NEAR-FIELD SOURCES LOCALIZATION : A MODEL-FITTING APPROACH

Hervé CHUBERRE, *Thierry* FILLEUL, *Jean-Jacques* FUCHS

IRISA/Université de Rennes I

Campus de Beaulieu - 35042 Rennes Cedex - France

ABSTRACT

Beamforming algorithms handle sources that are located in the near field by simply adapting the steering vector and having it depend not only on bearing but also on range. The corresponding algorithm, known as the focused beamformer, has poor performances even though the computational burden is quite important since it involves a 2-dimensional beam evaluation in the bearing-range plane. In order to localize such sources using a linear equispaced array in a narrowband context, we propose to apply a deconvolution approach [1] to the output of a beamformer performing a one-dimensional evaluation. For reasonably difficult scenarios, the performances we obtain are satisfactory (close to the Cramer-Rao bound) for a quite acceptable computational burden.

1. INTRODUCTION

We consider a linear array with equispaced sensors with an inter-sensor distance equal to half a wave-length ($\lambda/2$) at the temporal frequency f_t. The location of a source is defined by its bearing ϕ with respect to broadside and its range ρ both measured with respect to the center (of phase) of the array. We will assume that the sources are either in the far-field or the near-field but however at a range that is large enough for not having to consider a difference in the powers received at the different sensors of the array. We normalise all quantities : the temporal frequency f_t is taken equal to 0.5, the unit of length is taken to be $\lambda/2$ and the ranges will be expressed in this length-unit, the bearings will be transformed into spatial frequencies $\nu = \frac{1}{2}\sin\phi$ and thus vary between 0.5 and -0.5. A source is thus characterized by its power α, its spatial frequency ν and its range ρ. The corresponding unknowns will be denoted a, f and r.

We will assume that the propagation is well modeled by the usual model and that there is no mismatch between the direction vector $d(\nu_1, \rho_1)$, that is associated with a source, and the steering vector $d(f, r)$ used by the beamformer. The k-th component of the steering vector $d(f, r)$ when focused at $f = \nu_1$ and $r = \rho_1$ is then $exp\{2i\pi f_t \tau_k(\nu_1, \rho_1)\}$ where $\tau_k(\nu_1, \rho_1)$ is the propagation delay between the sensor k and the center of the array for the wave emitted by the source. Under the above defined normalizations, this component becomes $exp\{i\pi(\rho_{1,k} - \rho_1)\}$ where $\rho_{1,k}$ is the distance between the considered source and sensor k expressed in half-wave-lengths. The steering vector is thus given by :

$$d(\nu_1, \rho_1) = [e^{\{i\pi(\rho_{1,1}-\rho_1)\}} \quad e^{\{i\pi(\rho_{1,2}-\rho_1)\}} \quad ... \quad e^{\{i\pi(\rho_{1,N}-\rho_1)\}}]^T$$

For a P sources scenario in additive white noise of power σ_n^2, the covariance matrix of the snapshots is then given by :

$$R = \sum_{p=1}^{P} \alpha_p \, d(\nu_p, \rho_p) \, d(\nu_p, \rho_p)^* + \sigma_n^2 I$$

We define the output of the beamformer focused at f and r for this same scenario to be :

$$y(f, r) = \frac{1}{N^2} \, d(f, r)^* \, R \, d(f, r) \qquad (1)$$

where N is the number of sensors. Note that the covariance matrix R has no special structure : it is hermitian with a constant diagonal and depends upon $N(N-1)+1$ real degrees of freedom. For a single source in white noise the output of the focused beamformer becomes :

$$y(f, r) = \frac{1}{N^2} \, \alpha_1 \mid d(\nu_1, \rho_1)^* \, d(f, r) \mid^2 + \frac{\sigma_n^2}{N} \qquad (2)$$

2. NEAR-FIELD CHARACTERIZATION

Let us characterize the boundaries of the near-field domain and give an upper and a lower bound. Sources that are further away (than the upper bound) are said to be in the far-field, while for sources that are closer to the center of the array a different model would have to be considered.

