

# DECISION FEEDBACK NEURAL NETWORK COHERENT RECEIVERS FOR CONTINUOUS PHASE MODULATION BASED ON FREQUENCY DOMAIN

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## ABSTRACT

This paper presents a decision feedback neural network (NN) coherent receiver scheme for continuous phase modulation based on frequency domain. Through decision feedback pre-processing, the effect of the previous transmitted symbols can be removed for the present symbol decision. By employing Karhunen-Loève transform (KLT) or discrete cosine transform (DCT), the input data number of neural networks can be reduced significantly. To obtain more sufficient convergence of neural networks a modified "delta-bar-delta" BP learning algorithm is proposed. Despite of low complexity in NN training and implementation, computer simulation results show that our NN receivers can achieve near optimal demodulation performance.

## 1. INTRODUCTION

Continuous phase modulation (CPM) is an important class of nonlinear modulation schemes[1]. These schemes have two key features. One is their spectral economy, and the other is that a CPM signal has a constant envelope. Both these two are extremely important in satellite communication systems. To achieve optimal demodulation performance, the maximum-likelihood (ML) receivers require a bank of matched filters which need heavy computational complexity. In[2], Gustavo de Veciana and Avidesh Zakhori had developed a neural net-based continuous modulation receiver. Their motivation is to reduce the complexity of implementation by casting the demodulation task into the more general framework of a neural network classification task. However, their simulation results show a poor performance compared with optimal ML receivers. The decrease of the performance is not less than 3.5 db at the error probability  $P_e = 10^{-3}$ . From the works in [2], it can be confirmed that increasing the number of hidden nodes, the length of the

observation window, the sampling rates and training with noise can improve the performance. We think that the performance of a classifier depends not only on the classifier itself but also on the input signals. For a CPM signal, different input data selections may lead to different classification performance. For the present symbol decision, the observation signals lasting several symbol intervals also contain the information of the previous symbols and several later symbols. A simple decision feedback pre-processing discussed in section 2 can remove the part of the previous symbols. By doing so, the size of the signal sample set can be reduced and the signal patterns have better separability. Meanwhile, considering the excellent relativity of CPM signals with long observation windows and high sampling rates, KLT or DCT is employed to reduce the signal dimension in sections 3. From the point of pattern recognition, the feature extraction has the same importance as the classification. Also, in our work, we found that more sufficient convergence always lead to better demodulation performance and the convergence speed is important. So a modified "delta-bar-delta" BP learning algorithm, which has high convergence speed and good performance, is proposed in section 4.

## 2. DECISION FEEDBACK NEURAL NETWORK RECEIVER

A CPM signal can be expressed as

$$s(t) = \sqrt{\frac{2E}{T}} \cos(\omega_c t + 2\pi h \sum_i a_i q(t - iT) + \phi_0) \quad (1)$$

The data  $\{a_i\}$  are M-ary data symbols, M even, taken from the alphabet  $\pm 1, \pm 3, \dots, \pm(M-1)$ ; h is a modulation index; q(t) is the phase response function, with a corresponding frequency pulse lasting L symbol intervals; E is the energy per symbol;  $\phi_0$  is the phase offset and  $\omega_c$  is the carrier frequency. For coherent demodulation schemes,  $\phi_0$  can be set to zero.

In additive white Gaussian noise(AWGN) channel, the

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received signal can be modelled as

$$r(t) = s(t) + n(t) \quad (2)$$

where  $n(t)$  is white Gaussian noise with power spectral density  $N_0/2$ . For present symbol decision, a multiple interval observation is required to obtain optimal demodulation performance. For the sake of simplicity, we consider the present demodulated symbol as  $a_0$ . By setting the observation window to be  $[0, N_T T]$ , the discrete sampling visions of the received baseband signal quadrature components can be expressed as

$$I_r(n) = \sqrt{\frac{2E}{T}} \cos(\phi_b(n) + \phi_a(n) + \theta_0) + n_{rc}(n) \quad (3)$$

$$Q_r(n) = \sqrt{\frac{2E}{T}} \sin(\phi_b(n) + \phi_a(n) + \theta_0) + n_{rs}(n) \quad (4)$$

