THE APPLICATION OF WAVELET NEURAL NETWORKS TO ADAPTIVE TRANSFORM CODING OF ONE DIMENSIONAL SIGNALS

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ABSTRACT

The Inverse Receptive Field Partition (IRFP) algorithm developed from the Receptive Field Partition (RFP) algorithm, operates on a wavelet basis function network. This technique is a fast method of basis function selection . It begins with the highest resolution wavelet basis functions as its seed functions. Like RFP, IRFP then uses receptive field activation principle during training. This principle insures only wavelets within a specified interval are chosen as candidates. By way of the receptive activation principle, wavelets are selected with the best fit from a precalculated *development pool* and moved to the *main pool.* The selection of wavelet coefficients for the main pool is based on best fit across all resolutions. Overall functional fit can be controlled by global MSE and pruning thresholds.

1. INTRODUCTION

In recent years, multiresolutional analysis has been used extensively in pyrimidal encoding schemes. Specifically, pyrimdal schemes using quadrature mirror filters whose coefficients are wavelet coefficients have been used for image encoding [1] and subband speech encoding [2]. However, there has been no investigations of how to select the optimal set of wavelet coefficients by adaptive selection of scaling functions and resolution for quadrature mirror filters.

The only other approach to adaptive selection of wavelet coefficients was discovered by Coifman et al [3] and is called the best wavelet packet basis algorithm. This algorithm finds the best filter bank tree topology based on an efficient scheme using an entropy measure. However, there is an assumption of a preconfigured filter bank structure and as such there is a lack of reconfigurablity of the filter bank structure based on wavelet coefficient selection and number of resolutions chosen to describe provide optimal fit of signal. This paper will show that using a novel wavelet neural tree structure, the optimal wavelet coefficients can be chosen from different signal resolutions across different scaling functions (seed functions) as part of an adaptive encoding process.

The paper is broken into three sections. Section 2 describes the appropriate wavelet computational paradigm called wavelet neural networks and how the network is implemented as a data structure. Section 3 discusses the specific algorithm for wavelet neural networks called Inverse Receptive Field Partitioning (IRFP) and its application to wavelet basis functions. Lastly, Section 4 describes results of IRFP in terms of wavelet coefficient selectivity, signal reconstruction and sensitivity to signal characteristics.

2. BASIS FUNCTION NEURAL NETWORKS AND MULTIRESOLUTION WAVELETS

This section describes basis function neural networks in the context of wavelet multiresolutional analysis. This will provide the foundation for the Receptive Field Partitioning algorithm in the following section.

2.1 Basis Function Networks

Basis function networks are generally used for function approximation [4,5]. They can be expressed as:

$$f(x) = \sum_i \alpha_i B_i(x) \qquad \text{(Eq 1)}$$

where the B_i are the basis functions and the α_i are the corresponding coefficients. When compared to feedforward neural networks, the network structure has only one hidden layer, containing neurons whose activation functions are not sigmoid, but correspond to the chosen basis functions. The connection weights are taken to represent the associated coefficients of the basis functions. These coefficients can be updated any number of ways by methods commonly recognized in neural network application such as backpropagation. Likewise, as with most neural networks, the output layer performs the summation of the weighted outputs from the hidden layer, to generate the function approximation. The main problem with this approach is that of selecting an appropriate set of basis functions and their optimization parameters.

2.2 Wavelet Networks

Extending the concept of basis function networks to wavelets means we can develop a coding procedure which supports the

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concept of *receptive field activation principle* as described by Boubez and Peskin [6].

This principle is used in training of wavelet neural networks and centers around wavelet's local support property: only a small set of nodes out of all possible nodes in the basis function network need to be active at one time. For each training example, only wavelets whose support interval, or receptive field, includes that particular point are affected by the weight changes.

These neurons can be combined as a set of basis functions that can be used for encoding one dimensional signals. The weights can be thought of as the wavelets coefficients derived from the initial definition of basis function networks.

3. INVERSE RECEPTIVE FIELD PARTITION ALGORITHM

This section describes an algorithm based on the work of Boubez and Peskin [6]. The algorithm is different from previous work in that it has been modified to handle wavelet coefficients differently with a different the pruning strategy employed to limit scorable wavelet coefficient nodes in the wavelet network. The source code can be obtained by contacting the author.

