AN ALGORITHM FOR PREWHITENING A LARGE PARALLEL LINE ARRAY

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ABSTRACT

A steepest descent gradient algorithm prewhitens the signal received by a uniform planar array. Previously developed methods work only on single line arrays. A novel model facilitates algorithm development by reducing problem dimensionality associated with exact multi-dimensional autoregressive (AR) modeling. The discrete source model, based on a Kronecker product of the received signals between the vertical and horizontal elements of the array, agrees exactly with the classical sinusoidal model. The colored noise source Kronecker product model agrees approximately with a physical geometric one constructed from spherical surface harmonics. The algorithm uses a stacked vector parameterization of the vertical and horizontal AR parameters and optimizes them over a low order whiteness functional. Application of the algorithm with MUSIC demonstrates enhanced performance in terms of angular resolution and detection of low SNR sources. The algorithm allows extensibility and solves the general problem of the three-dimensional volumetric array with arbitrary geometry.

1. INTRODUCTION

The direction finding methods developed so far [1,2,3] require that the additive noise be spatially white (uncorrelated between sensors) or that the noise correlation matrix be known to within a constant scale factor [4]. Experimental and theoretical studies [5, Chapter 10] indicate that non-isotropic additive ambient sensor noise exists in the ocean. Ambient noise exhibits a frequency-dependent, vertical directionality: at low frequencies, distant shipping causes an increased noise about the horizontal; at high frequencies, wave action imparts an increased noise about the vertical [6]. A mixed spectral acoustic environment exists, consisting of discrete frequency sources (tonals) and continuous frequency sources (colored noise). Direction finders exhibit high false alarm rates in the presence of spatially colored ambient noise [7]. Standard methods applied in practice exhibit serious degradations in terms of bias, angular resolution, spurious peaks, nondetection of weak sources and estimate of the number of sources [4].

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A prewhitening filter preprocesses the correlation matrix of the observed signals to remove the effects of colored noise. Prewhitening filters rely on explicit parameterizations of the noise correlation matrix. Once estimated, these parameters construct a filter to preprocess the observed data. After whitening, conventional direction finders obtain source direction of arrival angles with minimal degradation.

A uniform planar or multiple parallel line array represents the principal geometry studied in this paper, as depicted in figure 1. Narrowband signals exist in an azimuthally uniform noise field sufficiently far from the array ($r \gg (N/M-1)d$) to allow a planar wavefront approximation. The N sensor array consists of M lines with N/M sensors per line, inter-sensor spacing d and inter-line spacing s, where, in general, $d \neq s$. A known number of narrowband stationary zero-mean sources propagate in an underwater medium with speed c centered at frequency ω and wavelength λ such that $\omega = 2\pi c/\lambda$ and $d = \lambda/2$. A stationary zero-mean random process correlated from sensor to sensor with an arbitrary and unknown covariance models the additive noise present. The array receives a signal from a far-field source, arriving from the direction-of-arrival angle (θ, ϕ) Discrete spatial sinusoids and continuous spatial noise

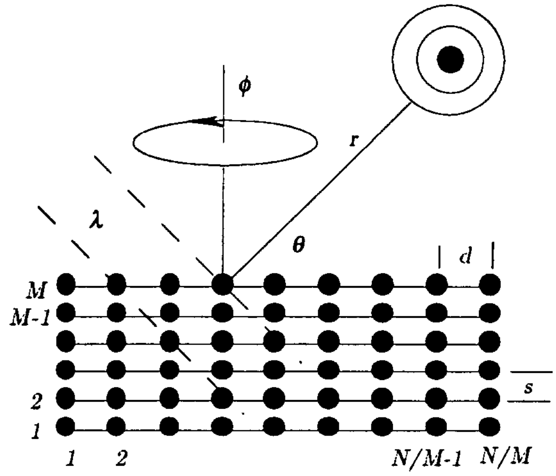

Figure 1: A Large Multiple Parallel Line Sensor Array

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components (colored noise) compose the signal.

