CORRELATION AMONG TIME DIFFERENCE OF ARRIVAL ESTIMATORS AND ITS EFFECT ON LOCALIZATION IN A MULTIPATH ENVIRONMENT

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ABSTRACT

A generalized covariance expression for time difference of arrival (TDOA) estimators is derived for an M-sensor arbitrary array in a two-path underwater environment. The resulting expression can be reduced to the variance and covariance expressions developed previously.

The correlation among the time difference of arrival estimators and its effect on a localization error are investigated for a two-sensor vertical array. The results show that the correlation among the multipath TDOA estimators is significant and the degree of correlation depends on the power spectral density of source signal and that of noise, and only on two multipath TDOAs. Because of this correlation, every TDOA estimate contributes information to localization and therefore a whole set of TDOA estimators should be used.

1. INTRODUCTION

The localization studied in this paper is a "sonobuoy"-type sensor system in an underwater acoustics environment. A bibliography of work done in this area is available in [1].

Research done in [2, 3, 4] has shown that localization in multipath dominant environments can yield more accurate estimates than localization in a direct-path-only propagation environment. For conventional estimators, the statistical uncertainty in the localization is closely related to the accuracy of time difference of arrival (TDOA) estimators. Therefore, calculation of the covariance matrix for the TDOA estimators is a key problem for determining the statistics of localization error.

A recent study of a two-sensor system [5] has shown that available expressions for variances of range and depth estimators can be in error by a significant factor if the correlation among the TDOA estimators is ignored. This paper investigates the correlation among the TDOA estimators and its effect on localization for an M-sensor arbitrary array in a two-path environment.

2. TIME DIFFERENCE OF ARRIVAL ESTIMATION

2.1. Signal and Noise Models

Consider a scenario where an M-sensor array is submerged in a direct and surface-reflected environment. It is assumed that the sound speed profile is constant and the ocean surface is a pure reflector. Under these assumptions, the surface-reflected paths can be viewed as the signals collected by virtual sensors located above the ocean surface.

The received signals at sensor m is given by

$$r_m(t) = g_{d_m} s(t - D_{d_m}) + g_{s_m} s(t - D_{s_m}) + n_m(t) \quad \text{for } m \in \{1, 2, ..., M\}, \tag{1}$$

where $s(t)$ is the signal radiating from the source, $n_m(t)$ is the noise received at sensor m, subscripts d_m and s_m indicate the direct and surface-reflected paths to sensor m, D_k is the time delay for the sound to travel from source to sensor along path k, and g_k is the attenuation experienced by the signal in traveling path k for $k \in \{d_1, d_2, ..., d_M, s_1, s_2, ..., s_M\}$. The source signal, $s(t)$, and the noise, $n_m(t)$, are assumed to be stationary, uncorrelated, zero mean, jointly Gaussian random processes with known broadband spectral densities $S(w)$ and $N_m(w)$ respectively.

The geometric information of the source location is fully encoded in the time differences among the path delays D_{d_1}, D_{d_2}, ..., D_{d_M}, D_{s_1}, D_{s_2}, ..., D_{s_M}. These TDOAs are denoted by a double subscripted D, for example $D_{s_1 d_1}$ for difference $D_{s_1} - D_{d_1}$.

2.2. Multipath TDOA Measurements

The TDOAs are the parameters in M autocorrelation functions and $\frac{M(M-1)}{2}$ cross-correlation functions for an M-sensor system. The autocorrelation function of the signal collected by sensor m is given by

$$R_{mm}(\tau) = E\{r_m(t) r_m(t-\tau)\}$$
$$= (g_{d_m}^2 + g_{s_m}^2) R_s(\tau) + g_{d_m} g_{s_m} \Big(R_s(\tau + D_{d_m s_m}) +$$

$$R_s(\tau - D_{d_m s_m})\Big) + R_{n_m}(\tau), \quad (2)$$

where $m \in \{1, 2, ..., M\}$, $E\{\bullet\}$ denotes the expected value, $R_s(\tau)$ and $R_{n_m}(\tau)$ are the autocorrelation functions for the source signal, $s(t)$, and noise, $n_m(t)$, respectively, and the TDOA $D_{d_m s_m}$ is given by the difference $D_{d_m} - D_{s_m}$.