A source is in the near-field when its wavefront can no longer be considered to be planar over the array aperture. It thus strongly depends upon the size of the array, we denote this length by l ($l = N - 1$ in our units). It is then easy to check that, for a given bearing ϕ, the discrepancy between the planar wavefront induced by a far-field source and the curved wavefront due to a near-field source with range ρ, is maximal for a source at broadside ($\phi = 0$) and that Δ, the corresponding difference in travel distance (along the array) is $\Delta \simeq (l cos \, \phi)^2/8\rho$. Indeed, as far as range estimation is concerned, $l \, cos \, \phi$ is the useful length of the array for a source with bearing ϕ. One can then consider, as in [1], that the limit between the far and near-field corresponds to

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$\Delta \simeq 1/8$ which leads to the usual values that fix this limit around N or $N/2$ times the length of the array.

$$\rho < lN \quad \text{or} \quad \rho < lN/2 \quad \text{upper limit}$$

Let us look now at what happens if the source is very close to the array. The solution to the wave equation [2] has always a factor of the form $\frac{1}{r}$ where r is the distance between the source and the considered point and the true steering vector is :

$$d(\nu_1, \rho_1) = [\frac{1}{\rho_{1,1}} e^{\{i\pi(\rho_{1,1}-\rho_1)\}} \ \ldots \ \frac{1}{\rho_{1,N}} e^{\{i\pi(\rho_{1,N}-\rho_1)\}}]^T$$

If a source is in the far-field the variation in the travel distances $\{\rho_{1,k}\}$ along the array is negligible and this term is omitted : the power is considered to be the same for all the sensors. The question is how long can this term be neglected ? The difference in travel distance is maximal for end-fire sources and never exceeds l. For a source with bearing ϕ it is roughly $l \sin\phi$ and if one is willing to neglect relative variations in power smaller than $\frac{1}{10}$ one obtains as a lower limit :

$$\rho > 10\ l\ \sin\phi \quad \text{lower limit} \qquad (3)$$

For sources that are closer to the (center of) array than this limit it is necessary to take into account the variation of power along the array in the estimation procedure (as well as in the simulations !).

3. GENERAL PRINCIPLE OF THE METHOD

Since the focused (bidimensional) beamformer has poor performances and is too time-consuming, we propose to use a one-dimensional beamformer followed by a model-fitting algorithm [3],[4]. Let us observe from the beginning that some information is lost when going from $\hat{R}$, the estimated covariance matrix which is a sufficient statistic for the data, to the outputs of a one-dimensional beamformer (1). Indeed R has $N^2 - N + 1$ real degrees of freedom and we will, in general, only form $4N - 1$ (real) beams.

The general idea of the procedure is as follows [3],[4].

• We start evaluating the output of the focused beamformer at a discrete set of spatial frequencies, say f_k, along a one-dimensional curve in the (f, r)-plane defined by $r = c(f)$. This curve is chosen so that for a single source scenario the output of the beamformer $F(f, c(f))$ exhibits, over f, a unique global maximum located at $f = \nu_1$ the true spatial frequency of the unique source present. This property has to hold for all possible locations of the unique source. To fix the ideas, we consider in the sequel that this curve is simply the straight line in the (f, r)-plane defined by $r = r_m$, a fixed range. Note that taking $r_m = \infty$, which corresponds to the standard beamformer (for far-field sources), is not a "line" that satisfies this property. Such a line exists however but more interesting curves should be considered. By linearity, one can expect that this property holds somehow for multiple sources scenarios. The idea behind this first step is to locate the interesting spatial sectors in which sources are potentially present by simply doing a one-dimensional search and thus to limit the computations. We thus evaluate for instance :

$$\hat{y}(f_k, r_m), \quad f_k = \frac{k}{4N-1} \quad k = 0, \pm 1, .. \pm (2N-1) \qquad (4)$$

with r_m a fixed intermediate range.

