PRELIMINARY COMPARISON BETWEEN TWO SPECTRAL ARRAY PRE-PROCESSORS FOR WIDEBAND BEAMFORMING

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ABSTRACT

Wideband beamformers based on cross-spectral matrices between array elements are examined. Two methods to estimate such matrices are described, a standard one, the Transform-and-Correlate (TC) pre-processor, and a novel one, indicated as Correlate-and-Transform (CT) pre-processor, which should be considered on an equal base as the former. Applications of CT are outlined in the estimation of correlated multipath, in acoustic Doppler current profiling, in interferometric seabed profiling and in the synthetic aperture sonar.

INTRODUCTION

The purpose of beamforming is to estimate the spatial parameters, such as the Directions Of Arrival (DOA) of the sources. Relative performances of two different beamformers are measured as follows:

- for a required performance in estimation accuracy, the better beamformer is the one which requires a shorter duration T of the data observation interval, and/or a smaller signal-to-noise ratio (SNR);
- for the same values of SNR and T the better beamformer is the one which achieves more accurate estimates.

The paper considers two pre-processors (Fig. 1.a,.b) in which:

- by processing the array output relative to a data observation interval, frequency bins are formed from a wide band;
- Cross-spectral Matrices (CM_i) among array elements are estimated for the bins, by two different algorithms.

After pre-processing, the CM_i matrices can be combined, coherently or not [1], to obtain a global beamformer. Some types of beamformers are Bartlett, Capon, MUSIC. For example, in MUSIC a global incoherent beamformer is ([2], p. 1510):

$$\underset{p}{\mathrm{MAX}}\left(\sum_i U_{i,p}^{+} N_i U_{i,p}\right)^{-1} \qquad (1)$$

$U_{i,p}$ is the steering vector relative to the i-th bin, which is the normalised complex array output for a hypothetical CW source of frequency f_i and spatial parameter p. In general, the current value p is not coincident with any of the unknown k-th source parameter p_k to be estimated. The value that the steering vector assumes in correspondence to p_k will be indicated as the propagation vector of the k-th source. The quantity N_i is the noise subspace matrix, derived by eigenanalysis of CM_i.

As the steering vectors are computed in correspondence of CW sources, employing the CM_i matrices for beamforming is fully motivated only when these have the same structures which occur in the case of CW sources [1].

The spectral method is considered an instrumental, intermediate step for the task of estimating the spatial parameters of the sources. The number of independent bins in a wideband depends on the structure of the pre-processor. In general, a loss occurs when recombining the bin outputs in a global beamformer [1]. By decreasing the number of bins, the "threshold" value of SNR in a bin (the value under which the estimation performance degrades rapidly) decreases, and the re-combination loss decreases too. The better performances are likely to be achieved by the pre-processor with the minor number of bins.

In sect. 1 the two pre-processors are examined for infinite duration of the observation interval. In sect. 2 the implementation of the two pre-processors for finite duration interval is shown and performances are preliminarily investigated. In sect. 3 some applications of the CT pre-processor are outlined.

LIST OF SYMBOLS

c	propagation speed
D	diameter of the array
f_0	centre frequency of the wideband signal
f_i	centre frequency of the i-th frequency bin
DOA	Direction Of Arrival
TOA	Time Of Arrival to an array element
$T_p = D/c$	time of wave propagation along the array
B	bandwidth of the wideband signal
B_b	bandwidth of the frequency bin
$q = B\,T_p$	decorrelation index
T	duration of the data observation interval
$y_i(t)$	complex envelope of the i-th array element
$+$	indicates conjugate transposition
CM_i	Cross-spectral Matrix for frequency bin f_i
$R_{CT}(\tau)$	cross-correlation matrix of CT at lag τ
$R_{CTij}(\tau) = E[y_i(t)y_j^{+}(t-\tau)]$	element i,j of $R_{CT}(\tau)$
$R_{TCi}(\tau)$	cross-correlation matrix of TC for bin f_i
$S_{CT}(z)$	Fourier Transform of $R_{CT}(\tau)$; $z = \exp(-j2\pi f)$
$S_{TCi} = R_{TCi}(0)$	cross-spectral matrix of TC for bin f_i
$S_{CTi} = S_{CT}(z_i)$	cross-spectral matrix of CT for bin f_i

1 OBSERVATION INTERVAL OF INFINITE DURATION

Assuming the ergodic hypothesis, the ensemble averages indicated in this section coincide with time-averages.