The cross-correlation function of two signals received at sensors m and n is

$$\begin{aligned} R_{mn}(\tau) &= E\{r_m(t) r_n(t-\tau)\} \\ &= g_{d_m} g_{d_n} R_s(\tau - D_{d_m d_n}) + g_{d_m} g_{s_n} R_s(\tau - D_{d_m s_n}) + \\ &\quad g_{s_m} g_{d_n} R_s(\tau - D_{s_m d_n}) + g_{s_m} g_{s_n} R_s(\tau - D_{s_m s_n}) \end{aligned} \quad (3)$$

for $m, n \in \{1, 2, ..., M\}$ with $m \neq n$. Since there is one unique TDOA parameter in each autocorrelation function and four unique TDOA parameters in each cross-correlation function, the total number of TDOAs is $M(2M-1)$ for an M-sensor array in a direct and surface-reflected (or bottom bounce) environment.

It is noted that the positions of the peaks in the auto- and cross-correlation functions will not correspond exactly to the TDOAs due to the superpositions of correlograms. The positions of the peaks are denoted by including superscript b on the TDOAs, for example, $D^b_{s_1 d_1}$, which indicates the peak is biased or shifted from $D_{s_1 d_1}$.

The time average auto- or cross-correlation functions, $\hat{R}_{mn}(\tau)$, are given by

$$\hat{R}_{mn}(\tau) = \frac{1}{T} \int_0^T r_m(t) r_n(t-\tau) dt \quad (4)$$

for $m, n \in \{1, 2, ..., M\}$. If the TDOAs are resolvable, then they can be estimated from (4). The average is taken over finite time and the noise will affect the positions of the peaks.

For a large time-bandwidth product, the variances of the TDOAs estimated from an auto- and cross-correlator are derived in [6, 7]. The expressions given in [6, 7] yield the $M(2M-1)$ variances which form the diagonal elements of the $M(2M-1) \times M(2M-1)$ TDOA covariance matrix for an M-sensor system in a two-path environment. The covariance expression for the TDOAs estimated from M auto- and $\frac{M(M-1)}{2}$ cross-correlators is derived in this paper.

III. CORRELATION AMONG MULTIPATH TIME DIFFERENCE OF ARRIVAL ESTIMATORS

3.1. Generalized Covariance Expression

To get an analytical expression for the covariance of the TDOA estimators, the derivative of the time average auto- or cross-correlation function is approximated with a first-order Taylor series expansion about the peak positions $D^b_{i_m j_n}$ for $i, j \in \{d, s\}$ and $m, n \in \{1, 2, ..., M\}$, which differ slightly from TDOAs $D_{i_m j_n}$ due to superpositions. The Taylor series approximation is set to zero and then solved to get an equation relating the error in the position of the peak to the first and second derivatives of the time average correlation function. The resulting expression for error is

$$\hat{D}^b_{i_m j_n} - D^b_{i_m j_n} \simeq \frac{-\frac{d\hat{R}_{mn}(\tau)}{d\tau}\big|_{\tau = D^b_{i_m j_n}}}{\frac{d^2 \hat{R}_{mn}(\tau)}{d\tau^2}\big|_{\tau = D^b_{i_m j_n}}}. \quad (5)$$

This error is given by the ratio of two random variables, however, the denominator has a small coefficient of variation.

Using (5) to get an approximation for the product of two errors and then taking the expectation produces a covariance expression which can be approximated by

$$\begin{aligned} &\mathrm{Cov}\left\{ (\hat{D}^b_{i_m j_n} - D^b_{i_m j_n}), (\hat{D}^b_{k_p l_q} - D^b_{k_p l_q}) \right\} \\ &\simeq \frac{E\left\{ \frac{d\hat{R}_{mn}(\tau_1)}{d\tau_1}\big|_{\tau_1 = D^b_{i_m j_n}} \frac{d\hat{R}_{pq}(\tau_2)}{d\tau_2}\big|_{\tau_2 = D^b_{k_p l_q}} \right\}}{E\left\{ \frac{d^2\hat{R}_{mn}(\tau_1)}{d\tau_1^2}\big|_{\tau_1 = D^b_{i_m j_n}} \frac{d^2\hat{R}_{pq}(\tau_2)}{d\tau_2^2}\big|_{\tau_2 = D^b_{k_p l_q}} \right\}} \end{aligned} \quad (6)$$

for $i, j, k, l \in \{d, s\}$ and $i_m j_n \neq k_p l_q$. Note that in this special circumstance, the expected value of the quotient is approximately equal to the quotient of expected values. The denominator is the product of two random variables each with a small coefficient of variation. Therefore the product has a vary small coefficient of variation and can be considered a constant. The value of this constant is given by the expected value of the product.