This research exploits the structure of the noise correlation matrix to allow a simple parameterization amenable to a vector parameter gradient algorithm. Future work addresses the general problem of the three-dimensional volumetric array with arbitrary geometry by uniformly sampling the acoustic field via array interpolation [8]. Section 2 discusses underwater noises and the mathematical models describing their correlation properties. Section 3 outlines the classical and Kronecker product received signal models. Section 4 formulates the prewhitening algorithm. Section 5 lists several useful properties of the Kronecker product. Section 6 employs the MUSIC [2] algorithm to demonstrate the prewhitening algorithm's performance. Finally, section 7 explains the simulation results and suggests future areas of research.

2. SOURCES OF UNDERWATER ACOUSTIC NOISE

Ambient noise due to natural phenomena and human-causes exists in the ocean. Two excellent references by Burdic [5, pages 297–302] and Wenz [9] describe common sources of acoustic ambient noise in the ocean. Natural physical sources of ambient noise include thermal agitation and hydrodynamic sources such as bubbles and surface waves caused by wind. The sound generated by distant oceanic traffic in shipping lanes constitutes the principal human contribution. Biological sources due to fish and ocean mammals also impact the ambient noise levels.

The Cox [6] model computes the correlation coefficients for both surface wave generated noise and distant shipping noise. This model describes these coefficients for any sensor array of arbitrary known geometry. When employing a two or three-dimensional uniform array, the model generates noise correlation matrices with block-Toeplitz structure. Cox defines a normalized directional density function for azimuthally uniform (uniform across ϕ) fields $F(\theta,\omega)$ and expands it into spherical harmonics

$$F(\theta,\omega)=\sum_{l=0}^{\infty}c_l(\omega)P_l(\cos\theta) \tag{1}$$

where $c_l(\cdot)$ represents the coefficient for the l-th term in the expansion, $P_l(\cdot)$ the Legendre polynomial of the first kind. Cox provides the coefficients, derived by curve-fitting to ocean acoustic data. The correlation $\mathbf{Q}_{mn}(d,\omega,\gamma)$ (normalized element of noise correlation matrix) between two sensors m and n separated by distance d with angle relative to the vertical γ at frequency ω equals

$$\mathbf{Q}_{mn}=\sum_{l=0}^{\infty}i^l c_l(\omega)P_l(\cos\gamma)j_l(\omega d/c) \tag{2}$$

where $j_l(\cdot)$ represents the l-th order spherical Bessel function of the first kind and i represents a complex number.

The multi-dimensional AR noise model considered in this paper appears to best fit [8] the surface noise correlation model suggested by Cox [6]. Similar models fit the correlation properties of other noise sources.

3. RECEIVED SIGNAL MODEL

Each i-th sinusoid in the acoustic field arrives from arrival angle (θ_i,ϕ_i) and has relative power σ_i. Define the two-dimensional steering vector for each signal as

$$\mathbf{d}_i^T(\theta_i,\phi_i)=\left[\begin{array}{cccc} e^{j\mathbf{k}_i^T\mathbf{p}_1} & e^{j\mathbf{k}_i^T\mathbf{p}_2} & \ldots & e^{j\mathbf{k}_i^T\mathbf{p}_N}\end{array}\right] \tag{3}$$

where the wavevector $\mathbf{k}_i=(\omega/c)\mathbf{u}_i$, the position vector of the l-th element $\mathbf{p}_l^T=[x\ y\ z]$ for $l=1,2,\ldots,N$ and the unit vector $\mathbf{u}_i^T=\left[\begin{array}{ccc}\sin\theta_i\cos\phi_i & \sin\theta_i\sin\phi_i & \cos\theta_i\end{array}\right]$.