• Localize the global maximum of this one-dimensional output and associate with it a spatial sector. A threshold is defined from a preliminary estimate of the power of the additive white noise and is used to associate with the current global maximum a limited spatial sector. While for an isolated source the width of this sector is in general limited to the main-lobe, for sources that are closely spaced in spatial frequency the width may be quite large and a sector encompass several sources. The objective here is to reduce the computational load and to avoid the emergence of local minima in the criterium used in the model-fitting scheme.

• The values of the beams belonging to this sector are collected in a vector, say V. Only an estimate $\hat{V}$ of this vector is available in practice. It is, asymptotically in the number of snapshots, a random gaussian vector with mean a *model-vector* $V(\theta_{ex})$, function of the parameters θ to be estimated and with covariance matrix $\Sigma(\theta_{ex})$.

For a single source scenario (see (2)), the unknowns are $\theta_{ex} = (\alpha_1\ \nu_1\ \rho_1\ \sigma_n^2)$ and the value of the component of the vector V at spatial frequency f_k belonging to the sector, is :

$$\frac{1}{N^2}\alpha_1 |d(\nu_1, \rho_1)^* d(f_k, r_m)|^2 + \frac{\sigma_n^2}{N} \qquad (5)$$

the corresponding component of the model-vector with unknowns $\theta = [a\ f\ r\ v]$ is :

$$\frac{1}{N^2} a |d(f_k, r_m)^* d(f, r)|^2 + \frac{v}{N} \qquad (6)$$

Note that the model-vector takes into account the fact that the one-dimensional search has been done along the curve $r = r_m$ and that for $\theta = \theta_{ex}$ the two components are identical. A consistent estimate $\hat{\Sigma}$ of the covariance matrix $\Sigma(\theta_{ex})$ can be obtained from the data without a preliminary estimation of $\hat{\theta}$. It is used to define a least-square type criterion between the observed vector $\hat{V}$ and the model vector $V(\theta)$:

$$C(\theta) = \|\hat{V} - V(\theta)\|^2_{\hat{\Sigma}^{-1}} \qquad (7)$$

This is a standard criterium that is justified here by the fact that the vector $\hat{V}$ is asymptotically gaussian

• The criterion is minimized for an increasing number P of sources, starting with $P = 0$. For a given P the number of unknowns is $3P + 1$, (a_i, f_i, r_i) for each source and the noise power v. The component at spatial frequency f_k of the model-vector $V(\theta)$ is then :

$$\frac{1}{N^2} \sum_{i=1}^{P} a_i |d^*(f_k, r_m).d(f_i, r_i)|^2 + \frac{v}{N}$$

The unknowns are obviously not of the same kind. The model is linear in the amplitudes $\mathbf{a} = [a_1..a_P]$ and v and non-linear in the frequencies $\mathbf{f} = [f_1..f_P]$ and the ranges $\mathbf{r} = [r_1..r_P]$. The model vector can thus be re-written as :

$$V(\theta) = V(\mathbf{a}, v, \mathbf{f}, \mathbf{r}) = D(\mathbf{f}, \mathbf{r})[\mathbf{a}, v] \qquad (8)$$

The criterion is evaluated on a coarse grid in the (f, r)-plane (corresponding to the non-linear unknowns) in order to obtain an initial estimate that is improved upon using in a quasi-Newton algorithm [5]. *Note that so far we have only performed a one-dimensional beam evaluation. The cost of the evaluations on the local two-dimensional grid has thus to be compared to the two-dimensional focused beamformer.*

• The value of the minimum denoted $C_{min}(\theta_P)$ decreases with P the number of fitted sources. An Akaike type test is then implemented to estimate the number of sources present in the spatial sector.

• Once a sector has been processed, the contribution of the sources estimated in this sector is substracted from all the beams. The same procedure is applied to the spatial sector associated with the new global maximum until no interesting sector is left.