The generalized covariance expression (6) can be expressed in terms of the power spectral densities of the signals received at sensors m, n, p, and q, using the approach given in [5]. The resulting expression is

$$\begin{aligned} &\mathrm{Cov}\left\{ (\hat{D}_{i_m j_n} - D_{i_m j_n}), (\hat{D}_{k_p l_q} - D_{k_p l_q}) \right\} \simeq \frac{2\pi}{T} \times \\ &\left(\int_{-\infty}^{\infty} w^2 S_{mp}(-w) S_{nq}(w) e^{-jw(D^b_{i_m j_n} - D^b_{k_p l_q})} dw - \right. \\ &\left. \int_{-\infty}^{\infty} w^2 S_{mq}(-w) S_{np}(w) e^{-jw(D^b_{i_m j_n} + D^b_{k_p l_q})} dw \right) \div \\ &\left(\int_{-\infty}^{\infty} w^2 S_{mn}(w) e^{jwD^o_{i_m j_n}} dw \times \right. \\ &\left. \int_{-\infty}^{\infty} w^2 S_{pq}(w) e^{jwD^b_{k_p l_q}} dw \right), \end{aligned} \quad (7)$$

where $i, j, k, l \in \{d, s\}$, $m, n, p, q \in \{1, 2, ..., M\}$, and the double subscripted S indicates the auto- or crosspower spectral density for $r_m(t)$, $r_n(t)$, $r_p(t)$, or/and $r_q(t)$.

Expression (7) is consistent with known expressions for the variance and covariance of TDOA estimators developed previously. It can be shown that if under the simplified condition that all paths experience the same attenuation, then equation (7) can be simplified to variance and covariance expressions derived by Hahn in 1975 (reprint given in [1]). Hahn's results is for a direct-path-only environment where the structure of the received signal power spectra is much simpler. Equation (7) can also be specialized to variance expressions for multipath TDOA estimators developed in [6, 7] and covariance expressions given in [5, 8] for a two-sensor array in a two-path environment. It is shown in [8] that the correlation among the TDOA estimators for a two-sensor array depends on only two multipath TDOAs, specifically the time difference of arrival between the direct and surface-reflected path for each of the two sensors.

IV. EXAMPLE CORRELATION AND EFFECT ON LOCALIZATION

The significance of correlation among the TDOA estimators is illustrated through an example. A two-sensor vertical array is used with sensor depths 200 and 400 meters. The source depth is taken to be 500 meters. The source range is varied from 500 meters to 10,000 meters. The acoustic source and ocean noise are assumed to have flat broadband power spectra with bandwidths 400 Hz. The ratio of source intensity (at the source location) to received noise intensity (SNR) is taken to be 80 dB. This yields an SNR of −2.2 dB for the direct path when the source range is 1,000 meters. The correlation coefficients are calculated from a Monte Carlo simulation with one hundred independent trials. A least-squared range estimator (given in [5]) is used to show the effect of correlation. Only 25 of the 100 trials are used to estimate the variance of the range estimator.

Figure 1 illustrates the covariances for TDOAs estimated from two autocorrelators for ranges from 500 to 10,000 meters. The simulation results are indicated by the point marked "o". The error bars associated with an estimate indicate the 95 percent confidence interval. The curve represents the theoretical calculation. The average difference between the theoretical and simulation-based coefficients over the 10 ranges is only 0.006 which suggests the simulation results support the theory.