For q sources, the discrete source model

$$\mathbf{P}=\sum_{i=1}^{q}\sigma_i\left(\mathbf{P}_{\uparrow,i}\otimes\mathbf{P}_{\rightarrow,i}\right) \tag{4}$$

where $\otimes$ represents the Kronecker product and

$$\mathbf{P}_{\uparrow,i}=\mathbf{d}_{\uparrow,i}(\theta_i,\phi_i)\,\mathbf{d}_{\uparrow,i}^H(\theta_i,\phi_i) \tag{5}$$

$$\mathbf{P}_{\rightarrow,i}=\mathbf{d}_{\rightarrow,i}(\theta_i,\phi_i)\,\mathbf{d}_{\rightarrow,i}^H(\theta_i,\phi_i) \tag{6}$$

where the vertical and horizontal steering vectors $\mathbf{d}_{\uparrow,i}$ and $\mathbf{d}_{\rightarrow,i}$ along the first row and column of sensors in the array come from the classical model. The Kronecker product model equals the classical model exactly for the case of sinusoidal signals. The principle of pattern multiplication [10, page 39] validates the use of this model.

For the classical formulation, each steering vector represents an column of the $N\times q$ steering matrix $\mathbf{A}$. The total signal correlation matrix $\mathbf{P}=\mathbf{ASA}^H$ where $\mathbf{S}=\mathrm{diag}\left(\begin{array}{cccc}\sigma_1 & \sigma_2 & \cdots & \sigma_q\end{array}\right)$.

Single dimensional AR processes effectively model ambient noise for the uniform line array case [4,7]. A multi-dimensional AR process [11, Chapter 15] concisely models an isotropic noise field when sampled uniformly by a planar grid of sensors. For simulation comparisons, the multi-dimensional Levinson algorithm [11, pages 462–3] computes the AR coefficients from the Cox generated correlations in a computationally efficient way. Such a parameterization, while accurate, when employed in a prewhitening filter suffers from high dimensionality. The feasibility of estimating $O(N)$ AR coefficients in a $M\times N/M$ array lacks practicality.

Writing a prewhitening algorithm for optimizing a whiteness functional requires an amenable parameterization for the inverse noise correlation matrix. Such a model should agree, at least approximately, with the physical noise model, provide a block Toeplitz matrix structure and reduce problem dimensionality far below $O(N)$. The noise correlation matrix

$$\mathbf{Q}=\mathbf{Q}_\uparrow\otimes\mathbf{Q}_\rightarrow \tag{7}$$

where the $M\times M$ and $N/M\times N/M$ dimensional Toeplitz matrices $\mathbf{Q}_\uparrow$ and $\mathbf{Q}_\rightarrow$ contain common power σ^2. The vector autoregressive coefficients $\mathbf{a}_\uparrow$ and $\mathbf{a}_\rightarrow$ parameterize $\mathbf{Q}_\uparrow$ and $\mathbf{Q}_\rightarrow$ via the well-known Gohberg-Semencul formula [12]. For a general Toeplitz correlation matrix the formula states that

$$\mathbf{Q}^{-1}=\frac{1}{\sigma^2}\left[\mathbf{A}_1\mathbf{A}_1^H-\mathbf{A}_3\mathbf{A}_3^H\right]. \tag{8}$$

The article by Cernuschi-Frias [12] contains concise definitions of the pertinent quantities. This parameterization reduces problem complexity to order $O(M+N/M)$.

Since the surface noise model [6] generates purely real correlations, real AR coefficients provide the necessary parameterization. This model produces a noise field broadside to the array, centered at $\theta = 90^\circ$ in bearing. Cox's model for distant shipping or noise sources steered off of broadside produce complex correlation coefficients, necessitating a complex AR parameterization [7][13, pages 184–6].

4. STACKED PARAMETER GRADIENT PREWHITENING ALGORITHM

The algorithm estimates the noise correlation matrix $\mathbf{Q}$ in the manner of LeCadre [4] by optimizing a whiteness functional over a low order surface parameterized by

$$\mathbf{A}_k^T = \left[\begin{array}{cccccc} \sigma_k^2 & a_{\uparrow,1}^k & \cdots & a_{\uparrow,p_\uparrow}^k & a_{\rightarrow,1}^k & \cdots & a_{\rightarrow,p_\rightarrow}^k \end{array} \right] \tag{9}$$

at iteration k and $p_\uparrow$ and $p_\rightarrow$ denote the respective lengths of the AR processes. The gradient parameter update equation

$$\mathbf{A}_{k+1} = \mathbf{A}_k - \rho \mathbf{G}_k \tag{10}$$

requires the computation of the i-th element of the gradient, where a_i^k represents the i-th element of $\mathbf{A}_k$,

$$\mathbf{G}_k(i) = \frac{\partial L}{\partial a_i^k} \tag{11}$$

for $i = 0, 1, \ldots, p_\uparrow + p_\rightarrow$. L represents the whiteness functional [4], ρ the algorithm step size. Figure 2 illustrates a typical functional surface contour for first order real parameterizations in the horizontal and the vertical. The figure depicts a low order modality of the optimization surface.