where  $n=0,1,\dots,N_T S$

$$\theta_0 = \pi h \sum_{i=-\infty}^{-L} a_i \quad (5)$$

$$\phi_b(n) = 2\pi h \sum_{i=-L+1}^{-1} a_i q((n-i*S)T/S) \quad (6)$$

$$\phi_a(n) = 2\pi h \sum_{i=0}^{N_T-1} a_i q((n-i*S)T/S) \quad (7)$$

and  $S$  is the number of samples per symbol interval  $T$ . If we denote

$$I_b(n) = \cos(\phi_b(n) + \theta_0) \quad (8)$$

$$Q_b(n) = -\sin(\phi_b(n) + \theta_0) \quad (9)$$

$$n_{ac}(n) = n_{rc}(n)I_b(n) + n_{rs}(n)Q_b(n) \quad (10)$$

$$n_{as}(n) = n_{rs}(n)I_b(n) - n_{rc}(n)Q_b(n) \quad (11)$$

$$I_a(n) = \sqrt{\frac{2E}{T}} \cos(\phi_a(n)) + n_{ac}(n) \quad (12)$$

$$Q_a(n) = \sqrt{\frac{2E}{T}} \sin(\phi_a(n)) + n_{as}(n) \quad (13)$$

then, by using simple trigonometric relations, the following relations can be obtained

$$I_a(n) = I_r(n)I_b(n) + Q_r(n)Q_b(n) \quad (14)$$

$$Q_a(n) = Q_r(n)I_b(n) - I_r(n)Q_b(n) \quad (15)$$

From eqs.(3) and (4), the observation signals  $I_r(n)$  and  $Q_r(n)$  not only depend on the previous symbol, but also contain the information of the initial state and  $N_T - 1$  later symbols. From the state space description of

CPM signals, the initial state is specified by the correlative state vector  $\vec{v} = [a_{-L+1}, \dots, a_{-1}]$  and the phase state  $\theta_0$ , and the entire state space contains  $pM^{L-1}$  states when the modulation index  $h$  is rational  $2k/p$ . So all the possible noiseless observation signals form a finite set with size  $pM^{L-1} * M^{N_T}$ . Through eqs.(14) and (15), the effect of the initial state can be removed. The generated signals  $I_a(n)$  and  $Q_a(n)$  only depend on the present symbol  $a_0$  and  $N_T - 1$  later symbols. The number of all possible generated noiseless signals is  $M^{N_T}$ . When the demodulation error probability is low enough, the previous decided symbols can serve as transmitted sequences to generate  $I_b(n)$  and  $Q_b(n)$  by a ROM table. Thus, a decision feedback NN receiver can be constructed as shown in Fig.1. Fig.1 also include a frequency domain processing which will be discussed in the next section.

### 3. IMPLEMENTATION IN FREQUENCY DOMAIN

As mentioned in section 2, increasing  $S$  and  $N_T$  always lead to a improvement of the demodulation performance. However, with large  $S$  and  $N_T$ , the complexity in NN training and implementation will become unbearable. Fortunately, in the case, excellent relativity exists in CPM signals as well as in the output signals of the decision feedback preprocessor. The signal relativity can be utilized to reduce the data number of NN input signals. So a proper transform, for example KLT or DCT, may be employed.

The KLT transform matrix can be generated by the sample set of noiseless signals  $I_a(n)$  and  $Q_a(n)$ . By deleting noise items in eqs.(12) and (13), the signals can be expressed as

$$I'_a(n) = \sqrt{\frac{2E}{T}} \cos(\phi_a(n)) \quad (16)$$

$$Q'_a(n) = \sqrt{\frac{2E}{T}} \sin(\phi_a(n)) \quad (17)$$

If we arrange the two signals into a vector notation

$$\vec{X}' = [I'_a(0)I'_a(1)\dots I'_a(N_T S)Q'_a(0)Q'_a(1)\dots Q'_a(N_T S)] \quad (18)$$

then, the signal sample set is described by a  $2(N_T S + 1)$  dimensional vector space with size  $M^{N_T}$ . To generate the KLT transform matrix, we define a matrix

$$\mathbf{R} = \frac{1}{M^{N_T}} \sum_{\vec{X}'} \vec{X}'^t \vec{X}' \quad (19)$$

where superscribe  $t$  denote transposition. Through eigenvalue and eigenvector solving, the KLT transform

matrix can be obtained.

For DCT, there are four kind selects, and many fast implementation algorithms can be obtained in literatures. From [3] [4] [5], only  $N/2 \log_2 N$  multiplications are required for  $N$ -point DCTII.