Figure 2 shows the coefficients among the 4 TDOAs estimated with a cross-correlator. The 4 TDOA estimates are denoted $\hat{D}_{d_1 d_2}$, $\hat{D}_{s_1 s_2}$, $\hat{D}_{s_1 d_2}$ and $\hat{D}_{d_1 s_2}$

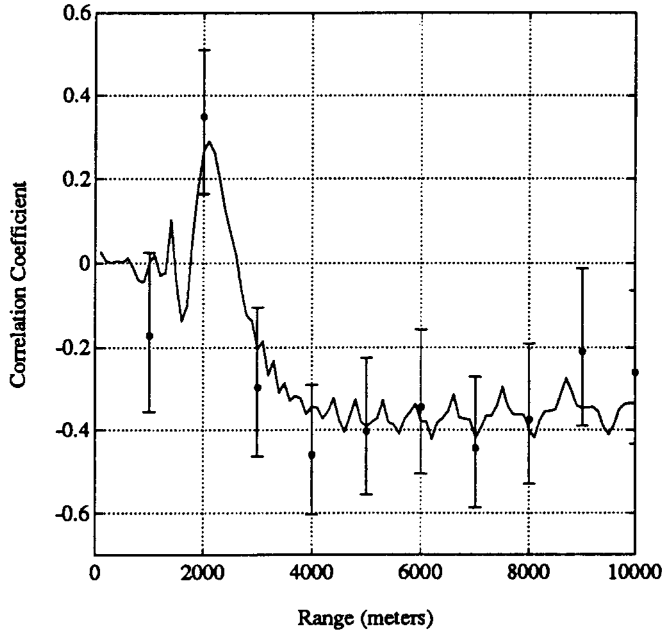

Figure 1: Correlation coefficients among the TDOAs estimated from the two autocorrelators.

($\hat{D}_{d_1 d_2}$ is the TDOA between the direct path to sensor 1 and the direct path to sensor 2, $\hat{D}_{s_1 s_2}$ is the TDOA between the surface-reflected path to sensor 1 and the surface-reflected path to sensor 2, etc.). The solid and dashed lines at the bottom represent the theoretical coefficients $\rho_{\hat{D}_{d_1 d_2}\hat{D}_{s_1 s_2}}$ and $\rho_{\hat{D}_{s_1 d_2}\hat{D}_{d_1 s_2}}$ respectively. The corresponding simulation results, indicated by marks "+" and "*" respectively, agree well with theory. The coefficients for these two pairs are large, up to −0.99. The other four curves which are coded by solid, dashed, dotted and dashdot lines express the theoretical coefficients $\rho_{\hat{D}_{s_1 s_2}\hat{D}_{d_1 s_2}}$, $\rho_{\hat{D}_{d_1 d_2}\hat{D}_{s_1 d_2}}$, $\rho_{\hat{D}_{d_1 d_2}\hat{D}_{d_1 s_2}}$ and $\rho_{\hat{D}_{s_1 s_2}\hat{D}_{s_1 d_2}}$ respectively. The corresponding simulation results are indicated by marks "o", "x", "+" and "*" respectively. The 95 percent confidence interval bars have not been plotted to keep the plot from becoming busy. Again, it can be seen from Figure 2 that the simulation results generally support the theory.

Figure 3 compares the theoretical variances with the simulation-based variance for a least-squared range estimator. The theoretical variances (solid and dashed lines) are calculated using full and diagonal covariance matrices respectively. The simulation-based variance (dotted line) includes the effects of correlation in the TDOA estimators. The simulation results, found at 100 meter interval, are connected by straight lines. It is shown in Figure 3 that the two theoretical variances dif-

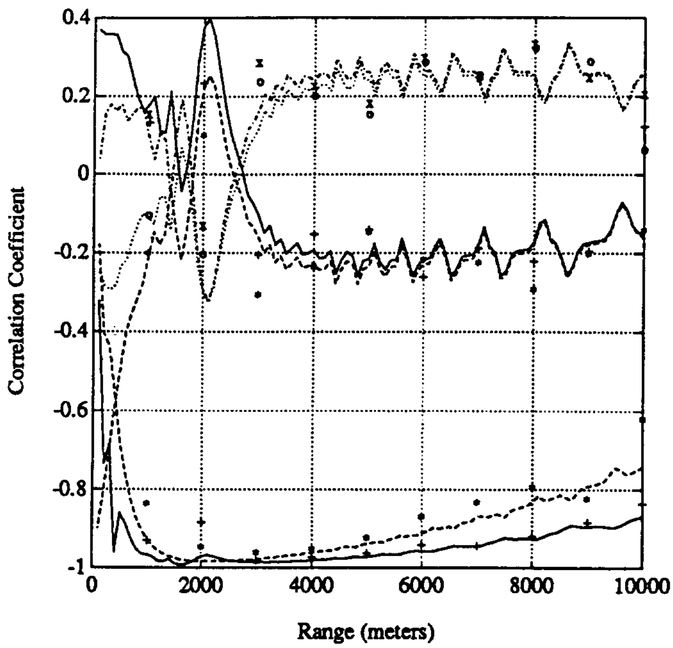

Figure 2: Correlation coefficients among the 4 TDOAs estimated from a cross-correlator.