4. SOME DETAILS OF THE METHOD

We now detail some of the important points of the approach.

4.1. THE CHOICE OF R_M

The deconvolution approach we propose uses as input the outputs of a one-dimensional beamformer in the frequency-range domain. We propose to evaluate the beams at a set of equispaced spatial frequencies f_k, along a curve $r = c(f)$. The idea behind this procedure is that an inspection of these beams allows to locate the spatial sectors in which sources are present, whatever their range, and thus to only process these potentially interesting sectors in the sequel.

In the simulations given below, we have evaluated the beams at $4N - 1$ equispaced spatial frequency and constant range, denoted r_m (4). This is certainly not the best choice. A probably better choice would be to consider points (in the frequency-range plane) that are on a circle tangent to the array at its center, under some simplifying assumptions one can show that the points on such a circle have constant range estimation variance [6] [7]. Note also that this choice is very important since it decides upon the quality of the information to be used in the sequel and thus directly affects the performances and detection threshold.

If one takes a closer look at the two terms (5,6) whose difference is *minimized* by the algorithm, one notes that if everything is known ($a = \alpha_1$, $f = \nu_1$, $v = \sigma_n^2$) but the range ρ_1 , and for f_k close to ν_1, the difference between these two terms is maximal if $r = r_m$ and decreases on both sides of this value of r. In general it thus has two minima, one on each side of r_m, the global minimum for $r = \rho_1$ and another one. This is a disadvantageous situation since an algorithm will converge towards to the minimum associated with its initialization point. One mean to circumvent this difficulty is to take r_m at the lower boundary of the domain of interest (3). It is this choice that we make in the simulations where we take $r_m = 4l$.

4.2. THE OPTIMIZATION ALGORITHM

Remember that the model (8) we fit to the "observations" is linear with respect to the amplitudes (sources and noise) and non-linear with respect to the frequency and range. The optimization is thus performed essentially with respect to the frequency and range variables, the associated optimal amplitudes being trivially obtained.

This is an important issue since the major drawback of a model-fitting approach is the computational load. Working on isolated spatial sectors is already a partial answer to this issue. In a given sector we start evaluating the criterion on a two-dimensional grid. This means that, if we assume, for instance, that there is a single source in the sector, we evaluate the criterion assuming successively the source to be at the different points of the grid. While the width of the mesh is a constant along the spatial frequency axis, along the range axis we take an unregular grid that takes into account the quick variation (of the order of r^2) of the standard-deviation of the range-estimate given by the Cramer-Rao bound[7]. In between the lower bound taken equal to r_m and the far-field we take six points in the grid. Note that the evaluation of the criterion on the grid is extremely time consuming since it has to be done in a given sector for an increasing number of potential sources for all possible source-position combinations.

The purpose of this first step is to furnish a good initialization point to the iterative optimization algorithm. We have implemented a quasi-Newton type algorithm : BFGS with a Wolfe's line search [5].

5. SIMULATION RESULTS

With our approach, a complete scenario is in general decomposed into a set of spatial sectors. Each sector is processed in turn, somehow independently of the others [3]. A good idea of the performances is thus already obtained by looking at what happens in one spatial sector. We present results that correspond to two-sources scenario, where the two sources are in the same sector.

We consider a linear array with $N = 31$ equispaced sensors. The number of snapshots is $T = 100$. We always start evaluating $4N - 1 = 123$ beams at equispaced spatial frequencies and (focused at) constant range $r_m = 120$. Remember that the unit of distance is $\lambda/2$, so that r_m is equal to 4 times the length of the array.

We present results obtained from 20 independent simulations for a number of different two sources scenarios. They have bearings closer than the (far-field) Rayleigh limit with different powers. A focused two-dimensional beamformer does not allow to distinguish them. In **table 1**, we indicate, for the different unknowns the estimated means and variances (averaged over the 20 independent trials) together with the corresponding Cramer-Rao bounds. For each scenario, we also give the number of sources detected by the Akaike-like criterion (in percentage).