As an example, we consider 3RC modulation with  $h = 0.8$ ,  $N_T = 9$  and  $S = 8$ . The minimum squared normalized Euclidean distance is 3.1707. For a given number of zoom sampling points in KLT and DCT domains, the corresponding minimum squared normalized Euclidean distance  $d_{min}^2(N_s)$  is plotted in Fig.2. From the point of minimum Euclidean distance demodulation, only 32 coefficients in KLT domain or 40 coefficients in DCT domain will be enough.

## 4. NN TRAINING

### 4.1. Generating the Training Set

The neural network training set can be generated by  $\bar{X}'$  through KLT or DCT and zoom sampling for noiseless training. The size of the complete set is  $M^{N_T}$  and the sample dimension  $N_S$  depends on practical modulation CPM signals. In the neural network training, noise may be added to the samples which can improve the performance.

### 4.2. Learning Algorithm

A one-hidden-layer BP neural network is used in our receiver scheme. As mentioned in section 1, more sufficient convergence always leads to better demodulation performance. To accelerate the convergence of the learning phase on a variety of problems, many methods have been proposed by researchers, such as the use of double-polar sigmoid function, adapting the learning rates and the use of the momentum term [6][7].

In the "delta-bar-delta" method [7], the learning rate is modified as:

$$\Delta_\epsilon(t) = \begin{cases} K & \text{if error decreases continuously,} \\ -\phi_\epsilon(t) & \text{if error increases,} \\ 0 & \text{otherwise.} \end{cases}$$

Where  $K$  is a constant. However, it is hard to select a proper value of  $K$ . The learning rate varies in a large range in the convergence process.  $K$  should be relatively large when the learning rate is a large value, or the learning rate can't be modified sufficiently. If the learning rate should be very small, a large  $K$  is unfit. So it is reasonable to adapt  $K$  during the searching process. We increase  $K$  as the error decreases continuously and reduce it if an "accident" (increase of the error) is encountered. This modified "delta-bar-delta" method is about fifty times faster than the standard

BP algorithm, and the neural network reaches its pitch after about forty iterations.

## 5. COMPUTER SIMULATION AND CONCLUSION

The decision feedback neural network coherent receiver was simulated by 3RC modulation signals with modulation index  $h=0.8$ . The parameters are  $N_T = 9$ ,  $S=8$ ,  $N_S = 32, 40$  for KLT, DCT respectively, and the number of hidden nodes  $H=16$ . The error probability was estimated by simulating the receiver until 200 errors were observed for various SNR. The results show that near optimal error performance can be achieved compared with optimal ML receivers as illustrated in fig.3. In the simulation, possible error spreading caused by decision feedback was not taken into account. But, because the demodulation error probability is low enough at high SNR, it will not lead to catastrophic results in practical TDMA systems.

So far we can conclude that our decision feedback neural network coherent receiver scheme has attractive error performance, low complexity in NN training and implementation especially with fast discrete cosine transform used. It offers an attractive alternative for CPM signal demodulation.

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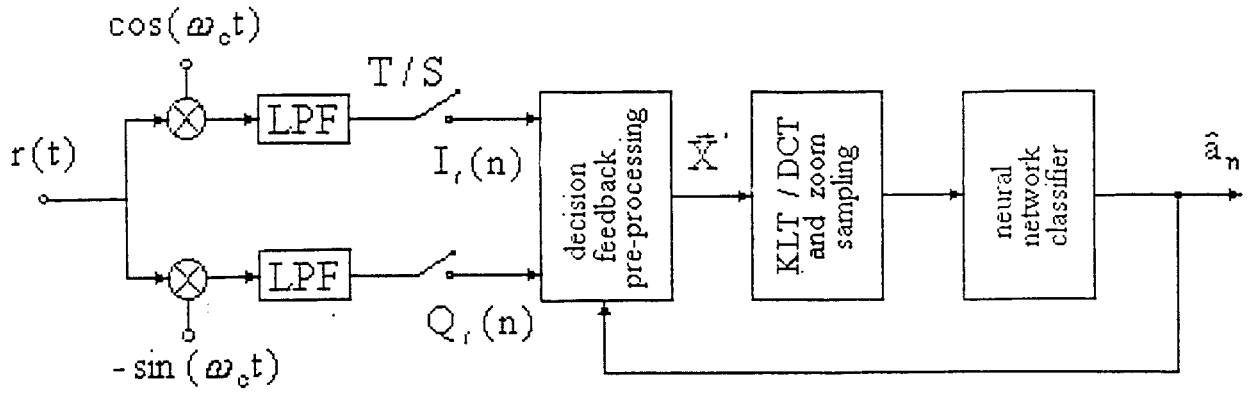