1.1 The Correlate-And-Transform Pre-Processor

The cross-spectral matrix estimated by CT is $S_{CT}(z)$, which is the Fourier Transform (FT) of $R_{CT}(\tau)$, the cross correlation matrix among the array elements at lag τ. Although a sampled data version of $S_{CT}(z)$ was introduced first in ([1], pp. 1513) it does not seem that the matrix properties which are illustrated by the following example have been fully recognised.

A planar wavefront associated with the broadband source random signal $x(t)$ impinges on a two-element array. The following quantities are defined:

$r(\tau) = \langle x(t)x^{+}(t-\tau)\rangle$	:autocorrelation of the source signal
$g(f) =$ FT of $r(\tau)$	:spectral power density of source signal
D	:distance between array elements
θ_0	:source DOA measured from broadside
$T_0 = D\sin(\theta_0)/c$	:difference in TOAs to array elements.

Neglecting delays common to both elements, the outputs are:

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$$y_1(t) = x(t) \quad ; \quad y_2(t) = x(t - T_0) \tag{2}$$

Assuming stationary signals in the observation interval, the cross correlation matrix among array elements is:

$$R_{CT}(\tau) = \begin{pmatrix} r(\tau) & r(\tau + T_0) \\ r(\tau - T_0) & r(\tau) \end{pmatrix} \tag{3}$$

After FT we have for a generic value f of the frequency

$$S_{CT}(z) = g(f)\begin{pmatrix} 1 & z^{T_0} \\ z^{-T_0} & 1 \end{pmatrix} = g(f)\begin{pmatrix} 1 \\ z^{-T_0} \end{pmatrix}\begin{pmatrix} 1 & z^{T_0} \end{pmatrix} \tag{4}$$

In deriving eq. (4) from (3), the theorem of FT has been applied, for which time delaying $r(\tau)$ by t implies multiplying the FT by z^{-t}. The rightmost member of eq. (4) indicates that $S_{CT}(z)$ has rank one, since it is given by the outer product of the unit-length vector $(1, z^{T_0})^+$ by itself. The vector is equal to the propagation vector of a CW source of frequency f.
In the case of P wideband mutually uncorrelated point sources, in the absence of noise, $S_{CT}(z)$ is the weighted sum of P outer products between unit-length propagation vectors, the weights being the source spectral power densities. In fact, for each source, the difference between TOAs relative to the array elements (ij), which appears in the argument of the scalar cross-correlation term $R_{CTij}(\tau)$ gives rise, after FT, to a product of three terms: two terms (phasors) have purely imaginary exponents which are proportional, but with opposite signs, to the i-th and to the j-th TOA, respectively; the third term, which is the source spectral power density, is the same for all the elements of $S_{CT}(z)$. The product of the two phasors is an element of the outer product of the source propagation vector by itself, then for each source a unit rank contribution to $S_{CT}(z)$ arises. This feature holds for any wavefront shape.
To illustrate the case of fully correlated multipath, consider two wavefronts due to the same source with DOAs (θ_1, θ_2). Let T_r be the delay between the wavefronts. We have:

$$\begin{aligned} y_1(t) &= x(t) + Ax(t - T_r) \qquad & T_1 &= (L/c)\sin\theta_1 \\ y_2(t) &= x(t - T_1) + Ax(t - T_r - T_2) \qquad & T_2 &= (L/c)\sin\theta_2 \end{aligned} \tag{5}$$

The quantity A is a complex number, which depends on the modality of propagation. It is easily proved that $S_{CT}(z)$ has rank one, with propagation vector $[\,1 + A^+ z^{T_r},\ z^{T_1} + A^+ z^{T_r + T_2}]^+$.
In conclusion, the $S_{CT}(z)$ matrix achieves the outer product structure required for beamforming exactly, for any value of source bandwidth. Hence CT would be able, in an observation interval of infinite duration, to solve the problem of decorrelation along the array arising from wideband signals.