6. CONCLUSION AND COMMENTS

We have proposed a model-fitting approach that localizes near-field as well as far-field sources. It works in narrowband on the outputs of a one-dimensional beamformer. The objective was to outperform the focused (two-dimensional) beamformer without a too important increase in computational cost. It is attained though the cost increases rapidly

when the number of sources in a spatial sector (clustered in spatial frequency) exceeds two.
This approach has several interesting features:

◇ it works locally on sector of limited width. This is interesting because it diminishes the computational burden and somehow avoids the presence of local minima.
◇ it includes a detection scheme and estimates the number of sources present together with all the characteristics of the sources : range, bearing and power.
◇ though there is some loss of information when going from the complete data to the focused beam outputs, it has good properties since it is based on the statistical properties of the focused beams.

To further increase the performances and improve the conditioning of the optimization problem one should probably evaluate some further focused beams along the range axis in each spatial sector and modify the criterion accordingly. This possibility has not been investigated.

One should also note the source motion is more perceptible for sources in the near-field : for a source with a given speed the variation in spatial frequency is much more important for a source in the near-field than for a source in the far-field. The time-span over which a source in the near-field can be considered as immobile is thus much shorter and this reduction in integration time in general not compensated for by the increase in SNR. The source-localization problem in thus intrinsically more difficult in the near-field. Including the motion as a additionnal parameter may help but further increases the (computational) complexity.

Table 1: Six scenarios. Two near-field sources with different power, N=31, $T = 100$, $r_m = 124$, 20 trials

dB_1 / dB_2	θ_1 / θ_2	ν_1 / ν_2	$\hat{\nu}_1$ / $\hat{\nu}_2$	var($\hat{\nu}_1$) / var($\hat{\nu}_2$)	cr(ν_1) / cr(ν_2)	ρ_1 / ρ_2	$\hat{\rho}_1$ / $\hat{\rho}_2$	var($\hat{\rho}_1$) / var($\hat{\rho}_2$)	cr(ρ_1) / cr(ρ_2)	a_1 / a_2	$\hat{a}_1$ / $\hat{a}_2$	var($\hat{a}_1$) / var($\hat{a}_2$)	cr(a_1) / cr(a_2)	1	2	3
0	5	.0436	.0435	.00029	.00025	140	137	8.67	3.01	1	.97	0.055	0.019		100	
-5	9	.0782	.0782	.00095	.00047	170	165	22.99	8.47	.316	.317	0.038	0.0349			
0	5	.0436	.0506	.0166	.00023	140	133	18.54	2.64	1	.77	0.321	0.103		95	5
-5	9.5	.0825	.0826	.00023	.0004	175	173	18.01	8.76	1	.96	0.107	0.1033			
0	5	.0435	.0505	.0167	.0002	140	131	23.53	2.64	1.	.77	0.321	0.1033	10	60	30
-10	10	.0868	.0799	.0169	.0008	180	195	98.61	16.85	0.1	.24	0.320	0.0134			
0	5	.0435	.0435	.0002	.0002	140	138	7.60	2.48	1	.95	0.093	0.10		90	10
-10	10.5	.0911	.0933	.0087	.0008	185	202	71.64	16.39	0.1	.094	0.028	0.013			
-5	5	.0435	.0436	.0006	.0004	140	132	19.25	4.23	.316	.326	0.046	0.004		75	25
-10	12	.103	.1043	.0008	.0008	200	178	54.46	17.38	0.1	.105	0.015	0.008			
-5	5	.043	.0436	.0003	.00043	140	138	22.71	4.28	.316	.309	0.020	0.0348	5	80	15
-10	12.5	.1082	.1081	.0009	.0008	210	217	38.34	19.52	0.10	.106	0.019	0.0132			

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