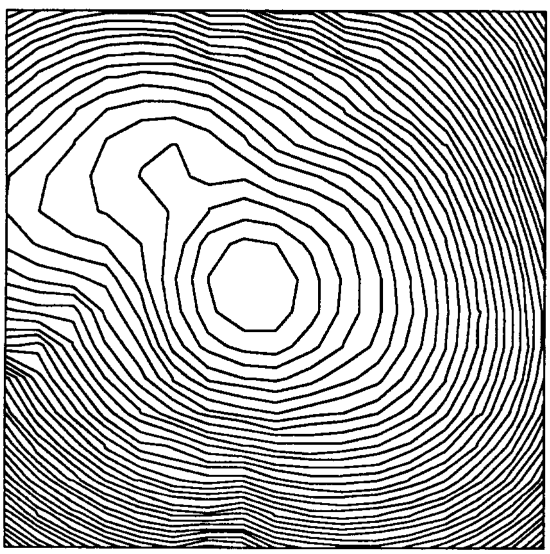

Figure 2: Typical Whiteness Functional Surface Contour

For algorithm implementation, use equations 9-15 in conjunction with the detailed procedure of LeCadre [4,7]. At each step in the iteration compute an appropriate step size via a line search over a range of ρ of the whiteness functional expressed as a function of the eigenvalues. The range expression effectively bounds the computed real or complex coefficients within the unit circle.

5. PROPERTIES OF THE KRONECKER PRODUCT

Several useful properties of the Kronecker product aid in algorithm development.

Definition 1 *Given the $M \times M$ matrix $\mathbf{A}$ and the $N/M \times N/M$ matrix $\mathbf{B}$ define the $N \times N$ Kronecker product matrix $\mathbf{C} = \mathbf{A} \otimes \mathbf{B}$ as*

$$\mathbf{C} = \begin{bmatrix} a_{11}\mathbf{B} & \cdots & a_{1M}\mathbf{B} \\ \vdots & \ddots & \vdots \\ a_{M1}\mathbf{B} & \cdots & a_{MM}\mathbf{B} \end{bmatrix} \tag{12}$$

where a_{ij} represents the (i,j)-th element of matrix A.

Lemma 1 *If $\mathbf{A}^{-1}$ and $\mathbf{B}^{-1}$ exist, then*

$$\mathbf{C}^{-1} = \mathbf{A}^{-1} \otimes \mathbf{B}^{-1} \tag{13}$$

Lemma 2 *If $\mathbf{Q}^{-1} = \mathbf{Q}_\uparrow^{-1} \otimes \mathbf{Q}_\rightarrow^{-1}$ and $\mathbf{a}_\uparrow$ and $\mathbf{a}_\rightarrow$ parameterize $\mathbf{Q}_\uparrow^{-1}$ and $\mathbf{Q}_\rightarrow^{-1}$ respectively*

$$\frac{\partial \mathbf{Q}^{-1}}{\partial a_{\uparrow,i}} = \frac{\partial \mathbf{Q}_\uparrow^{-1}}{\partial a_{\uparrow,i}} \otimes \mathbf{Q}_\rightarrow^{-1} \tag{14}$$

$$\frac{\partial \mathbf{Q}^{-1}}{\partial a_{\rightarrow,i}} = \mathbf{Q}_\uparrow^{-1} \otimes \frac{\partial \mathbf{Q}_\rightarrow^{-1}}{\partial a_{\rightarrow,i}} \tag{15}$$

for the i-th element.