3.1 Input and Initialization: Wavelet Basis Function Creation

The algorithm begins with the generation of wavelet basis functions. The basis functions are generated for an entire parameter space for real-valued wavelets with two to six coefficients [7].

Unlike previous implementations of RFP [6], this constructive method starts with bases at the highest level (highest resolution) and refines the approximation by adding units from lower approximations (lower resolution) using an MSE fit criterion which establishes a goodness of fit. This constructive method is used because the fit criterion partitioned the signal space based on goodness of fit rather than lack of fit criteria and as such will be called the *Inverse* RFP (IRFP). Like RFP, the objective is facilitated by the localized properties of the wavelet functions used.

The algorithm starts by generating an initial set of bases, the seed bases, consisting of the generating functions and the bases at the highest resolution, and maintains two virtual pools of bases throughout the training. The first pool, the *main pool*, contains all the bases that are being used in the current computation and constitutes the actual neural network. The second pool, the *development pool*, contains all the bases that have not been refined yet, and is a subset of the first. Any basis that is selected for refinement get removed from this pool and its children are added to both pools.

The pools discussed above are virtual, in that the two pools are managed within a single data structure with the appropriate flags as to whether the detail and approximation basis signal at a particular resolution belong to either main or development pool. This increases speed and efficiency of the management of wavelet coeifficients.

The efficiency is further increased by organizing the basis functions in a binary tree data structure. In this structure, the basis functions are arranged by MSE with respect to the input signal so the smaller MSE basis function are at the top of the binary tree while larger MSE basis functions are at the bottom. Thus, each basis function is represented as a node in the binary tree and the traversal of the tree is done by MSE scores. Pruning of nodes in the binary tree are discussed in the following section.

The input and initialization routine is summarized below:

INPUT: 1-D signal, levels of resolution, alpha/beta range, alpha/beta step, pruning threshold and score target.

INITIALIZE:

1. Compute the seed bases.
2. Create main and development pools.
3. Compute coefficients for the seed bases.
4. Compute the classification score.

3.2 Main Body: Wavelet Coefficient Selection and Scoring

The constructive process of adding new basis to the main pool is guided by the overall MSE score. During the process of growing the main pool, the MSE score is calculated for each basis function added to it. This process ends only when the desired classification score has been achieved or all of the basis functions from the development pool have been examined.

With the binary tree structure discussed above, it is necessary to prune the tree, if the number of basis functions at each node exceed a given threshold they can be pruned or deleted from the development pool. The critera for pruning the nodes is based on marginal MSE such that the most likely candidates for pruning are those nodes which contribute the most to MSE (worst fit).

The main body is summarized as follows:

MAIN LOOP:
While (score < target_score)
1. Select basis function with least MSE and extract its subbases from the development pool.
2. Change binary status flag of effected nodes.
3. if (size(main_pool) is large) the prune.
4. Compute the MSE of all main pool nodes.
End While

Prune Network

3.3 Output: Wavelet Network Structure and Coefficients

The resulting main pool is output so the signal can be correctly reconstructed from the output. Therefore, the file includes the total signal length followed by a series of records which give the coefficients used for signal decomposition (the same for reconstruction) as well as the detail and approximation signal for a given level. This allows for complete signal reconstruction according to the above discussion.

4. RESULTS

The results discussed below are based on fixed multiresolution externsion of the wavelet network to 4 levels of resolution. In addition, the range of α and β was limited to two. Therefore, the results show how IRFP selects wavelet coefficients for the neurons in the wavelet network given MSE and pruning constraints and how well it reconstructs signals in terms of comparisions to plots of original signals and MSE learning curves.

4.1 Wavelet Coefficient Selectivity

Tables 1 and 2 show that the IRFP algorithm successfully selected those wavelet scaling/dialation functions in those neurons which minimized the LOF criteria. The results show that all seed bases functions are chosen from the development pool and placed in the main pool to form the wavelet network. This is consistent with the fact that these basis functions also have the lowest LOF or best 'fit' for the original signal. In addition, it is apparent in general, the larger differential MSE (DMSE) correlates with those basis functions selected. Although this may not always be true in the case of filter 1, level 1 and filter 1, level 2.