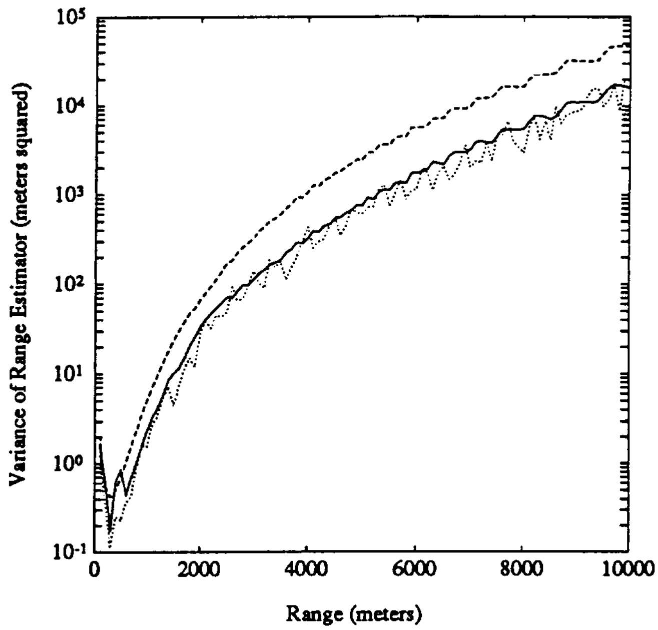

Figure 3: Variance of a range estimator: the solid and dashed lines are the theoretical calculation using full and diagonal covariance matrices respectively, and the dotted line is the simulation-based variance.

fer by as much as a factor of 3.3. The simulation results agree with the theoretical curve calculated with the full covariance matrix. The ripple in the simulation-based variance curve is primarily due to the small number of trials (which is 25) used to calculate each point.

V. CONCLUSIONS

The generalized covariance expression derived in this paper can be used to compute the covariance matrix of the TDOA estimators associated with an M-sensor arbitrary array in a two-path underwater environment as well as in a direct-path-only environment.

It is shown for a two-sensor system in a direct and surface-reflected environment that the correlation among the TDOA estimators depends on the power spectral density of the source signal and that of the ocean noise, and only two multipath TDOAs. In the example of a flat low-pass source signal given here, the correlation among the TDOA estimators is significant and is a complicated function of range showing marked variation.

1. REFERENCES

[1] G. C. Carter, ed., *Coherence and Time Delay Estimation - An Applied Tutorial for Research, Development, Test, and Evaluation Engineers.* Piscataway: IEEE Press, 1993.

[2] M. Hamilton and P. M. Schultheiss, "Passive ranging in multipath dominant environments, part i: Known multipath parameters," *IEEE Trans. Signal Processing*, vol. 40, pp. 1–12, Jan. 1992.

[3] M. J. D. Rendas and J. M. F. Moura, "Cramer-rao bound for location systems in multipath environments," *IEEE Trans. Signal Processing*, vol. 39, pp. 2593–2610, Dec. 1991.

[4] B. Friedlander, "Accuracy of source localization using multipath delays," *IEEE Trans. Aerospace and Electronic Systems*, vol. 24, pp. 346–359, July 1988.

[5] Y. X. Yuan and J. E. Salt, "Range and depth estimation using a vertical array in a correlated multipath environment," *IEEE Journal on Oceanic Engineering*, vol. 18, pp. 500–507, Oct. 1993.

[6] J. P. Ianniello, "Large and small error performance limits for multipath time delay estimation," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. 34, pp. 245–251, April 1986.

[7] B. L. F. Daku and J. E. Salt, "Quality of underwater source localization in a multipath environment," *J. Acoust. Soc. Am.*, vol. , pp. 957–964, Feb. 1992.

[8] Y. X. Yuan, *Passive Localization of an Underwater Acoustic Source using Directional Sensors.* PhD thesis, University of Saskatchewan, Saskatoon, Saskatchewan, Canada, S7N 0W0, 1994.