6. SIMULATION RESULTS

A 3 line array of inter-line spacing $s = 0.75$ m with 8 sensors/line with inter-sensor spacing $d = 1.5$ m receives signals propagated from 2 sinusoidal sources immersed in a surface generated noise field of SNR 0 dB generated from the Cox noise model. The far-field sources of temporal frequency 500 Hz impinge on the array from arrival angles $(\theta = 70^\circ, \phi = 20^\circ)$ and $(\theta = 110^\circ, \phi = 60^\circ)$ and with SNR (-8.5 dB, -11.1 dB) through a medium with propagation speed $c = 1498$ m/s. Figure 3 represents a contour plot of the two-dimensional MUSIC [2] spectra before whitening, figure 4 after whitening. MUSIC and the whiteness functional use a signal subspace dimension of 2, the vertical and horizontal AR order equals 2. Figure 5 depicts the normalized eigenvalue spectra for the correlation matrix of the whitened (dashed) and non-whitened (solid) data.

7. CONCLUSIONS

Figure 4 illustrates enhanced performance in terms of angular resolution and detection of low SNR sources. The whitened response detects the source at $(\theta = 110^\circ, \phi = 60^\circ)$. In figure 5, the whitened eigenvalues display a flatter spectrum, as expected. The modulo-8 characteristic of the 3×8 array contributes to the choppy descent of the two plots. Further research on this topic studies the effects of source number underdetermination, applies the algorithm to an arbitrary three-dimensional volumetric array and employs a complex AR coefficient model.

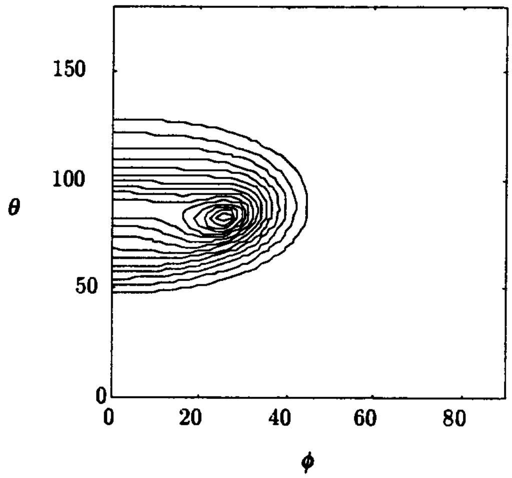

Figure 3: MUSIC Response without Whitening

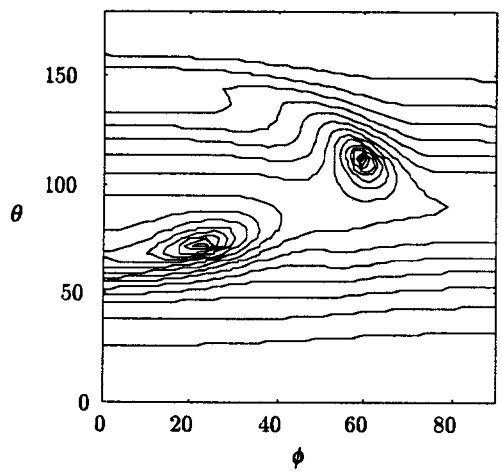

Figure 4: Whitened MUSIC Response

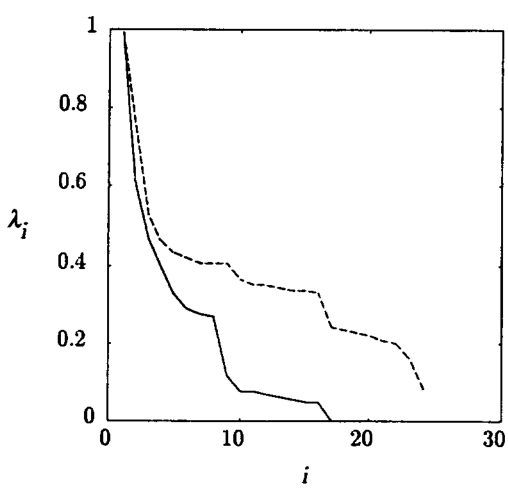

Figure 5: Normalized Eigenvalue Spectra

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