4.2 Signal Reconstruction

Figures 1 and 2 show the effect of pruning thresholds on signal reconstruction. It was shown through additional runs that MSE thresholding has an equivalent effect: increases of these thresholds increase the fidelity of the signal reconstruction by increasing the number of wavelet nodes (neurons) in the main pool

4.3 Signal MSE Learning Curves

Figure 2 shows the MSE learning curves for both development and main pools. It is apparent that the main pool mimics the development pool learning curve. The extent of the main pool learning curve is controlled by the MSE/Pruning thresholds.

5. CONCLUSION

The method of Inverse Receptive Field Partitioning (IRFP) successfully selected optimal wavelet coefficients under MSE/Pruning constraints. It can be used to create a sparse wavelet coefficient space to selectively fit signals based on an adjustable LOF constraint.

IRFP needs to be further investigated as it may be applicable to adaptive transform coding techniques. In this scheme, IRFP will provide source encoding by generating the highest resolution basis functions which can be vector quantized prior to channel encoding.

6. REFERENCES

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Table 1. Wavelet Nodes Selected For MSE Threshold of 2.5 and Pruning Threshold of 7

Filter	Level	α	β	LOF	DMSE	Selected?
0	0	1.570796	1.570796	0.347272	0.347272	Yes
1	0	1.570796	1.570796	0.618975	0.271702	Yes
2	0	1.570796	1.570796	0.774221	0.155247	Yes
3	0	1.570796	1.570796	0.887388	0.113167	Yes
0	1	1.570796	3.141593	1.044499	0.000000	Yes
1	1	1.570796	3.141593	1.391664	0.347165	No
0	2	3.141593	1.570796	1.257181	0.000000	No
1	2	3.141593	1.570796	1.339506	0.285180	No
2	1	1.570796	3.141593	1.436296	0.044632	No
3	1	1.570796	3.141593	1.480356	0.044060	No
2	2	3.141593	1.570796	1.512853	0.096133	No
3	2	3.141593	1.570796	1.601572	0.012733	No
0	3	3.141593	3.141593	1.998573	0.000000	No
1	3	3.141593	3.141593	2.837063	0.306421	No
2	3	3.141593	3.141593	3.492339	0.151922	No
3	3	3.141593	3.141593	4.037576	0.113047	No

Table 2. Wavelet Nodes Selected For MSE Threshold of 2.5 and Pruning Threshold of 11

Filter	Level	α	β	LOF	DMSE	Selected?
0	0	1.570796	1.570796	0.347272	0.347272	Yes
1	0	1.570796	1.570796	0.618975	0.271702	Yes
2	0	1.570796	1.570796	0.774221	0.155247	Yes
3	0	1.570796	1.570796	0.887388	0.113167	Yes
0	1	1.570796	3.141593	1.044499	0.000000	Yes
1	1	1.570796	3.141593	1.391664	0.347165	Yes
0	2	3.141593	1.570796	1.257181	0.000000	Yes
1	2	3.141593	1.570796	1.339506	0.082325	No
2	1	1.570796	3.141593	1.436296	0.044632	Yes
3	1	1.570796	3.141593	1.480356	0.044060	Yes
2	2	3.141593	1.570796	1.512853	0.173347	No
3	2	3.141593	1.570796	1.601572	0.088720	No
0	3	3.141593	3.141593	1.998573	0.000000	No
1	3	3.141593	3.141593	2.837063	0.306421	No
2	3	3.141593	3.141593	3.492339	0.151922	No
3	3	3.141593	3.141593	4.037576	0.113047	No

Fig 1.Selected Wavelet Basis functions and Original Signal

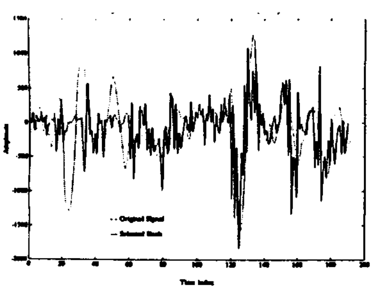

Fig 2. MSE Learning Curve for Development and Main Pools