA and TT estimates due to observation noise given by the matrix $\mathbf{\Delta X_g}$:

$$\Delta\theta_p = \frac{\Re[\beta_p^* \mathbf{\Delta X_g}^* \alpha_{1p}]}{\gamma_p} \qquad (9)$$

$$\Delta T_p = \frac{\Re[\beta_p^* \mathbf{\Delta X_g}^* \alpha_{2p}]}{\gamma_p} \qquad (10)$$

with $\beta_p = \mathbf{V_S \Lambda_S^{-1} U_S^* a}$ and

$$\alpha_{1p} = \frac{\mathbf{a}_T^* \mathbf{U_O U_O^* a}_T}{\Re[\mathbf{a}_\theta^* \mathbf{U_O U_O^* a}_T]} \mathbf{U_O U_O^* a}_\theta - \mathbf{U_O U_O^* a}_T$$

$$\alpha_{2p} = \frac{\mathbf{a}_\theta^* \mathbf{U_O U_O^* a}_\theta}{\Re[\mathbf{a}_\theta^* \mathbf{U_O U_O^* a}_T]} \mathbf{U_O U_O^* a}_T - \mathbf{U_O U_O^* a}_\theta$$

$$\gamma_p = \frac{\mathbf{a}_T^* \mathbf{U_O U_O^* a}_T \mathbf{a}_\theta^* \mathbf{U_O U_O^* a}_\theta}{\Re[\mathbf{a}_\theta^* \mathbf{U_O U_O^* a}_T]} - \Re[\mathbf{a}_\theta^* \mathbf{U_O U_O^* a}_T]$$

where $\mathbf{a}_\theta = \frac{\partial \mathbf{a}}{\partial \theta}(\theta_p, T_p)$ and $\mathbf{a_T} = \frac{\partial \mathbf{a}}{\partial T}(\theta_p, T_p)$

In the case of a zero-mean, spatially and temporally white noise with variance σ^2, the mean-squared error expression is :

$$E(\Delta\theta_p{}^2) = \frac{||\beta_p||^2 ||\alpha_{1p}||^2 \sigma^2}{2\gamma_p^2} \qquad (11)$$

$$E(\Delta T_p{}^2) = \frac{||\beta_p||^2 ||\alpha_{2p}||^2 \sigma^2}{2\gamma_p^2} \qquad (12)$$

In the case of one-path propagation, it has been shown analytically that the variances of the estimates reach the Cramer-Rao bounds.

3.3. Numerical Example

The configuration of the experiment is a four-element uniform line array with one path at 0.0 *rad* and $T = 6$ *ms*. The emitted signal is composed of 3 frequencies. Twenty snapshots of array data were taken for 1 hundred trials. Figure 2 shows statistical performance with respect to different number of sensors.

In the figures displayed here, the lines are theoritical predictions and discrete symbols are simulation measurements.

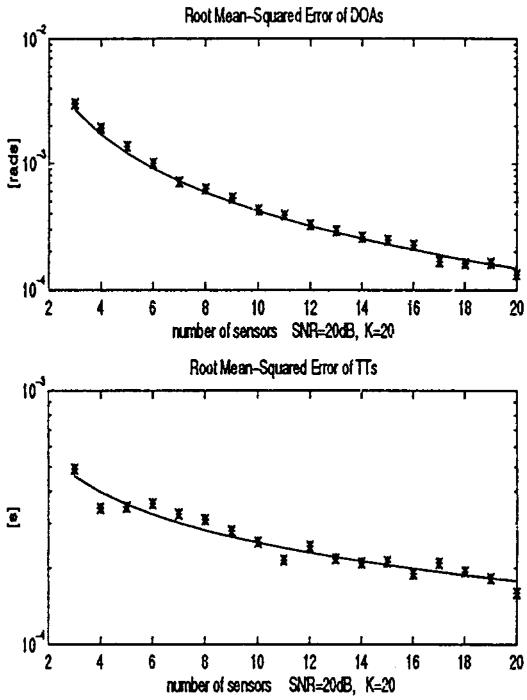

Figure 2: Root Mean-Squared Error of DOAs and TTs versus number of sensors

4. CONCLUSION

In this paper, a high resolution method for the joint estimation of directions of arrival and travel times using the a priori information of the transmitted signal has been presented. Its statistical performance have been derived and mean-squared error of the DOA and TT estimations have been given.

5. REFERENCES

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