1.2 The Transform-And-Correlate Pre-Processor

Figure 1.a shows the Transform-and-Correlate pre-processor for estimating the cross-spectral matrix for the i-th bin, indicated by S_{TCi}. The observation interval is partitioned in nonoverlapping subintervals, so that, after FT in each subinterval, the condition $B_b T_p << 1$ holds and the bin vector data have quasi-CW structure; in a sub-interval one outer vector product is computed for each bin, then time uncorrelated vector products, derived by distinct subintervals, are averaged bin by bin to obtain statistical stable estimates of S_{TCi}. With reference to eq. (3) we have for the two-element array for frequency bin f_i:

$$S_{TCi} = R_{TCi}(0) = \begin{pmatrix} r_i(0) & |r_i(-T_0)| z_i^{T_0} \\ |r_i(T_0)| z_i^{-T_0} & r_i(0) \end{pmatrix} \quad ; \quad |r_i(T_0)| = |r_i(-T_0)| \tag{6}$$

where $r_i(\tau)$ is the scalar autocorrelation function at the output of an array element. As for non-CW signals $|r_i(T_0)| < r_i(0)$, the S_{TCi} matrix is not proportional to the outer product of the source propagation vector by itself as in the S_{TC} matrix (eq. (4)). Because a steering vector has a CW structure, employing S_{TCi} for beamforming is fully motivated only when this has the "right" outer product structure, which occurs for CW signals but not wideband signals. The S_{TCi} matrix would achieve the required structure only asymptotically for $B_i \to 0$, as in this case $|r_i(-T_0)| \to r_i(0)$. Hence, in TC the problem of decorrelation along the array due to wideband signals is attenuated by dividing the bandwidth in frequency bins, but not solved as in CT.

1.3 Comparison of the Pre-processors

A question arises: assumed that the main requirement for beamforming is the estimation of the source spatial parameters and not spectral estimation, why is Fourier Transform employed in both pre-processors? An answer is that, as beamformers valid for CW signals are available, it is convenient to try to extend the methods valid for CW signals to wideband. A requirement for the extension is that the CM_i have the proper structure which is a weighted sum of outer products between propagation vectors.
The technique adopted in TC to fulfil the requirement employs two steps: the first is obtaining quasi-CW data by dividing the wideband in frequency bins, the second is estimating the bin matrices. As a consequence of the first step, which is not a requirement from beamforming but it is instrumental to the second step, the matrices quasi achieve the required structure. The TC processing criterion is sufficient but not necessary in order that the bin cross-spectral matrices estimated by processing the array data (quasi) achieve the required structure. In fact, a requirement arises from spectral beamforming only for the cross-spectral matrix structures, but not for the structures of the bin data after FT.
The CT technique consists in looking for a functional transform of $R_{CT}(\tau)$ able to generate bin matrices with the proper outer product structures, and this functional transform is a FT. In fact in the case of wideband and for a generic source-array scenario *only* the matrix $S_{CT}(z)$ and not $R_{CT}(\tau)$ has the required outer product structure. In CT it is not necessary to obtain CW or quasi-CW data as an intermediate step for beamforming.

2 FINITE OBSERVATION INTERVAL

The discrete implementation of the two pre-processors (Fig. 1.a,b), shows that the two matrix estimates perform the same operations, but not in the same sequence. The operations are:
- estimating the cross-correlation terms: computing elementary outer products and time averaging of the products in an interval of duration T to obtain stable matrix estimates,
- Fourier Transform.

Although the operations are the same, the pre-processors are not identical, as shown by eqs. (4), (6): an outer product is a non-linear operation, so changing the sequence changes the result.
The TC and CT pre-processors are generalisations to arrays of two scalar estimates of the spectral power density of a stochastic process: a) the Periodogram, and b) the Blackman-Tukey spectral estimator ([3], sect. 14.2). In most of the applications of wideband spectral beamformers found in technical literature, the pre-processor employed is TC. Preliminary analysis indicates that the number of bins in CT is smaller than in TC. Simulation indicates that the MUSIC beamformer based on CT has significantly better performance than the Frost beamformer [4] in the task of discriminating the DOAs of a weak and a strong source, and in the minimal operational signal-to-noise ratio.