Fig. 1 Decision feedback neural network coherent receiver scheme

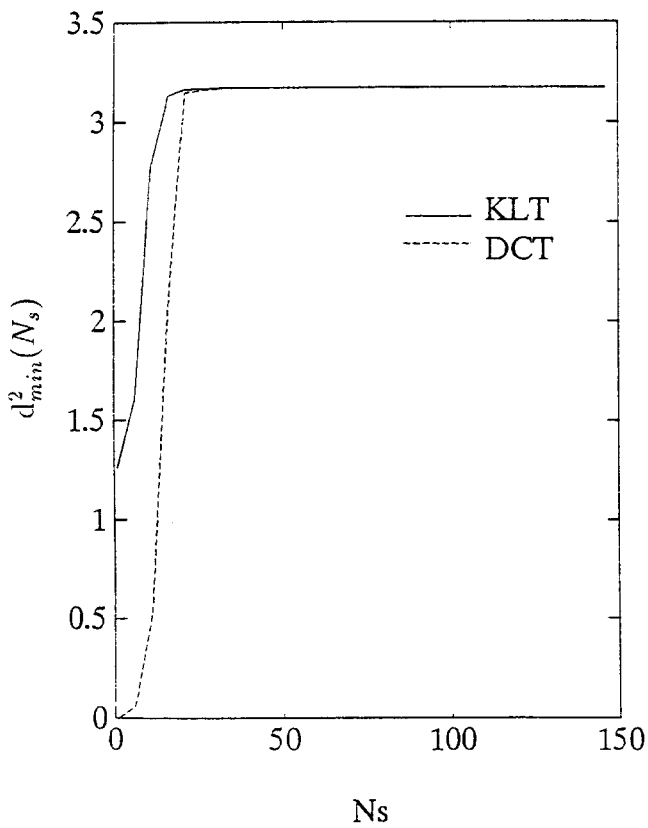


Fig. 2 Minimum distance vs.  $N_s$  for 3RC with  $h = 0.8$ .

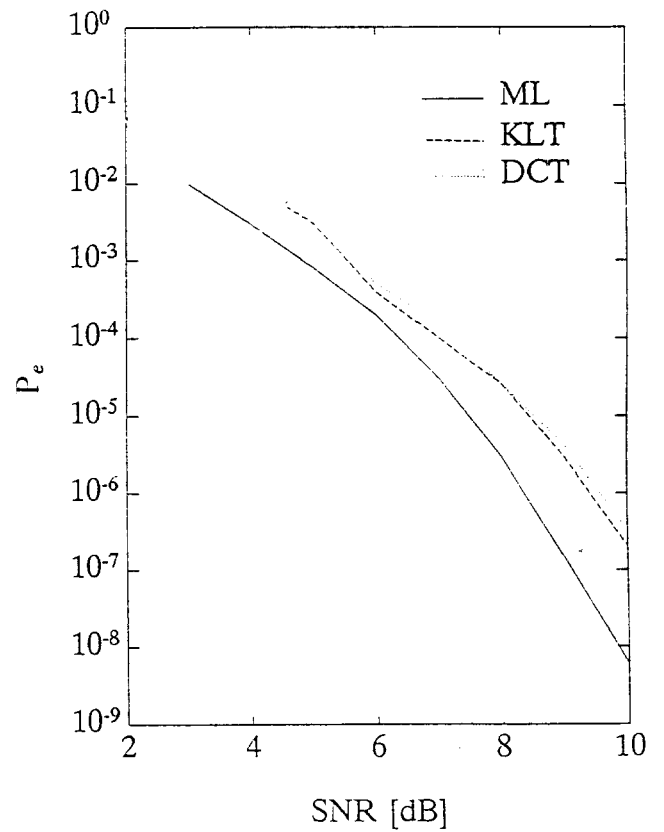


Fig. 3 Comparison of error performance for 3RC with  $h = 0.8$