2.1 Transform-and-correlate pre-processor

A requisite that determines partitioning of the finite interval in subintervals is indicated below for the two-element array: if a short sub-interval duration is taken, averaging over the subintervals gives S_{TCi} matrix estimates with small statistical fluctuations; but the ratio $|r_i'(T_0)/r_i'(0)|$ between the matrix element *estimates* (the symbol ' indicates average value in a finite interval) would be significantly smaller than one, so the deviation of the matrix from a unit rank matrix would be significant (see eq. (6)). By taking a long sub-interval duration the ratio of the expected values would be closer to one but the statistical fluctuation due to the small number of sub-intervals available for averaging would give rise again to a deviation, of statistical nature, between the estimated correlation matrix and a unit rank matrix. Hence a compromise choice for the sub-interval duration is required. The bin bandwidth B_b is given by:

$$B_b = B_b T_p \cdot \frac{1}{T_p B} \cdot B = \frac{B}{a_p q} \quad ; \qquad a_p \equiv \frac{1}{B_b T_p} > 1 \tag{7}$$

Note that $B_b T_p < 1$, as indicated at the beginning of sect. 1.2

2.2 Correlate-and-transform pre-processor

The processing [2] extends the method valid for the scalar case ([3], ch. 14.2). Two vector data samples, one sample being taken with a lag τ with respect to the other, are (outer) multiplied, then an average of the uncorrelated outer products contained in the observation interval is performed to obtain a $R_{CT}(\tau)$ estimate. The latter is multiplied by a scalar triangular tapering function $w(\tau)$ which is one at lag zero and zero for $|\tau| \geq \tau_0$; the lag τ_0 is chosen so that for $|\tau| > \tau_0$ the *actual* value of $R_{CT}(\tau)$ is negligible. The $S_{CT}(z)$ estimate, derived by applying FT to the estimate of $(R(\tau)w(\tau))$ in the interval $(-\tau_0 \leq \tau \leq \tau_0)$, has the following characteristics:

- the standard deviation of the scalar spectral power density estimate divided by the value of the actual spectral power density is about $(2\tau_0/T)^{1/2}$ ([3] eq. (14.61)); a triangular tapering function is sufficient to guarantee a non-negative spectral power density estimate;
- the bin bandwidth is $1/\tau_0$. Setting to zero the correlation estimate for $|\tau| \geq \tau_0$ degrades the spectral resolution and introduces spectral bias, which are negligible for large τ_0.

Assuming that the spectral bias of the cross-spectral matrix estimate is not a problem for beamforming, the value of τ_0 should be chosen as indicated below:

- if τ_0 is too small, enough uncorrelated outer product samples can be averaged in the data interval to obtain a stable estimate of $R_{CT}(\tau)$. However the amplitude bias which affects this matrix is significant and especially the off-diagonal elements of $R_{CT}(\tau)$, the ones bearing information on the DOAs, are severely distorted. As a consequence it is likely that the $S_{CT}(z)$ estimate outer product structure, which is relevant to beamforming, is severely distorted too;
- if τ_0 is too large the amplitude bias of $R_{CT}(\tau)$ is negligible, but the number of uncorrelated outer products available in the interval for averaging is too small to obtain a stable estimate of $R_{CT}(\tau)$ and, consequently, of $S_{CT}(z)$.

So an optimal choice of the value τ_0 is likely to exist. At this stage of analysis, a simple criterion leading to an optimal value of τ_0 has been not found. Qualitatively, by assuming in the infinite interval two-element array example the worst case value for T_0, i.e., $T_0 = T_p$ in eq. (3), the off-diagonal element $r(\tau - T_p)$ becomes negligible when $(\tau - T_p) \geq a_0/B$. The number a_0 should be taken so large that the scalar autocorrelation function is negligible for $|\tau| \geq \tau_0 = T_p + a_0/B$. Thus it can be assumed for an N element array:

$$\tau_0 = \frac{a_0}{B} + T_p = \frac{a_0 + q}{B} \qquad (a_0 \geq 4) \tag{8}$$

By eq. (7) the ratio p between the frequency bin bandwidths of the CT and TC pre-processors is:

$$p = \frac{B_i(CT)}{B_i(TC)} = \frac{q a_p}{q + a_0} \tag{9}$$

when $q \gg a_0$ we have $p \approx a_p$.

3 APPLICATIONS OF THE CT PRE-PROCESSOR

3.1 Simultaneous Estimation of the DOAs and Relative TOAs between Correlated Emissions

The comments to eq. (5) indicate that by processing the $S_{CT}(z)$ eigenvectors of a N-element array (N>2) the estimates of T_1, T_2, T_r can be obtained. Phase ambiguities might be minimised by processing simultaneously the $S_{CT}(z)$ of all the bins. Referring to the two-element example, and assuming $BT \gg 1$, we have for the required data observation interval duration T:

- TC case: for the two wavefronts to appear correlated in a bin, its bandwidth must be chosen so that $B_b T_r < 1$; a number M of subintervals for averaging are required to achieve stable matrix estimates in a bin, so T cannot be less than MT_r;
- CT case: to get a rough indication of T consider only the first of eq. (5) , with A=1, and assume that the aim is obtaining a stable estimate of $r(\tau)$ at lag T_r, where a secondary peak of the autocorrelation occurs. As the elementary products between pairs of samples of the element outputs spaced T_r seconds, which are averaged to estimate $r(T_r)$ are almost uncorrelated for a sampling interval $1/B$, the interval duration is of the order of $(T_r + M/B)$, which is significantly shorter than that required by TC.

We will consider now continuous extended targets, modelled as spatial random processes, in the cases of backscattering from the transmission medium (as in an acoustic Doppler current profiler [5] using an array antenna) or from the sea bottom (as in an interferometric profiler [6] or in a synthetic aperture sonar).

3.2 Array Acoustic Doppler Current Profiler (ADCP)

The transmitted waveform is composed of a number N_t of identical wideband time-adjacent sub-pulses, each of duration T; adjacent sub-pulses are separated by a short transmission gap of duration τ_0. A number of array elements N=3 is assumed. A linear evenly spaced array is examined, for a 'target' which is the backscattering volume defined by the range and azimuth cells.

V: target radial velocity to be estimated; $d \equiv 1+2V/c$
L: distance between adjacent sensors
θ_1: Direction of Arrival of the source
$T_1 = L\sin\theta_1/c$: TOA difference between adjacent array elements
T_0: interval between transmitted sub-pulses
T: sub-pulse duration; we take $T_0 = T + \tau_0$ (eq. (8))
B: sub-pulse bandwidth; we assume $BT \gg 1$.

By taking N_t=2, we have:

$$y_1(t) = x(t); y_2(t) = x(t - T_1); y_3(t) = x(t - 2T_1)$$
$$y_4(t) = y_1(t - T_0); y_5(t) = y_2(t - T_0); y_6(t) = y_3(t - T_0) \tag{10}$$

The observed vector has dimension $(N \cdot N_t)$. The $S_{TC}(z)$ has dimensions $(N \cdot N_t\ ,\ N \cdot N_t)$. The observation interval is assumed equal to $N_t T$. The target is not point-like as in sect.2, but has a finite DOA distribution: it is conjectured that the DOA distribution affects mainly the higher order eigenvectors (those corresponding to smaller eigenvalues). The rank of the ensemble average of $S_{TC}(z)$ is 'nearly' one, and the propagation vector is

$$(\ 1,\ z^{dT_1},\ z^{2dT_1},\ z^{dT_0},\ z^{d(T_0+T_1)},\ z^{d(T_0+2T_1)}\ )^+. \tag{11}$$

In the presence of a turbulence-distorted wavefront the first eigenvector of $S_{TC}(z)$ bears the imprinting of the wavefront shape and of the Doppler shift; by processing the eigenvector the shape and the shift can be estimated. The refocused spectral power is contained in the first eigenvalue. When only one array element is present, the processing reduces to the pulse-pair processing for velocity estimation [5].

The same considerations apply to the aerial counterpart of the ADCP, which is the SODAR

3.3 The Interferometric Sea Bottom Profiler

A high-BT pulse is transmitted. The range and DOA estimates are derived, respectively, by the time elapsed between transmitted and received signals, and by exploiting the differences in echo TOAs to the array elements. A small value of T is required because the DOA of the bottom echo varies rapidly during a receiving cycle.
It is assumed that the required range resolution r, i.e. the range cell, is that corresponding to pulse duration T, which is $r = cT/2$. After pulse compression, employed to increase the SNR, since the time interval between uncorrelated time samples is still $1/B$, the uncorrelated array vector data comprised in an interval of duration T, whose number is BT, are processed to estimate $R_{CT}(\tau)$, from which the cross-spectral matrices $S_{CT}(z_i)$ are estimated; then they are employed to obtain a global beamformer (fig. 1.b) for the range cell. The range resolution equals r.
By employing a high-BT transmitted pulse, followed by the averaging of the uncorrelated outer products comprised in the interval T, reduction of the angular scintillation (glint) of the DOA estimate is likely to be achieved.
The DOA estimation is achieved here by processing the array element outputs not in pair-wise mode (as it is customary in most of today's systems) but globally, via the CT pre-processor. The first eigenvector, for each range resolution cell, can provide an accurate DOA estimate. Example: r=2m, wavelength=0.015m, B/f_0=0.4, BT=100.

3.4 Synthetic Aperture Sonar

The processing is the same as in sect. 3.3, but here the processed samples are the ones relative to the along-track synthetic antenna, while in sect. 3.3 the samples are the element outputs of the real array for across-track profiling. For each rangexfrequency bin the components of the first eigenvector have a phase pattern containing, beyond the usual quadratic part associated to range migration, a residual part due to uncompensated platform motion. It is conjectured that:
- by processing simultaneously the eigenvector components phase structures of all the frequency bins pertaining to the same range bin, the residual uncompensated platform motion might be, at least in part, estimated (the phase histories can be unwrapped) and compensated before along-track beamforming. By doing so the requirements on platform attitude control might be released;
- the problems of target decorrelation due to the changing of the aspect angle as seen by the platform might be attenuated by first-eigenvector-based beamforming.

4 CONCLUSIONS

Between the pre-processors for estimating the cross-spectral matrices in wideband spectral beamforming, the TC pre-processor is more widespread than CT. Although for infinite observation interval the two pre-processor outputs are nearly the same, their performance indexes may differ, as the operation sequence, not the same for the two, and averaging on a finite interval may give rise to distinct statistics. Preliminary analysis indicates promising performance for CT, so the pre-processors should deserve an extensive comparative evaluation, and should be considered on equal ground for possible applications.

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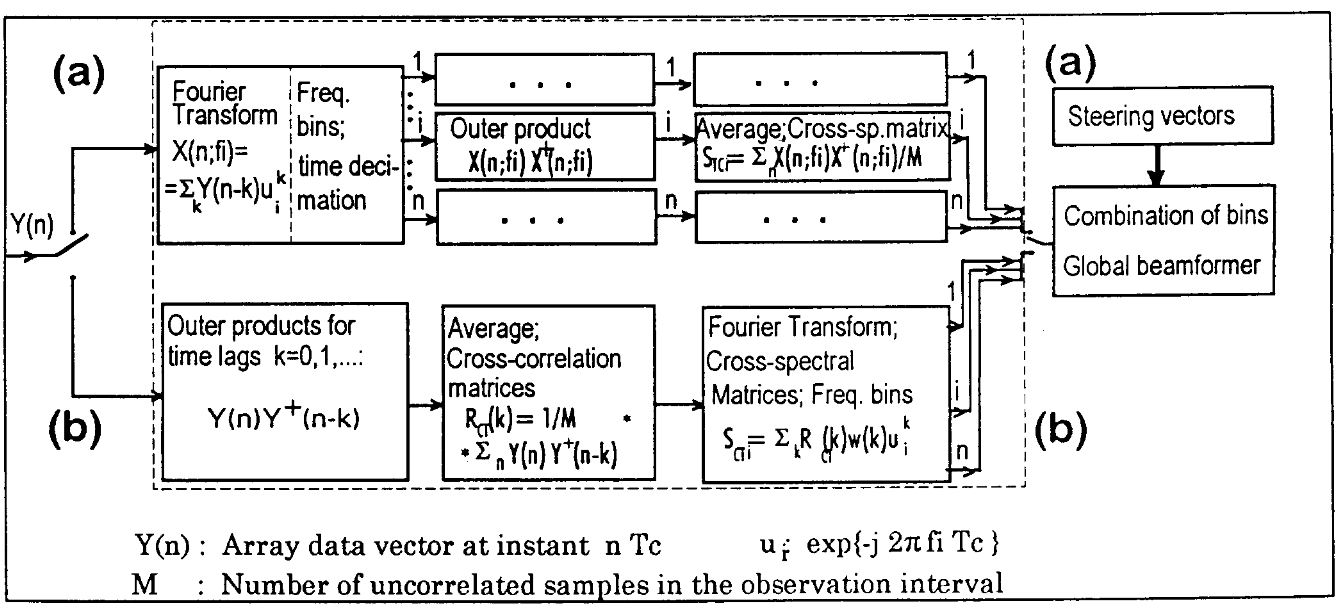

Fig. 1. Estimating cross-spectral matrices from array data: (a) Transform-and-Correlate and (b) Correlate-and-Transform pre-processors.